Journal of Systems Engineering and Electronics
Vol. 28, No. 3, June 2017, pp.503 – 513

Combination method of conflict evidences based on evidence similarity

Wenhao Bi, An Zhang*, and Yuan Yuan

School of Aeronautics, Northwestern Polytechnical University, Xi'an 710129, China

Abstract: Aiming at the problem that the traditional Dempster-Shafer (D-S) evidence theory cannot deal with conflicted evidences effectively and correctly, this paper points out that the key issue of this problem is to measure the degree of conflict between evidences correctly after analyzing various improved methods. The existing evidence conflict measure methods are analyzed, and a new evidence conflict measure method called evidence similarity measure based on the Tanimoto measure is proposed, while a new evidence combination method is proposed on the basis of evidence similarity measure. Firstly, the conflict degrees between evidences are obtained through the evidence similarity measure. Then the evidence sources are modified based on the credibility of different evidences and the weights of conflicted parts of evidences on different focal elements are determined. Finally, the fusion result is obtained by this method. Numerical examples show that the proposed method can effectively fuse evidences when evidences are consistent or highly conflicted, and it has a fast convergence speed, a high degree of accuracy and good adaptability.

Keywords: evidence theory, evidence conflict, conflict measure, evidence similarity, information fusion.

DOI: 10.21629/JSEE.2017.03.10

1. Introduction

As a kind of uncertainty reasoning method, the evidence theory can well deal with uncertain problems caused by randomness and fuzziness without prior probability and conditional probability density. It has been widely applied to the multi-source information fusion [1 – 3].

However, the process of evidence combination can sometimes produce counterintuitive conclusions, when there is a conflict between evidences. Aiming at this defect, many scholars have actively analyzed and discussed, and successively put forward various improved methods [4 – 13]. In general, the existing improved methods can be divided into two categories. One is to modify the classic combination rules of the evidence theory and to reallocate the conflict. For example, Yager [5] thought that the conflict could not provide useful information so that he regarded all the conflict information to be unknown to reduce the conflict. This method denies the conflict between evidences and is too conservative. It increases the uncertainty of the fusion evidences and could not meet the purpose of evidence reasoning. Lefèvre et al. [6,7] believed that a part of the conflict between evidences can be utilized and the conflict information should be proportionally distributed to the focal element sets of all the evidences. Guo et al. [8,9] proposed improved algorithms. These methods mainly focuse on to which specific proposition and in what proportion the conflict information should be allocated. The other is to modify conflict evidences before the fusion without changing the classic combination rules. Murphy [10] took the average of the basic probability assignment involved in the synthesis of evidences, and then got a new evidence model and synthesized evidences using the Dempster combination rule. This is just a simple arithmetic average of evidences without considering the importance of evidence and the result is not ideal. Deng et al. [11] introduced the concept of evidence support based on the Jousselme distance function and took a weighted average of all the evidences, and some improved algorithms [12 – 15] were presented to determine the evidential weights. These methods mainly study which means could be used to preprocess evidences before the combination.

All the above methods make up for the deficiency of the evidence theory to some extent and improve the fusion results of conflict evidences from different angles. However, the existing improvements on the evidence theory do not focus on how to measure evidence conflicts. Only after making full use of the information of the original evidence to estimate the degree of evidence conflict and determining the measurement index of the evidence conflict, can the evidence credibility be correctly and quantitatively described and the original evidence be reasonably modified.

Manuscript received February 29, 2016.
*Corresponding author.
This work was supported by the National Natural Science Foundation of China (61573283).

Thus the problem of conflict evidence fusion can be effectively solved.

To effectively solve the problem of conflict evidence combination, this paper proposes a new algorithm to measure the evidence conflict, and presentes a new method of evidence combination based on this algorithm.

2. Background material

Definition 1 Let $\Theta = \{\theta^1, \theta^2, \ldots, \theta^n\}$ be a non-empty finite set in which the elements are incompatible with each other, then Θ is called the framework of identification. θ^i $(1 \leqslant i \leqslant n)$ is called the single subset of Θ. The power set of Θ is defined as 2^Θ, which is the set of all the subsets of Θ.

Definition 2 Basic probability assignment function

Let Θ be the framework of identification, $A \subseteq \Theta$. If the function $m : 2^\Theta \to [0,1]$ satisfies the conditions $m(\varnothing) = 0$ and $\sum m(A) = 1$, then m is called the basic probability assignment function within Θ, and $m(A)$ is called the basic probability assignment for A and it reflects the support degree of the evidence to the proposition A in the framework of identification [16,17]. If $m(A) > 0$, then A is called a focal element. The union of all the focal elements of the evidence is called the nucleus $\boldsymbol{X}$ of the evidence and the evidence can be expressed as $\{\Theta, X, m\}$.

Definition 3 Dempster combination rule

If there are two independent evidences in the same framework of identification Θ and their focal elements are B_i $(i = 1, 2, \ldots, m)$ and C_j $(j = 1, 2, \ldots, n)$ respectively, where m and n are the numbers of the focal elements of these two evidences and the basic probability assignment functions are m_1 and m_2, then the Dempster combination rule [16,17] is defined as

$$\begin{cases} m_{\oplus}(A) = \dfrac{\displaystyle\sum_{B_i \cap C_j = A} m_1(B_i) m_2(C_j)}{1 - \displaystyle\sum_{B_i \cap C_j = \varnothing} m_1(B_i) m_2(C_j)}, \quad A \neq \varnothing \\ m(\varnothing) = 0 \end{cases} \tag{1}$$

where $\oplus$ is the fusion identifier and $k = \displaystyle\sum_{B_i \cap C_j = \varnothing} m_1(B_i) m_2(C_j)$ is the conflict coefficient. The Dempster combination rule is the core of the evidence theory. In order to maintain the normalization of the basic probability assignment function, the Dempster combination rule abandons the product part of focal elements with empty intersection.

3. Evidence conflict measure algorithms

With the following numerical experiments, several existing evidence conflict measure algorithms are analyzed.

3.1 Some evidence conflict measure algorithms

3.1.1 Conflict coefficient

Conflict coefficient k $(k \in [0,1])$ is used to measure the conflict degree between original evidences without extra computing and is very simple and convenient. The bigger the k, the higher the conflict degree between evidences. However, when evidences are completely incompatible, that is, $k = 1$, the Dempster combination rule cannot be used. There are still some problems, such as, the consistent evidences are misjudged, and the conflict degree represented by the conflict coefficient is not correspondent with practice.

Example 1 Give two pieces of evidences $E_1 : m_1 = (0.4, 0.3, 0.3)$ and $E_2 : m_2 = (0.4, 0.3, 0.3)$, we can obtain the conflict coefficient $k = 0.66$. According to k, the conflict between these two evidences is relatively high while actually the two evidences are exactly the same without any conflict.

3.1.2 Evidence distance

Jousselme et al. [18] introduced a vector space to the evidence theory and came up with the concept of evidence distance.

Definition 4 Within the framework of identification Θ, the Jousselme distance between two basic belief assignment functions m_1 and m_2 is

$$d(m_1, m_2) = \sqrt{\frac{1}{2}(\boldsymbol{m}_1 - \boldsymbol{m}_2)^{\mathrm{T}} \underline{\underline{D}}((\boldsymbol{m}_1 - \boldsymbol{m}_2))} \tag{2}$$

where $\underline{\underline{D}}[i,j] = |A_i \cap B_j| / |A_i \cup B_j|$ is the positive definited matrix of $2^{|\Theta|} \times 2^{|\Theta|}$, A_i and B_j are elements in the power set 2^Θ of m_1 and that of m_2 respectively. The bigger the distance between two evidences, the higher the conflict and the lower the similarity. Evidence distance can measure the conflict degree between evidences and solve the misjudgment problems caused by conflict coefficient in case of consistent evidences. However the calculation process is complex and the algorithm complexity is high.

Example 2 Let $\Theta = \{a, b, c\}$,

$$E_1 : m_1(a) = 0.5, \quad m_1(b) = 0.3, \quad m_1(c) = 0.2$$

$$E_2 : m_2(a) = 0.2, \quad m_2(b) = 0.3, \quad m_2(c) = 0.5$$

$$E_3 : m_3(a) = 0.0, \quad m_3(b) = 0.1, \quad m_3(c) = 0.9.$$

From intuitive analysis we can see that the evidence E_1 supports a while the evidence E_2 and the evidence E_3 support c. Thus a conclusion can be obtained that the conflict between E_2 and E_1 is higher than the conflict between E_2 and E_3. However, according to evidence distance, we can calculate that $d_{12} = 0.3$ and $d_{23} = 0.35$, where d_{ij} is the evidence distance between the evidence i and the evidence j. From the calculation results, the conflict between E_2 and E_3 is higher than that between E_2 and E_1, which does not agree with the previous intuitive analysis. This is because the focal elements of the two evidences are independent and thus the matrix $\underline{\underline{D}}$ is a diagonal matrix. This magnifies the effect of the relatively big focal elements on the conflict measure to some extent, that is, the evidence distance cannot correctly reflect the divergence when different evidences support different focal elements.

3.1.3 Pignistic probability distance

Liu [19] proposed the pignistic probability distance in 2006.

Definition 5 Let m be the basic probability assignment function within the same framework of identification, then the pignistic probability function [20] is

$$BetP_m(A) = \sum_{B\subseteq\Theta,\forall A\subseteq\Theta} \frac{|A\cap B|}{|B|}\cdot\frac{m(B)}{1-m(\phi)} \tag{3}$$

where $|A|$ is the number of elements included in set A and $BetP_m$ describes the support degree of the subsets of each proposition over the power set 2^{Θ} of the basic probability assignment function.

Definition 6 The pignistic probability distance is defined [19] as

$$difBetP_{m_1}^{m_2} = \max_{A\subseteq\Theta}(|BetP_{m_1}(A) - BetP_{m_2}(A)|). \tag{4}$$

The pignistic probability distance describes the maximum divergence of the support that evidences can give to different focal elements. The bigger the pignistic distance is, the higher the conflict between m_1 and m_2 is. Since different evidences intensively supporting different focal elements can cause conflict, the pignistic probability distance can effectively describe the evidence conflict. However there are also some deficiencies.

Example 3 There are two evidences $m_1 = (0.7, 0.15, 0.15)$ and $m_2 = (0.15, 0.7, 0.15)$. We can obtain the pignistic probability distance between m_1 and m_2 is $difBetP_{m_1}^{m_2} = 0.55$. Although the pignistic probability distance between the two evidences is not big, the focal element those evidence supports are different.

3.1.4 Cosine similarity

On the basis of the vector space introduced by Jousselme, Zhang et al. [21] used the cosine of the angle between vectors to measure the evidence conflict.

Suppose that Θ is the framework of identification. Establishing a R^n space based on the basic probability assignment (BPA) of every evidence within Θ, where n is the number of the propositions, the evidence vector in this space can be defined as

$$\boldsymbol{m}_i = (m_i(A_1), m_i(A_2), \dots, m_i(A_k), \dots, m_i(A_n))^{\mathrm{T}}$$

where $1 \leqslant i \leqslant N$. All the evidences can be seen as a set with N different n-dimensional vectors.

Definition 7 The cosine similarity between two evidence vectors $\boldsymbol{m}_1$ and $\boldsymbol{m}_2$ is

$$\cos(\boldsymbol{m}_1, \boldsymbol{m}_2) = \frac{\boldsymbol{m}_1^{\mathrm{T}}\cdot\boldsymbol{m}_2}{\|\boldsymbol{m}_1\|\cdot\|\boldsymbol{m}_2\|} = \frac{\boldsymbol{m}_1^{\mathrm{T}}\cdot\boldsymbol{m}_2}{\sqrt{(\boldsymbol{m}_1^{\mathrm{T}}\cdot\boldsymbol{m}_1)(\boldsymbol{m}_2^{\mathrm{T}}\cdot\boldsymbol{m}_2)}} \tag{5}$$

where $\boldsymbol{m}_1^{\mathrm{T}}\cdot\boldsymbol{m}_2 = \sum_{k=1}^{n} m_1(A_k)\cdot m_2(A_k)$. The value range of cosine similarity is between [0,1] and the bigger this value is, the lower the conflict between two evidences is. However, the calculation process is complex and the algorithm complexity is high.

3.1.5 Relative coefficient

Within the framework of identification Θ, the focal elements of two evidences are $a_1, \dots, a_k$ and $b_1, \dots, b_k$ respectively and the corresponding basic probability assignments are $p_1, \dots, p_k$ and $q_1, \dots, q_k$. Thus we can regard the two evidences as two random variables X and Y.

$$X = \begin{Bmatrix} a_1, & \cdots, & a_K \\ p_1, & \cdots, & p_K \end{Bmatrix}, \quad Y = \begin{Bmatrix} b_1, & \cdots, & b_K \\ q_1, & \cdots, & q_K \end{Bmatrix}.$$

Definition 8 The partial entropy [22] of random variable X over random variable Y is

$$H_Y(X) = -\sum_{k=1}^{K} q_k \log_2 p_k$$

$$H(X) = -\sum_{k=1}^{K} q_k \log_2 q_k. \tag{6}$$

Definition 9 The relative coefficient [22] between random variable X and random variable Y is

$$r(X,Y) = \frac{H(X)+H(Y)}{H_Y(X)+H_X(Y)}. \tag{7}$$

The value range of the relative coefficient is between [0,1] and the bigger this value is, the lower the conflict between two evidences is, the higher the similarity is. Whereas in the calculation process there are conditions when $\log_2 0 = -\infty$ and this will make the relative coefficient to be 0. If there are items in one evidence equal to 0 and the corresponding items in the other evidence are not equal to 0, even though there is no conflict between these two evidences, the calculated relative coefficient will tend to be 0 and in this condition the conflict between evidences cannot be correctly estimated. To solve this problem, Deng et al. [23] changed the partial entropy to be

$$H_Y(X) = \sum q_k \cdot \mathrm{e}^{-5p_k}, \quad H(X) = \sum q_k \cdot \mathrm{e}^{-5q_k} \quad (8)$$

where e^{-5p_k} is the result close to $\log$ gained through checking computations.

3.1.6 Analysis of others algorithms

On the basis of common evidence conflict measure algorithms, many scholars have proposed improved algorithms.

To solve the problem of conflict management, Deng [24] put forward the generalized evidence theory (GET) in an open world, provided the generalized combination rule (GCR), and proposed the generalized conflict model based on the generalized evidence distance (GED) to measure the conflict between evidences. Jiang et al. [25] comprehensively considered the amount of information and the consistency of the evidence, introduced the concept of Deng entropy to measure information volume of BPAs directly, and weighted averaged the evidence to apply the evidence theory in multi-sensor data fusion in fault diagnosis. Reference [26] studied the generalized evidence theory in open world. It used training samples to build triangular fuzzy number models, which determined the generalized basic probability assignment (GBPA), and provided the difference degree to measure the distance of evidence conflict.

Zhang et al. [13] used the Mahalanobis distance to describe the evidence conflict. Liu [19] utilized a binary group of conflict coefficient and a pignistic probability distance to estimate the evidence conflict. Jousselme et al. [27] discussed and analyzed composite distances, Minkowski family of distances, inner product family of distances, fidelity family of distances, information-based distances and two-dimensional distances, and provided a general mathematical formula composed of a cosine. Hu et al. [28] compared the distance functions, such as Jousselme distance, Euclidean distance, Mahalanobis distance, and evidence average distance. Destercke et al. [29] measured the conflict by examining consistency and conflict on sets. Loudahi et al. [30] introduced a new bba distance based on Dempsterian specialization matrices. Sarabi-Jamab et al. [31] reviewed different dissimilarity measures based on measuring any of the conflict, non-specificity, or unified total uncertainty, and proposed the set of most discriminative dissimilarity measures to describe the evidence conflict degree. Yang et al. [32] provided the degree of disagreement based on the Jousselme distance to measure the evidence conflict. Huang et al. [33] used the pignistic probability transformation and the Jousselme distance to measure the evidence conflict.

3.2 New evidence conflict measure algorithm

After the brief analysis of several evidence conflict measure algorithms, we can see that each measure algorithm has its own advantages and disadvantages, but these algorithms cannot comprehensively and correctly describe the conflict between evidences. Based on the pignistic probability function of single subset, this paper introduces the concept of Tanimoto measure and comes up with a new algorithm to calculate the conflict degree, that is, evidence similarity.

In the process of evidence fusion, the basic probability assignment function m of original evidence is transformed by the pignistic probability transformation to get the new basic probability assignment function m', then the Tanimoto measure is introduced, and the similarity between each evidence is calculated by m'.

Definition 10 Pignistic probability function over single subset

Let $\Theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$ be the framework of identification and there are N independent pieces of evidences. A_k is the focal element of one certain evidence in the framework of identification Θ.

$$BetP_m(\theta_i) = \sum_{\theta_i \in A_k} \frac{1}{|A_k|} \cdot m(A_k), \quad i = 1, 2, \ldots, n. \quad (9)$$

Then $BetP_m(\theta_i)$ is called the pignistic probability function of single subset θ_i under the basic probability assignment function m [20]. Its essence is to divide the basic probability assignment of the focal element A_k equally and to give it to the single subset focal element.

If the subsets that two evidences have are inconsistent, let the mass value of the absent subsets of each evidence be 0. Then the basic probability assignment function m is converted to

$$m' = (BetP_m(\theta_1), BetP_m(\theta_2), \ldots, BetP_m(\theta_n)) = (m'(A'_1), m'(A'_2), \ldots, m'(A'_n)).$$

Definition 11 Evidence similarity

In order to comprehensively consider the inclusiveness and difference between evidences, Tanimoto measure, which is a reference to the Tanimoto coefficient in the clustering algorithm is introduced and the evidence similarity between two evidences of which the basic probability assignment functions are m'_1 and m'_2 respectively is

$$sim(m'_1, m'_2) = \frac{\sum_{i=1}^{n} m'_1(A'_i) \cdot m'_2(A'_i)}{\sum_{i=1}^{n} m'_1(A'_i)^2 + \sum_{i=1}^{n} m'_2(A'_i)^2 - \sum_{i=1}^{n} m'_1(A'_i) \cdot m'_2(A'_i)}. \quad (10)$$

The value range of evidence similarity is in [0,1], and the bigger this value, the higher the similarity between two evidences. The evidence similarity and evidence conflict are antonym. The higher the similarity between two evidences, the lower the conflict between them.

3.3 Comparison and analysis of some algorithms

Compared with other algorithms, using the evidence similarity to describe the evidence conflict can achieve better results. The superiority of this algorithm is illustrated by using the examples mentioned above.

In Example 1, the evidence similarity between two evidences is $sim^{(1)} = 1.0$. This means they are identical and there is no conflict between them. In Example 2, $sim_{12}^{(2)} = 0.617$, $sim_{23}^{(2)} = 0.667$. This means that the support of two evidences which are seriously conflicted with each other to the same target is very different and their evidence similarity is much lower. In Example 3, the conflict between two evidences is high, so their evidence similarity is relatively low and $sim^{(3)} = 0.278$.

In order to easily compare the above evidence conflict measure algorithms, the example in [19] is applied to illustrate the universal validity of evidence similarity proposed in this paper.

Example 4 The framework of identification is $\Theta = \{1, 2, 3, \ldots, 20\}$ and two basic probability assignments are as follows:

$$\text{Evidence 1} : m_1(2,3,4) = 0.05, \quad m_1(7) = 0.05$$
$$m_1(\Theta) = 0.1, \quad m_1(A) = 0.8$$
$$\text{Evidence 2} : m_2(1,2,3,4,5) = 1$$

where A changes according to $\{1\}, \{1,2\}, \{1,2,3\}, \cdots, \{1,2,3,\cdots,20\}$.

Fig. 1 shows the variation trend of the conflict coefficient k, evidence distance d, pignistic probability distance $difBetP$, cosine similarity cos, relative coefficient r and evidence similarity sim with the change of A.

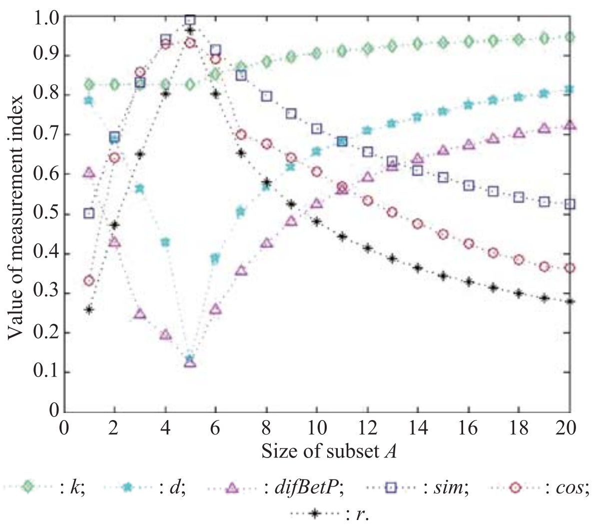


Fig. 1 Comparison between different evidence conflict measure algorithms

Based on the commonsense analysis, when the size of subset A is five, the focal element $A = \{1, 2, 3, 4, 5\}$ of the original evidence 1 is the same with the focal element of the original evidence 2 and $m_1(1, 2, 3, 4, 5) = 0.8$, $m_2(1, 2, 3, 4, 5) = 1$. Thus the conflict degree between these two evidences should be the lowest.

From Fig. 1, we can see that evidence distance d and pignistic probability distance $difBetP$ are positively related to the evidence conflict and the bigger these values, the higher the conflict. While cosine similarity cos, relative coefficient r and evidence similarity sim are negatively related to evidence conflict and the bigger these values, the lower the conflict. When the subset A ranges from small to big, the conflict coefficient k first remains unchanged and then gradually increases. The relatively high value indicates that the conflict is high and obviously it does not fit with the common sense. The values of d, $difBetP$, cos, r and sim significantly change with the change of the subset A and the change trends representing the conflict degree are similar. When A changes from 1 to 5, d, $difBetP$, cos, r and sim can properly express the gradual decrease of the evidence conflict and the change trend of sim is smoother. When $A = \{1, 2, 3, 4, 5\}$, d, $difBetP$, cos, r and sim can correctly show that the conflict degree is the lowest. When A changes from 5 to 20, d, $difBetP$, cos, r and sim can accurately indicate that the evidence conflict is gradually increasing and the increase trends become slower and slower.

By comparison, we can conclude that the proposed similarity can thoroughly measure the evidence conflict degree and the similarity value can be used directly in the evidence credibility calculation without further processing.

4. Evidence combination method based on evidence similarity

4.1 General framework of evidence combination

We think that modifying the evidential source and improving the combination rule at the same time could solve the problem of conflict evidence combination more effectively. Thus, this paper introduces the general framework of evidence combination.

Suppose that $\Theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$ is the framework of identification and there are N pieces of independent evidences $\{\Theta, X_i, m_i\}$ $(i = 1, 2, \ldots, N)$.Then the model to modify the evidence can be expressed as follows:

$$\begin{cases} \widetilde{m}_i(A_i) = Cred_i \cdot m_i(A_i), \quad A_i \neq \Theta \\ \widetilde{m}_i(\Theta) = Cred_i \cdot m_i(\Theta) + (1 - Cred_i) \end{cases} \tag{11}$$

where $Cred_i (0 \leqslant Cred_i \leqslant 1)$ is the credibility of evidence i. Evidence credibility is the degree of the evidence supported by other evidences and is negatively related to the conflict between evidences. It can be obtained by the evidence conflict measure algorithms.

Then the modified basic probability assignment function $\widetilde{m}_i$ is obtained. Based on this, the evidence combination model is defined.

$$\begin{cases} m(\varnothing)=0 \\ m(A)=\sum\limits_{\cap A_i=A}\prod\limits_{1\leqslant j\leqslant n}\widetilde{m}_j(A_i)+K\cdot\delta(A,\widetilde{m}), \quad A\neq\varnothing \end{cases} \tag{12}$$

where $K=\sum\limits_{\cap A_i=\phi}\prod\limits_{1\leqslant j\leqslant n}\widetilde{m}_j(A_i), A_i\in X_i$ is the total conflict of the modified evidence model, $\delta(A,\widetilde{m})$ is the weight of K over the focal element A and $\sum\limits_{A\in\Theta}\delta(A,\widetilde{m})=1$.

Proof For $\forall A\subseteq\Theta, A\neq\varnothing$,

$$\sum_{A\subseteq\Theta}m(A)=\sum_{A\subseteq\Theta}\left(\sum_{\cap A_i=A}\prod_{1\leqslant j\leqslant n}\widetilde{m}_j(A_i)\right)+\sum_{A\subseteq\Theta}(K\cdot\delta(A,\widetilde{m}))=$$

$$\sum_{A\subseteq\Theta}\sum_{\cap A_i=A}\prod_{1\leqslant j\leqslant n}\widetilde{m}_j(A_i)+K\cdot\sum_{A\subseteq\Theta}\delta(A,\widetilde{m})=$$

$$\sum_{\cap A_i\neq\varnothing}\prod_{1\leqslant j\leqslant n}\widetilde{m}_j(A_i)+K=1-K+K=1.$$

□

The proof shows that the fusion results meet the requirements of the basic probability assignment function.

Under the general framework of evidence combination, the specific evidence combination method can be achieved as long as the evidence credibility $Cred_i$ and the distributing weight $\delta(A,\widetilde{m})$ are defined. For example, when $Cred_i=1$ and $\delta(A,\widetilde{m})=\left(\sum\limits_{\cap A_i=A}\prod\limits_{1\leqslant j\leqslant n}\widetilde{m}_j(A_i)\right)\Big/(1-K)$, the framework is the D-S evidence combination method. When $Cred_i=1$ and $\delta(A,\widetilde{m})=\begin{cases}1, & A=\Theta\\ 0, & A\neq\Theta\end{cases}$, the framework is the Yager evidence combination method.

From the general framework of evidence combination, we can know that every piece of evidence, according to their credibility, will distribute a fraction of the basic probability assignment of every focal element to the universal set. In the modified evidence, the lower the credibility of the evidence is, the more uncertain information the evidence has, thus reducing the effect that the evidences with small credibility cause to the fusion results and improving the accuracy and speed of the fusion.

4.2 Combination method based on the evidence similarity

The proposed method can be described as follows:

Suppose that Θ is the framework of identification and there are N pieces of independent evidences.

Step 1 Use (9) to do the pignistic probability function conversion and calculate the similarity $sim(m'_i,m'_j)$ between evidence i and evidence j according to (10).

Step 2 Build an $N\times N$ similarity matrix $\boldsymbol{SIM}$ after obtaining the similarities between every two evidences.

$$\boldsymbol{SIM}=\begin{bmatrix} 1 & sim(m'_1,m'_2) & \cdots & sim(m'_1,m'_N) \\ sim(m'_2,m'_1) & 1 & \cdots & sim(m'_2,m'_N) \\ \vdots & \vdots & \ddots & \vdots \\ sim(m'_N,m'_1) & sim(m'_N,m'_2) & \cdots & 1 \end{bmatrix}. \tag{13}$$

Step 3 Determine the support degree and credibility of each evidence.

The support degree $Sup(m'_i)$ that all other evidences give to evidence i is

$$Sup(m'_i)=\sum_{j=1,j\neq i}^{N}sim(m'_i,m'_j), \quad i=1,2,\dots,N. \tag{14}$$

Evidence credibility can be obtained after the normalization of support degree. The credibility of evidence i $Cred_i$ is

$$Cred_i=\frac{Sup(m'_i)}{\max\limits_{1\leqslant i\leqslant N}[Sup(m'_i)]}. \tag{15}$$

Step 4 Modify the evidence.

Use the credibility $Cred_i$ as the weight of evidence and do a weighted correction to the basic probability assignment function m_i by applying (11), then the new basic probability assignment function $\widetilde{m}_i$ is

$$\begin{cases}\widetilde{m}_i(A_i)=Cred_i\cdot m_i(A_i), \quad A_i\neq\Theta \\ \widetilde{m}_i(\Theta)=Cred_i\cdot m_i(\Theta)+(1-Cred_i)\end{cases}.$$

Step 5 Determine the distribution weight of each focal element.

The relative credibility $Ccred_i$ of the evidence i is

$$Ccred_i=\frac{Sup(m'_i)}{\sum\limits_{i=1}^{N}Sup(m'_i)}. \tag{16}$$

The distribution weight $\delta(A,\widetilde{m})$ of the focal element A is

$$\delta(A,\widetilde{m})=\sum_{i=1}^{N}Ccred_i\cdot\widetilde{m}_i(A). \tag{17}$$

Proof For $\forall A\subseteq\Theta, A\neq\varnothing$,

$$\sum_{A\in\Theta}\delta(A,\widetilde{m})=\sum_{A\subseteq\Theta}\left(\sum_{i=1}^{N}Ccred_i\cdot\widetilde{m}_i(A)\right)=$$

$$\sum_{i=1}^{N}\sum_{A\subseteq\Theta} Ccred_i\cdot\widetilde{m}_i(A)=$$

$$\sum_{i=1}^{N} Ccred_i\sum_{A\subseteq\Theta}\widetilde{m}_i(A)=$$

$$\sum_{i=1}^{N} Ccred_i=\frac{\sum\limits_{i=1}^{N} Sup(m_i')}{\sum\limits_{i=1}^{N} Sup(m_i')}=1.$$

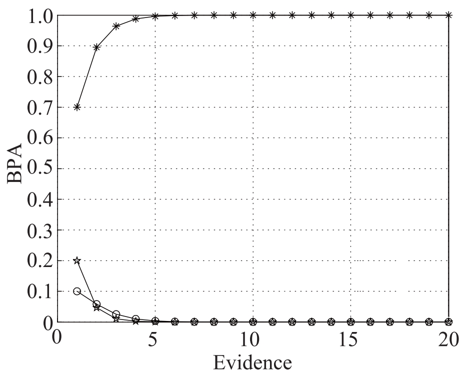
Step 6 Use (12) to do the evidence combination and get the final fusion results.

$$\begin{cases} m(\varnothing)=0\\ m(A)=\sum\limits_{\cap A_i=A}\prod\limits_{1\leqslant j\leqslant n}\widetilde{m}_j(A_i)+K\delta(A,\widetilde{m}),\ A\neq\varnothing \end{cases}\cdot$$

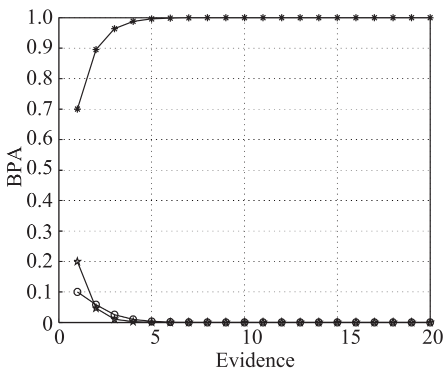
5. Comparison and analysis of numerical examples

5.1 Numerical example 1

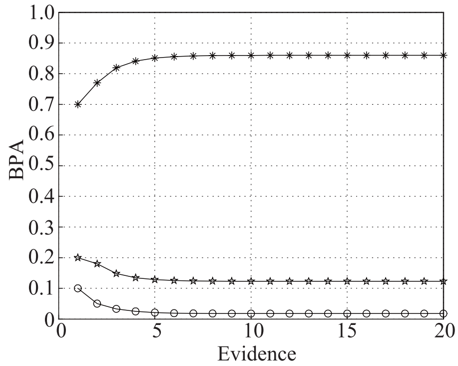
Suppose the framework of identification is $\Theta=\{A,B\}$. D-S method [17], Yager method [5], Murphy method [10] and the proposed method in this paper are used to fuse 50 identical evidences and BPAs of the evidence are $m(A)=0.7$, $m(B)=0.1$, $m(\Theta)=0.2$. Results can be seen in Fig. 2.



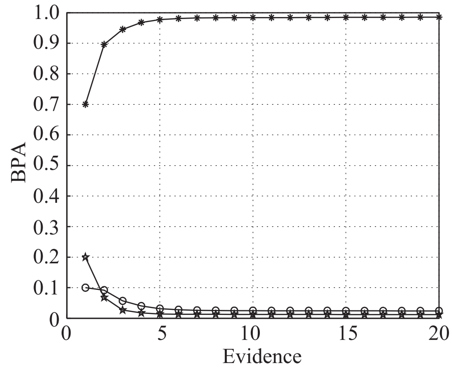
(a) Fusion results of D-S method



(b) Fusion results of Murphy method



(c) Fusion results of Yager method



(d) Fusion results of the proposed method

: $m(A)$; : $m(B)$; : $m(\Theta)$.

Fig. 2 Fusion results of different methods

From Fig. 2, it is obvious that when fusing multiple identical evidences, with the increase of evidence number, D-S method, Murphy method, Yager method and the proposed method in this paper can all effectively fuse evidences. And the fusion results focus on the superior focal element A at a fast convergence speed as well as the uncertain information $m(\Theta)$ decreases. Meanwhile, the convergence speed is approximately the same, that is, relatively stable results can be obtained by fusing at least five evidences. However there are still some differences between these methods in details. Murphy method and D-S method have similar results and their fusion results rapidly converge to $(m(A)=1, m(B)=0, m(\Theta)=0)$. Yager method gives all the conflict information to the framework of identification Θ and this makes the uncertainty information $m(\Theta)$ unable to promptly converge to 0, reducing its ability to deal with the uncertain information. The fusion results of 20 evidences are $m(A)=0.859\,6$, $m(B)=0.017\,6$, $m(\Theta)=0.122\,8$. The suggested method in this paper proportionally distributes the conflict to each focal element and enables the fusion results to rapidly converge to the expected value $(m(A)\rightarrow 1, m(B)\rightarrow 0, m(\Theta)\rightarrow 0)$. The fusion results of 20 evidences are $m(A)=0.985\,3$, $m(B)=0.023\,8$, $m(\Theta)=0.011\,9$.

5.2 Numerical example 2

To verify the universal validity of the proposed method and its superiority in dealing with the highly conflicted evidences, an example is given below and the proposed method is compared with D-S method [17], Murphy method [10], Yager method [5] and Liu's method [15].

There are five sensors with different properties observing and recognizing one certain target in the air. Let the framework of identification be $\Theta = \{a : \mathrm{Civilaircraft}, b : \mathrm{Bomberaircraft}, c : \mathrm{Fighteraircraft}\}$.

We can get five evidences within Θ after converting the information obtained by five sensors to the basic probability assignments, that is

$$E_1 : m_1(a) = 0.5, \quad m_1(b) = 0.2, \quad m_1(c) = 0.3$$

$$E_2 : m_2(a) = 0.65, \quad m_2(b) = 0.1, \quad m_2(a, c) = 0.25$$

$$E_3 : m_3(a) = 0.8, \quad m_3(b) = 0.1, \quad m_3(c) = 0.1$$

$$E_4 : m_4(a) = 0.75, \quad m_4(b) = 0.1, \quad m_4(c) = 0.15$$

$$E_5 : m_5(a) = 0.55, \ m_5(b) = 0.1, m_5(a, c) = 0.35.$$

The five evidences show that all evidences are likely to identify the target as a, and the conflict between these evidences are relatively low.

The five evidences are fused using five different methods respectively and results are shown in Table 1.

Table 1 Fusion results of five methods under low conflict

Method	E_1, E_2	E_1, E_2, E_3	E_1, E_2, E_3, E_4	E_1, E_2, E_3, E_4, E_5
D-S method	$m(a) = 0.825\,7$	$m(a) = 0.974\,3$	$m(a) = 0.995\,1$	$m(a) = 0.998\,3$
	$m(b) = 0.036\,7$	$m(b) = 0.005\,4$	$m(b) = 0.000\,8$	$m(b) = 0.000\,1$
	$m(c) = 0.137\,6$	$m(c) = 0.020\,3$	$m(c) = 0.004\,1$	$m(c) = 0.001\,6$
Murphy method	$m(a) = 0.828\,6$	$m(a) = 0.969\,1$	$m(a) = 0.993\,8$	$m(a) = 0.997\,5$
	$m(b) = 0.039\,3$	$m(b) = 0.005\,9$	$m(b) = 0.000\,8$	$m(b) = 0.000\,1$
	$m(c) = 0.104\,8$	$m(c) = 0.023\,6$	$m(c) = 0.005\,3$	$m(c) = 0.002\,3$
	$m(a, c) = 0.027\,3$	$m(a, c) = 0.001\,4$	$m(a, c) = 0.000\,1$	$m(a, c) = 0.000\,1$
Yager method	$m(a) = 0.45$	$m(a) = 0.724$	$m(a) = 0.674\,6$	$m(a) = 0.755$
	$m(b) = 0.02$	$m(b) = 0.047\,5$	$m(b) = 0.022\,3$	$m(b) = 0.029\,1$
	$m(c) = 0.075$	$m(c) = 0.053$	$m(c) = 0.034\,3$	$m(c) = 0.012\,0$
	$m(a, c) = 0$	$m(a, c) = 0$	$m(a, c) = 0$	$m(a, c) = 0.094\,1$
	$m(\Theta) = 0.455$	$m(\Theta) = 0.175\,5$	$m(\Theta) = 0.268\,8$	$m(\Theta) = 0.109\,8$
Liu's method	$m(a) = 0.825\,7$	$m(a) = 0.969\,0$	$m(a) = 0.990\,8$	$m(a) = 0.997\,3$
	$m(b) = 0.036\,7$	$m(b) = 0.007\,7$	$m(b) = 0.002\,1$	$m(b) = 0.000\,2$
	$m(c) = 0.137\,6$	$m(c) = 0.023\,3$	$m(c) = 0.007\,1$	$m(c) = 0.002\,5$
Proposed method	$m(a) = 0.828\,6$	$m(a) = 0.965\,1$	$m(a) = 0.992\,5$	$m(a) = 0.997\,5$
	$m(b) = 0.039\,3$	$m(b) = 0.008\,3$	$m(b) = 0.001\,3$	$m(b) = 0.000\,16$
	$m(c) = 0.104\,8$	$m(c) = 0.025\,3$	$m(c) = 0.006\,1$	$m(c) = 0.002\,3$
	$m(a, c) = 0.027\,3$	$m(a, c) = 0.001\,3$	$m(a, c) = 0.000\,1$	$m(a, c) = 0.000\,04$

As can be seen from the fusion results in Table 1, when the conflict between evidences that are provided by each sensors are relatively low, with the increasing number of evidences that support focal element a, the performance of D-S method, Murphy method, Liu's method and the proposed method are all ideal, and these methods all can fuse evidences effectively and get the correct fusion result, as well as converge to the expected value $(m(a) \to 1,\ m(b) \to 0,\ m(c) \to 0)$ at a relatively fast convergence speed. Yager method result is $m(a) > m(\Theta) > (m(b), m(c))$, however, the value of $m(a)$ is far less than other four methods, meanwhile, all of the conflicted information is classified as uncertain $m(\Theta)$, which increases the uncertainty of the combined evidences, and is contrary to the original intention of dealing with the uncertainty of evidence theory.

5.3 Numerical example 3

In the real world battlefield, information acquired by these sensors can easily be highly conflicted due to natural factors, human disturbance, or sensor failure. Assume in the same framework of identification, the evidences E_1, E_2, E_4, E_5 obtained by sensors 1, 2, 4, 5 are the same of those in Numerical example 2, while sensor 3 is disturbed and the corresponding evidence is $E_3' : m_3'(a) = 0.0, m_3'(b) = 0.8, m_3'(c) = 0.2$.

The five evidences are as follows:

$$E_1 : m_1(a) = 0.5, \quad m_1(b) = 0.2, \quad m_1(c) = 0.3$$

$$E_2 : m_2(a) = 0.65, \quad m_2(b) = 0.1, \quad m_2(a, c) = 0.25$$

$$E_3' : m_3'(a) = 0.0, \quad m_3'(b) = 0.8, \quad m_3'(c) = 0.2$$

$$E_4 : m_4(a) = 0.75, \quad m_4(b) = 0.1, \quad m_4(c) = 0.15$$

$$E_5 : m_5(a) = 0.55, \quad m_5(b) = 0.1, \quad m_5(a, c) = 0.35.$$

It can be seen that evidences E_1, E_2, E_4, and E_5 all support the target to be a and the conflicts between them are relatively small. The evidence 3 strongly supports b and it is the abnormal evidence is highly conflicted with other evidences.

Table 2 Fusion results of five methods under high conflict

Method	E_1, E_2	E_1, E_2, E_3'	E_1, E_2, E_3', E_4	E_1, E_2, E_3', E_4, E_5
D-S method	$m(a) = 0.825\ 7$	$m(a) = 0$	$m(a) = 0$	$m(a) = 0$
	$m(b) = 0.036\ 7$	$m(b) = 0.516\ 1$	$m(b) = 0.415\ 6$	$m(b) = 0.168\ 9$
	$m(c) = 0.137\ 6$	$m(c) = 0.483\ 9$	$m(c) = 0.584\ 4$	$m(c) = 0.831\ 1$
Murphy method	$m(a) = 0.828\ 6$	$m(a) = 0.608\ 8$	$m(a) = 0.886\ 7$	$m(a) = 0.975\ 0$
	$m(b) = 0.039\ 3$	$m(b) = 0.297\ 0$	$m(b) = 0.086\ 07$	$m(b) = 0.013\ 7$
	$m(c) = 0.104\ 8$	$m(c) = 0.090\ 7$	$m(c) = 0.027\ 07$	$m(c) = 0.011\ 0$
	$m(a, c) = 0.027\ 3$	$m(a, c) = 0.003\ 5$	$m(a, c) = 0.000\ 16$	$m(a, c) = 0.000\ 3$
Yager method	$m(a) = 0.45$	$m(a) = 0$	$m(a) = 0.385\ 5$	$m(a) = 0.584\ 6$
	$m(b) = 0.02$	$m(b) = 0.38$	$m(b) = 0.089\ 4$	$m(b) = 0.052\ 2$
	$m(c) = 0.075$	$m(c) = 0.106$	$m(c) = 0.093$	$m(c) = 0.032\ 6$
	$m(a, c) = 0$	$m(a, c) = 0$	$m(a, c) = 0$	$m(a, c) = 0.151\ 2$
	$m(\Theta) = 0.455$	$m(\Theta) = 0.514$	$m(\Theta) = 0.432\ 1$	$m(\Theta) = 0.179\ 5$
Liu's method	$m(a) = 0.825\ 7$	$m(a) = 0.644\ 4$	$m(a) = 0.919\ 5$	$m(a) = 0.976\ 7$
	$m(b) = 0.036\ 7$	$m(b) = 0.153\ 0$	$m(b) = 0.024\ 0$	$m(b) = 0.002\ 3$
	$m(c) = 0.137\ 6$	$m(c) = 0.202\ 6$	$m(c) = 0.056\ 5$	$m(c) = 0.021\ 0$
Proposed method	$m(a) = 0.828\ 6$	$m(a) = 0.805\ 5$	$m(a) = 0.959\ 8$	$m(a) = 0.986\ 1$
	$m(b) = 0.039\ 3$	$m(b) = 0.053\ 4$	$m(b) = 0.007\ 5$	$m(b) = 0.000\ 9$
	$m(c) = 0.104\ 8$	$m(c) = 0.114\ 6$	$m(c) = 0.032\ 2$	$m(c) = 0.012\ 8$
	$m(a, c) = 0.027\ 3$	$m(a, c) = 0.026\ 5$	$m(a, c) = 0.000\ 4$	$m(a, c) = 0.000\ 2$

As can be seen from Table 2, when the abnormal evidence 3 occurs, the fusion results of these five methods are different. Since $m(a) = 0$ in evidence 3, though the subsequent evidences strongly support a, the fusion result of the D-S method still is $m(a) = 0$. In Yager method, the value of $m(\Theta)$ is greater than that of $m(a)$ during the fusion of the first four evidences, and $m(a) = 0.584\ 6$ becomes greater than $m(\Theta) = 0.179\ 5$ only after fusing all five evidences, which shows that Yager method is not suitable for decision-making because its convergence precision is not high enough and its convergence rate is slow, as well as the value of $m(\Theta)$ is relatively large. None of Murphy method, Liu's method or the proposed method changes the Dempster combination rule, they all modify evidence sources to effectively reduce the influence of abnormal evidence 3, so the basic probability assignment of each target converges to the expected value $(m(a) \to 1, m(b) \to 0, m(c) \to 0)$.

However, there are still some differences. As can be seen from the five evidences, $m(b)$ is always lower than $m(c)$ except abnormal evidence 3, thus we can conclude that $m(b)$ would be lower than $m(c)$ in the fusion result. However, the fusion result of Murphy method is $m(a) \gg m(b) > m(c)$, which shows that Murphy method just results in a simple average value of evidences and it ignores the credibility of evidences, thus it needs more credible evidences to offset the impact of abnormal evidence. The Liu's method is an improvement of Murphy method, it can fuse all the evidences correctly and can result in $m(a) \gg m(c) > m(b)$, however, when calculating the weight of evidence credibility, the calculation process is complex, and the complexity of this method is relatively high. The proposed method is relatively simple and convenient, and $m(a) = 0.805\ 5$ after three evidences including the abnormal evidence 3 are fused, which is greater than that of other methods, while $m(b) = 0.053\ 4$ is lower than that of other methods, which means the proposed method can deal with the conflicted evidences more effectively and can reduce the impact of abnormal evidences. Meanwhile, the fusion results can rapidly converge to the expected value with the increasing number of evidences, $m(a) = 0.959\ 8$ after four evidences are fused, and $m(a) = 0.986\ 1$ after five evidences are fused. The convergence speed is fast and the accuracy is high, and the method is relatively stable while the fluctuation of $m(a)$ value is small. Meanwhile, $m(a, c)$ promptly converges to 0, which fits the reality of original evidence, and it indicates the strong capacity of the evidence theory to deal with the uncertain information.

6. Conclusions

This paper analyzes the relevant achievements of the D-S evidence theory in recent years. Aimed at the problem that the existing algorithms cannot comprehensively and correctly describe the conflict degree between evidences, a new conflict measure algorithm is proposed. Based on this new conflict measure algorithm, a new evidence combination method is established within the general framework of evidence combination. Examples results show that the proposed method can fully take the global information of evidences such as evidence credibility into account and can minimize the effect that conflict evidences have on the process of evidence fusion, reducing the influence of conflict information on the final fusion results. It improves the reliability and rationality of the fusion results and has a relatively fast convergence speed.

Based on the analysis of the causes of evidence conflict, the future work is to discuss if there is a better conflict measure algorithm and if there is a more reasonable evidence combination rule based on the new conflict measure algorithm. These problems are worth to further research.

References

[1] B. Khaleghi, A. Khamis, F. O. Karray, et al. Multisensor data fusion: a review of the state-of-the-art. *Information Fusion*, 2013, 14(1): 28 – 44.

[2] B. Shen, Y. Liu, J. S. Fu. An integrated model for robust multisensor data fusion. *Sensors*, 2014, 14(10): 19669 – 19686.

[3] W. T. Sung, K. Y. Chang. Evidence-based multi-sensor information fusion for remote healthcare systems. *Sensors and Actuators A: Physical*, 2013, 204(24): 1 – 19.

[4] M. E. Cattaneo. Belief functions combination without the assumption of independence of the information sources. *International Journal of Approximate Reasoning*, 2011, 52(3): 299 – 315.

[5] R. R. Yager. On the Dempster-Shafer framework and new combination rules. *Information Science*, 1987, 41(2): 93 – 137.

[6] E. Lefèvre, O. Colot, P. Vannoorenberghe. Belief function combination and conflict management. *Information Fusion*, 2002, 3(2): 149 – 162.

[7] E. Lefèvre, Z. Elouedi. How to preserve the conflict as an alarm in the combination of belief functions? *Decision Support Systems*, 2013, 56(1): 326 – 333.

[8] K. H. Guo, W. L. Li. Combination rule of D-S evidence theory based on the strategy of cross merging between evidences. *Expert Systems with Applications*, 2011, 38(10): 13360 – 13366.

[9] Y. Leung, N. N. Ji, J. H. Ma. An integrated information fusion approach based on the theory of evidence and group decision-making. *Information Fusion*, 2013, 14(4): 410 – 422.

[10] C. K. Murphy. Combining belief functions when evidence conflicts. *Decision Support Systems*, 2000, 29(1): 1 – 9.

[11] Y. Deng, W. K. Shi, Z. F. Zhu, et al. Combining belief functions based on distance of evidence. *Decision support systems*, 2004, 38(3): 489 – 493.

[12] J. Schubert. Conflict management in Dempster Shafer theory using the degree of falsity. *International Journal of Approximate Reasoning*, 2011, 52(3): 449 – 460.

[13] Z. J. Zhang, T. H. Liu, W. Y. Zhang. Novel paradigm for constructing masses in dempster-Shafer evidence theory for wireless sensor network's multisource data fusion. *Sensors*, 2014, 14(4): 7049 – 7065.

[14] D. Q. Han, Y. Deng, C. Z. Han, et al. Weighted evidence combination based on distance of evidence and uncertainty measure. *Journal of Infrared & Millimeter Waves*, 2011, 30(5): 396 – 468.

[15] Z. G. Liu, J. Dezert, Q. Pan, et al. Combination of sources of evidence with different discounting factors based on a new dissimilarity measure. *Decision Support Systems*, 2011, 52(1): 133 – 141.

[16] A. P. Dempster. Upper and low probabilities induced by a multi-valued mapping. *Annuals of Mathematical Statistics*, 1967, 38(6): 325 – 339.

[17] G. A. Shafer. Mathematical theory of evidence. NJ, USA: Princeton University Press: Princeton, 1976.

[18] A. L. Jousselme, D. Grenier, É. Bossé. A new distance between two bodies of evidence. *Information Fusion*, 2001, 2(2): 91 – 101.

[19] W. R. Liu. Analyzing the degree of conflict among belief functions. *Artificial Intelligence*, 2006, 170(11): 909 – 924.

[20] P. Smets. Decision making in the TBM: the necessity of the pignistic transformation. *International Journal of Approximate Reasoning*, 2005, 38(2): 133 – 147.

[21] Z. J. Zhang, T. H. Liu, D. Chen, et al. Novel algorithm for identifying and fusing conflicting data in wireless sensor networks. *Sensors*, 2014, 14(6): 9562 – 9581.

[22] W. Q. Wang, X. L. Xin. Hybrid partial entropy and relative entropy. *Fuzzy Systems and Mathematics*, 2005, 19(4): 60 – 65.

[23] Y. Deng, D. Wang, Q. Li, et al. A new method to analyze evidence conflict. *Control Theory & Applications*, 2011, 28(6): 839 – 844.

[24] Y. Deng. Generalized evidence theory. *Applied Intelligence*, 2015, 43(3): 530 – 543.

[25] W. Jiang, B. Wei, C. Xie, et al. An evidential sensor fusion method in fault diagnosis. *Advances in Mechanical Engineering*, 2016, 8(3): 1 – 7.

[26] W. Jiang, J. Zhan, D. Zhou, et al. A method to determine generalized basic probability assignment in the open world. *Mathematical Problems in Engineering*, 2016, (2): 1 – 11.

[27] A. L. Jousselme, P. Maupin. Distances in evidence theory: comprehensive survey and generalizations. *International Journal of Approximate Reasoning*, 2012, 53(2): 118 – 145.

[28] L. F. Hu, X. Guan, Y. Deng, et al. Measuring conflict functions in generalized power space. *Chinese Journal of Aeronautics*, 2011, 24(1): 65 – 73.

[29] S. Destercke, T. Burger. Toward an axiomatic definition of conflict between belief functions. *IEEE Trans. on Cybernetics*, 2013, 43(2): 585 – 596.

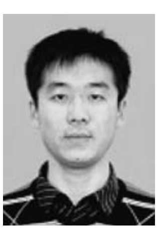
[30] M. Loudahi, J. Klein, J. M. Vannobel, et al. New distances between bodies of evidence based on Dempsterian specialization matrices and their consistency with the conjunctive combination rule. *International Journal of Approximate Reasoning*, 2014, 55(5): 1093 – 1112.

[31] A. Sarabi-Jamab, B. N. Araabi, T. Augustin. Information-based dissimilarity assessment in Dempster-Shafer theory. *Knowledge-Based Systems*, 2013, 54: 114 – 127.

[32] Y. Yang, D. Q. Han, C. Z. Han. Discounted combination of unreliable evidence using degree of disagreement. *International Journal of Approximate Reasoning*, 2013, 54(8): 1197 – 1216.

[33] S. Y. Huang, X. Y. Su, Y. Hu, et al. New decision-making method by incomplete preferences based on evidence distance. *Knowledge-Based Systems*, 2013, 56(3): 264 – 272.

Biographies



Wenhao Bi was born in 1986. He is a Ph.D. candidate in Northwestern Polytechnical University. His research interests are complex system modeling and effectiveness evaluation, aviation weapon firepower control technology.
E-mail: wenhao_bi@hotmail.com

An Zhang was born in 1962. He received his Ph.D. degree from Northwestern Polytechnical University in 1999. His research interests are complex system modeling and effectiveness evaluation, aviation weapon firepower control technology, intelligent command and control engineering, advanced aircraft aviation electronic integration technology platform.
E-mail: zhangan@nwpu.edu.cn

Yuan Yuan was born in 1990. She is a graduate student in Northwestern Polytechnical University. Her research interests are system engineering, intelligent system modeling and simulation, system analysis and performance evaluation of aircraft cockpit.
E-mail: yuanyuannpu@163.com