Adaptive partition intuitionistic fuzzy time series forecasting model

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Abstract: To enhance the accuracy of intuitionistic fuzzy time series forecasting model, this paper analyses the influence of universe of discourse partition and compares with relevant literature. Traditional models usually partition the global universe of discourse, which is not appropriate for all objectives. For example, the universe of the secular trend model is continuously variational. In addition, most forecasting methods rely on prior information, i.e., fuzzy relationship groups (FRG). Numerous relationship groups lead to the explosive growth of relationship library in a linear model and increase the computational complexity. To overcome problems above and ascertain an appropriate order, an intuitionistic fuzzy time series forecasting model based on order decision and adaptive partition algorithm is proposed. By forecasting the vector operator matrix, the proposed model can adjust partitions and intervals adaptively. The proposed model is tested on student enrollments of Alabama dataset, typical seasonal dataset Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) and a secular trend dataset of total retail sales for social consumer goods in China. Experimental results illustrate the validity and applicability of the proposed method for different patterns of dataset.

Keywords: intuitionistic fuzzy set, time series, forecasting, vector operator matrix, order deciding, adaptive partition.

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1. Introduction

Time series analysis is an availability method for prediction. In reality, conventional methods cannot deal with uncertain information such as linguistic variable. Song et al. proposed a fuzzy time series (FTS) forecasting model by combining the fuzzy set theory with time series analysis [1,2]. Chen et al. proposed a simplified arithmetic replaced max-min composition operations, which improves the operation efficiency [3]. Hwang et al. presented a successful FTS prediction model to forecast the enrollments of the University of Alabama [4]. In [5], Huarng et al. proposed

a method by adding a heuristic function to improve the previous model. The application of neural networks and particle swarm optimization (PSO) in FTS prediction were discussed in [6,7], respectively, both models were used to forecast Taiwan Futures Exchange (TAIFEX). Song analyzed the linear FTS by defining fuzzy plus and minus operators, and proposed a forecasting model based on autocorrelation functions [8]. Cheng et al. presented a fuzzy time series forecasting method based on entropy and trapezoid fuzzification [9]. Considering about the influence of multiple attribute on input, a multivariate stochastic fuzzy model [10] and an adaptive expectation forecasting model were proposed [11].

For reforming the forecasting accuracy, several literature researched the high-order fuzzy time series models instead of the first-order model. In [12], Chen constructed a high-order fuzzy time series model, then Own et al. proposed another heuristic high-order method to improve it [13]. Lee et al. considered more factors for prediction and distinguished the main factor from the second factor, then proposed a two-factor high-order FTS model [14]. Then several different high-order FTS forecasting models were presented [15–19]. On the other hand, it is shown that the intervals length of universe of discourse influences the prediction accuracy greatly. Fuzzy clustering is the most popular intervals partition method [17,18,20–23]. PSO [24,25], tabu search [26], information granules [27,28], and ant colony optimization [29] have been applied to seek appropriate intervals. In general, the typical time series patterns can be sorted as secular trend, seasonal variation trend, cyclical variation trend and irregular variation trend [30]. Unlike the stationary of trend time series pattern, Liu et al. proposed an improved forecasting model focusing on forecasting seasonal time series [31], which demonstrated that the universe of discourse partition and relationship generated different performance in different patterns. Li et al. presented a deterministic forecasting model based on the fuzzy *c*-means (FCM) clustering method to address

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the two-factor forecasting problem [32]. Liu proposed an incorporate clustering method and introduced the α factor method to adjust forecasting output value [11]. In [33], a hierarchical clustering algorithm was presented to deal with non-linear time series. In a word, the order deciding and the universe of discourse partition are two important aspects influencing the forecasting accuracy.

Taking a close examination on existing methods, we can conclude that more accurate partition can lead to better forecasting results. No matter what equal partition or clustering algorithm was adopted, these models need to discuss the whole universe of discourse [3,4,11,12,16,18], and the computational complexity will grow along with the scale of data. Usually, these methods are applied to small scale, stable range or seasonal variation model, such as enrollments and temperature of weather data, which illuminates that the applicability and generality of these models are limited. As an extension of fuzzy sets, the intuitionistic fuzzy sets theory proposed by Atanassov can depict the fuzzy object more clearly [34]. In [35], Castillo et al. introduced the intuitionistic fuzzy reasoning to time series analysis. After that, some intuitionistic fuzzy time series (IFTS) methods were successively proposed [36,37]. Zheng et al. constructed a kind of IFTS models [38] to handle long-term forecasting problem [39]. Above researches indicate that the fusion of intuitionistic fuzzy sets and time series prediction is a new development trend. To address the difficulty in optimal order number determination and the limitation of partition in existent models, in this paper, we attempt to propose a novel IFTS forecasting model, with the purpose of increasing the forecasting accuracy and improving the model adaptability. The proposed model offers an intuitionistic fuzzification method of historical data and the appropriate order number determination algorithms. Meanwhile the proposed IFTS model based on adaptive partition is able to deal with conventional FTS problems and overcome the limitation of the universe of discourse. Our model will enhance the generalization ability of FTS forecasting.

The rest of this paper is organized as follows. In Section 2, some definitions of intuitionistic FTS and related work are given. Then, the order deciding and adaptive partition algorithms are presented particularly. In Section 3, an integrated adaptive partition IFTS forecasting model is constructed and compared with other methods by being applied to enrollments of Alabama. In Section 4, the proposed model is implemented on two different dataset patterns, seasonal pattern, TAIEX and secular trend pattern, the monthly total retail sales of social consumer goods (TRSSCG). Finally, some conclusions are drawn in Section 5.

2. IFTS and algorithms

2.1 IFTS definitions

In [1,2], the FTS and basic theory have been proposed. Here, some definitions are introduced briefly.

Definition 1 Assume A is a fuzzy set defined in universe of discourse U , which is represented as

$$A = f(\mu_1)/A_1 + f(\mu_2)/A_2 + \cdots + f(\mu_n)/A_n \quad (1)$$

where $f(\mu_i)$ is the membership function of A , $f(\mu_i) \rightarrow [0, 1]$ ($1 \leq i \leq n$) is the connection sign.

Definition 2 Let $\{Y(t), t = 0, 1, \dots, n\}$, a subset of $\mathbf{R}$, be the universe of discourse, on which the fuzzy set $f_i(t)$ ($i = 1, 2, 3, \dots, n; f_i(t) \in [0, 1]$) is defined and $F(t)$ is a collection of $f_1(t), f_2(t), \dots, f_n(t)$, where $F(t)$ can be denoted as the linguistic variable, then $F(t)$ is called a fuzzy time series defined on $Y(t)$.

Definition 3 Assume that an FTS $F(t)$ is decided by $F(t-1)$ only, then $F(t) = F(t-1) \circ \mathbf{R}(t, t-1)$, is a fuzzy synthetic operator, which is called a first-order model of $F(t)$, where $\mathbf{R}(t, t-1)$ is the fuzzy relationship matrix. If for any t , $\mathbf{R}(t, t-1) = \mathbf{R}(t-1, t-2)$, $F(t) = F(t-1) \circ \mathbf{R}$ is called a time-invariant fuzzy series, otherwise it is a time-variant fuzzy series.

Definition 4 Assume that an FTS $F(t)$ is decided by $F(t-1), F(t-2), \dots, F(t-m)$,

$$F(t) = F(t-1) \times F(t-2) \times \cdots \times F(t-m) \circ \mathbf{R}(t, t-m) \quad (2)$$

where $F(t)$ is called an m -order fuzzy time series model, $\mathbf{R}(t, t-m)$ is an m -order composition fuzzy relationship, $\times$ is the Cartesian product.

Similarly, based on the theories of FTS, some definitions of intuitionistic fuzzy time series are presented particularly as follows:

Definition 5 Let $\{Y(t), t = 0, 1, \dots, n\}$ be a time series on universe of discourse U , and A be a division set of U , namely $\{A_i, i = 1, 2, \dots, k; \bigcup_{i=1}^k A_i = U\}$, where A_i is the language variable. If a time series $F(t)$ composed by $\{A_i\}$ has a pair of membership and non-membership function $\mu_i(Y(t)), \gamma_i(Y(t))$, $F_I(t)$ is marked as an intuitionistic fuzzy time series on $Y(t)$ and expressed by the following expression.

$$\begin{aligned} F_I(t) = & \langle \mu_1(Y(t)), \gamma_1(Y(t)) \rangle / A_1 + \\ & \langle \mu_2(Y(t)), \gamma_2(Y(t)) \rangle / A_2 + \cdots + \\ & \langle \mu_n(Y(t)), \gamma_n(Y(t)) \rangle / A_n \end{aligned} \quad (3)$$

where $f_i(t) = \langle \mu_i(Y(t)), \gamma_i(Y(t)) \rangle / L_i$ denotes the membership and non-membership functions of IFS $Y(t)$ with respect to A_i .

Definition 6 Assume that an IFTS $F_I(t)$ is merely decided by $F_I(t-1)$, then $F_I(t) = F_I(t-1) \circ \mathbf{R}_I(t, t-1)$ is called a first-order IFTS, where $\mathbf{R}_I(t, t-1)$ is the intuitionistic fuzzy relationship (IFR) matrix with $\mathbf{R}_{ij} = \langle \mathbf{R}(\mu_{ij}), \mathbf{R}(\gamma_{ij}) \rangle = F_I^T(t) \circ F_I(t-1) = [R_{ij}]_{r \times r}$. If the membership and non-membership functions of $F_I(t)$ and $F_I(t-1)$ are $\langle \mu_{1i}, \gamma_{1i} \rangle$ and $\langle \mu_{2i}, \gamma_{2i} \rangle$, respectively, $\mathbf{R}(\mu_{ij})$ and $\mathbf{R}(\gamma_{ij})$ can be calculated by the following expression.

$$\mathbf{R}(\mu_{ij}) = \bigvee_{k=1}^r (\mu_{1ik} \wedge \mu_{2ik}), \mathbf{R}(\gamma_{ij}) = \bigwedge_{k=1}^r (\gamma_{1ik} \vee \gamma_{2ik}). \quad (4)$$

For any t , if $\mathbf{R}_I(t, t-1) = \mathbf{R}_I(t-1, t-2)$, then $F_I(t) = F_I(t-1) \circ \mathbf{R}_I$ is called a time-invariant intuitionistic fuzzy series; otherwise, it is called a time-variant intuitionistic fuzzy series.

Definition 7 Assume that an IFTS $F_I(t)$ is decided by $F_I(t-1), F_I(t-2), \dots, F_I(t-m)$, we can get a high-order intuitionistic fuzzy time series relationship expression as

follows:

$$F_I(t) = F_I(t-1) \times F_I(t-2) \times \dots \times F_I(t-m) \circ \mathbf{R}_I(t, t-m) \quad (5)$$

where $\mathbf{R}_I(t, t-m)$ is an m -order composition fuzzy relationship. Furthermore, the intuitionistic fuzzy relationship can be obtained as

$$\mathbf{R}_I(t, t-m) = f_I(t) \times f_I(t-1) \cup f_I(t-1) \times f_I(t-2) \cup \dots \cup f_I(t-m+1) \times f_I(t-m). \quad (6)$$

In [1–5], some FTS forecasting models have been constructed. In general, an FTS forecasting model includes several aspects: (i) the universe of discourse partition; (ii) historical time series data fuzzification; (iii) established fuzzy relationship; (iv) forecasting and defuzzification. According to prototype, a novel adaptive IFTS forecasting model is presented in this paper and the framework is shown in Fig. 1. The proposed model includes two parts, data training and data forecasting. The number m is decided by the order deciding algorithm and the universe of discourse is divided by the adaptive partition algorithm.

Fig. 1 Framework of the adaptive partition IFTS model

2.2 Order deciding algorithm

In the data training part, the m -order deciding algorithm is used to decide an appropriate order. Detailed procedure is presented stepwise as follows and the flow chart is presented in Fig. 2.

Step 1 To compare with other methods clearly, we use equal intervals as the initial partition. Define the universe of discourse $U = [D_{\min} - D_1, D_{\max} - D_2]$, where $D_{\min}$

and $D_{\max}$ are minimum and maximum of historical data. In order to highlight fuzzy attribute of U , which is divided into n overlap parts, $U = u_1 \cup u_2 \cup \dots \cup u_n$, the midpoint of current interval is regarded as the start point of next interval.

Step 2 Let A_i represent the linguistic variable, then, $A_i = \sum_{j=1}^n \langle \mu_{ij}, \gamma_{ij} \rangle / u_j$, where μ_{ij} and γ_{ij} are mem-

bership and non-membership of A_i belong to u_i . The μ_{ij} and γ_{ij} can be calculated as

$$\begin{aligned} &< \mu_{ij}, \gamma_{ij} > = \\ &\begin{cases} < 1 - \left| \frac{2d_i - (A+B)}{\lambda(B-A)} \right|, \left| \frac{2d_i - (A+B)}{B-A} \right| >, \\ &\quad d_i \in u_j \\ < 0, 1 >, \quad \text{otherwise} \end{cases} \quad (7) \end{aligned}$$

where A and B are boundaries of interval, d_i is the historical data, λ is an adjustment factor of hesitancy degree.

Step 3 The linguistic fuzzification result based on interval which the data fall into, and then, intuitionistic fuzzy logical relationships (IFLR) are obtained. We can get a relationship $A_i, A_j \rightarrow A_{i+1}, A_{j+1}$, here, a pair of results A_i and A_j is called the current state of intuitionistic fuzzy logical relationship and A_{i+1} and A_{j+1} is called the next state of intuitionistic fuzzy logical relationship.

Step 4 m (initial $m = 1$) intuitionistic fuzzy relationship pairs are obtained, namely, $A_{t-m} \rightarrow A_{t-m+1}, A_{t-m+1} \rightarrow A_{t-m+2}, \dots, A_{t-1} \rightarrow A_t$. According to (4), we can calculate $R_k = < R(\mu_{ij}), R(\gamma_{ij}) >$ ($k = 1, 2, \dots, m$), respectively. Finally, the intuitionistic fuzzy relationship is calculated by

$$R_I = < \bigcup_{k=1}^m R_k(\mu_{ij}), \bigcap_{k=1}^m R_k(\gamma_{ij}) >. \quad (8)$$

Step 5 These IFLRs are gathered to groups. If there is one current state corresponding to several next states, the IFLRs are regarded as one group. When forecasting, the next state is decided by the mean current relationship in intuitionistic fuzzy logical relationship groups (IFLRG). For example, if the current intuitionistic fuzzy logical relationship group is $< A_4 A_5 \rightarrow A_3 A_4, A_4 A_5 \rightarrow A_4 A_5, A_4 A_5 \rightarrow A_5 A_6 >$, the next state is forecasted as $A_4 A_5$.

Step 6 Forecasted output $F_I(t+1)$ is calculated based on (5). The nonzero $< \mu_i, \gamma_i > / u_i$ belonging to the relevant intuitionistic fuzzy relationship is chosen according to the triangular membership function and the precise output value is calculated by the following formula.

$$\begin{aligned} &def(F(t+1)) = \\ &\frac{1}{n} \sum_{i=1}^n \left(B_i - \frac{B_i - A_i}{4} (1 + \mu_i - \gamma_i) \right). \quad (9) \end{aligned}$$

Based on the defuzzification method, the outputs rely on nearby two intervals, here $n = 2$.

Step 7 The intuitionistic fuzzy distance square error (IFDSE) $d(\mu(m), \gamma(m))$ is calculated by (10).

$$d(\mu(m), \gamma(m)) =$$

$$\frac{1}{l} \sum_{i=1}^l \sum_{j=1}^n [(\mu_{ij} - \mu'_{ij})^2 + (\gamma_{ij} - \gamma'_{ij})^2] \quad (10)$$

where μ_{ij} and γ_{ij} are calculated by (7), μ'_{ij} and γ'_{ij} are calculated by (5). The former pair of membership and non-membership functions is actual values and the later pair is forecasted values. l is the capacity of training data. Update $m = m + 1$, according to (11), we choose m from the minimum of intuitionistic fuzzy mean distance square error or the variety between $d(m)$ and $d(m+1)$ which is small enough, where ε is permissible error.

$$\begin{cases} d(\mu(m-1), \gamma(m-1)) \leq d(\mu(m), \gamma(m)) \leq \\ d(\mu(m+1), \gamma(m+1)) \\ \text{or} \\ d(\mu(m), \gamma(m)) - d(\mu(m+1), \gamma(m+1)) \leq \varepsilon \end{cases} \quad (11)$$

Step 8 Output an appropriate value m .

Fig. 2 Flow of the m -order deciding algorithm

2.3 Adaptive partition algorithm

During the procedure of IFTS model construction, the establishment of fuzzy relationship is crucial to the accuracy of forecasting. In [1,2], the current fuzzy relationship was synthesized by previous m relationships and regarded as next relationship for prediction. The virtue of this method is convenient, but the precision of forecasting is not high. In [3,11,32,36], the presented methods improved the accuracy of forecasting by the fuzzy relationship obtained from fuzzy relationship groups. However, such relationship groups are prior information, then the forecasting error will be very large when the next state does not appear in groups before. The applicability of these methods is limited. Furthermore, type of dataset impacts universe of discourse partition. For example, the global universe of a linear time series dataset is divided into parts that may cause explosive growth of fuzzy relationship library. The accuracy is positively correlated to the computational complexity. In this situation, constructing relationship groups are not appropriate.

To overcome these shortages, we propose a new method to calculate the intuitionistic fuzzy relationship in a forecasting model based on vector operator matrix $O(t)$.

Definition 8 Assume that an IFTS $F_I(t)$ is decided by $F_I(t-1), F_I(t-2), \dots, F_I(t-m), R_I(t, t-m)$ is calculated based on (4) and m intuitionistic fuzzy logic relationships $A_i \rightarrow A_j$ are available, then we have

$$F_I(t+1) = F_I(t) \times R_I(t-m) \times O(t) \quad (12)$$

$$O(t) = \begin{cases} D^r, & d > 0 \\ E, & d = 0 \\ D^{T(-r)}, & d < 0 \end{cases} \quad (13)$$

$$D = [a_{k,k+1} = 1]_{p \times p}, \quad k = 1, 2, \dots, p-1 \quad (14)$$

$$r = \left\lceil \frac{1}{m} \sum_{t=1}^m i - j \right\rceil \quad (15)$$

where D is a direction vector matrix, E is the unit matrix, r is the forecasting index and p is the partition value.

For example, assume that there is a 3-order IFTS forecasting model, of which the partition value is five and the fuzzy relationships $\langle A_1 \rightarrow A_3, A_3 \rightarrow A_3, A_3 \rightarrow A_4 \rangle$ are available, then, we can calculate $r=1$,

$$O(t) = D^1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

For illustration, assume $A_4 = 0/u_1 + 0/u_2 + 0.5/u_3 + 1/u_4 + 0/u_5$, $R_I(t-m) = E$, finally, $F_I(t+1) = A_5 = 0/u_1 + 0/u_2 + 0/u_3 + 0.5/u_4 + 1/u_5$ is forecasted based on (5).

In this section, the forecasting part is constructed and the adaptive partition algorithm is shown as follows, and the flow chart is presented in Fig. 3.

Step 1 Set the initial partition p . The initial universe of discourse U is divided into p parts with equal interval ΔA , namely, $U = A_1 \cup A_2 \cup \dots \cup A_p$, then the midpoint of current interval is set as the start point of next interval.

Step 2 Calculate the linguistic fuzzification results by the interval which data drop into. μ_{ij} and γ_{ij} can be calculated by the same Step 2 of the order deciding algorithm.

Step 3 Get the relationships $A_i, A_j \rightarrow A_{i+1}, A_{j+1}$ by historical time series data, calculate r and $O(t)$ according to (15) and (13), respectively. The relationship matrix R_μ and R_γ are calculated by (6).

Step 4 The IFTS forecasting result $F(t+1)$ is calculated based on (12). Forecast fuzzy output $A_i, A_j \rightarrow A_{i+r}, A_{j+r}$ and identify the actual result by the fuzzification. The defuzzification value is calculated based on (9). Here,

choose the membership and non-membership functions of $\langle A_{i+r}, A_{j+r} \rangle$ and $n = 2$.

Step 5 Update partition ΔA and limit by trend of forecasting result. When $r > 0$, increase the upper limit and lower limit of universe of discourse, in contrast, when $r < 0$, decrease the upper limit and lower limit simultaneously. Keep it unchanged when $r = 0$. If the fuzzification results belong to the first or the last interval, we should set $p = p + 1$, which indicates that the universe of discourse is not larger enough. If the scale of universe changes drastically, set $\Delta A = 2^n \Delta A$ or $\Delta A = (1/2)^n \Delta A$, here, $n = 1, 2, \dots$

Step 6 Jump to Step 4 and calculate the rest fuzzification and defuzzification values.

Fig. 3 Flow of the adaptive partition algorithm

There are two cases of defuzzification. Taking membership degree function as an example, in Case 1, the fuzzy output F only locates in the first or the last interval, such as F_1 and F_n in Fig. 4, and we utilize one membership defuzzification. In Case 2, the fuzzy output F_2 locates in overlap intervals, and we utilize the mean of two precise values calculated by $\langle \mu_i, \gamma_i \rangle$ respectively as the final output value.

Fig. 4 Cases of defuzzification

3. Adaptive partition IFTS forecasting model

In this section, the adaptive partition IFTS forecasting model is presented by implementing on widely used data, student enrollments of Alabama from 1971 to 1992 particular. The effect of the m -order deciding and adaptive partition algorithm is validated.

3.1 Data training

The enrollments from 1971 to 1992 are shown in Table 1. In data training part, we utilize the former 12 data as the training specimen to decide number m , others as testing data for testifying validity of the order deciding algorithm.

Table 1 Specimen of enrollments

Year	Value	Fuzzification	Year	Value	Fuzzification
1971	13 055	$A_1, /$	1982	15 433	A_4, A_5
1972	13 563	A_1, A_2	1983	15 497	A_4, A_5
1973	13 867	A_1, A_2	1984	15 145	A_4, A_5
1974	14 696	A_3, A_4	1985	15 163	A_4, A_5
1975	15 460	A_4, A_5	1986	15 984	A_5, A_6
1976	15 311	A_3, A_4	1987	16 859	A_7, A_8
1977	15 603	A_5, A_6	1988	18 150	A_{10}, A_{11}
1978	15 861	A_5, A_6	1989	18 970	A_{11}, A_{12}
1979	16 807	A_7, A_8	1990	19 328	A_{12}, A_{13}
1980	16 919	A_7, A_8	1991	19 337	A_{12}, A_{13}
1981	16 383	A_6, A_7	1992	18 876	A_{11}, A_{12}

Firstly, define the universe of discourse $U = [13\ 000, 20\ 000]$ and partition it as follows: $A_1 = [13\ 000, 14\ 000]$, $A_2 = [13\ 500, 14\ 500]$, $A_3 = [14\ 000, 15\ 000]$, $A_4 = [14\ 500, 15\ 500]$, $A_5 = [15\ 000, 16\ 000]$, $A_6 = [15\ 500, 16\ 500]$, $A_7 = [16\ 000, 17\ 000]$, $A_8 = [16\ 500, 17\ 500]$, $A_9 = [17\ 000, 18\ 000]$, $A_{10} = [17\ 500, 18\ 500]$, $A_{11} = [18\ 000, 19\ 000]$, $A_{12} = [18\ 500, 19\ 500]$, $A_{13} = [19\ 000, 20\ 000]$. The linguistic fuzzification result based on interval which enrollments fall into and intuitionistic fuzzy logical relationships are obtained shown in Table 1.

Between 1973 and 1974, we can get a relationship $A_1, A_2 \rightarrow A_3, A_4$, and cluster these fuzzy logical relationships to groups, if there is one current state corresponding to several next states, regard them as one group. For example, the current intuitionistic fuzzy logical relationship group is $\langle A_4 A_5 \rightarrow A_3 A_4, A_4 A_5 \rightarrow A_4 A_5, A_4 A_5 \rightarrow A_5 A_6 \rangle$, and the next state is forecasted $A_4 A_5$. The logical relationships of enrollments and the logical relationship groups are shown in Table 2 and Table 3.

Table 2 Intuitionistic fuzzy logical relationships of enrollments

Relationship groups	
$A_1 \rightarrow A_1, A_2$	$A_1, A_2 \rightarrow A_1, A_2$
$A_1, A_2 \rightarrow A_3, A_4$	$A_3, A_4 \rightarrow A_4, A_5$
$A_4, A_5 \rightarrow A_3, A_4$	$A_3, A_4 \rightarrow A_5, A_6$
$A_5, A_6 \rightarrow A_5, A_6$	$A_5, A_6 \rightarrow A_7, A_8$
$A_7, A_8 \rightarrow A_7, A_8$	$A_7, A_8 \rightarrow A_6, A_7$
$A_6, A_7 \rightarrow A_4, A_5$	$A_4, A_5 \rightarrow A_4, A_5$
$A_4, A_5 \rightarrow A_4, A_5$	$A_4, A_5 \rightarrow A_4, A_5$
$A_4, A_5 \rightarrow A_4, A_5$	$A_4, A_5 \rightarrow A_5, A_6$
$A_5, A_6 \rightarrow A_7, A_8$	$A_7, A_8 \rightarrow A_{10}, A_{11}$
$A_{10}, A_{11} \rightarrow A_{11}, A_{12}$	$A_{11}, A_{12} \rightarrow A_{12}, A_{13}$
$A_{12}, A_{13} \rightarrow A_{11}, A_{12}$	

Table 3 Intuitionistic fuzzy logical relationship groups

Relationship groups		
$A_1 \rightarrow A_1, A_2$		
$A_1, A_2 \rightarrow A_1, A_2$	$A_1, A_2 \rightarrow A_3, A_4$	
$A_3, A_4 \rightarrow A_4, A_5$	$A_3, A_4 \rightarrow A_5, A_6$	
$A_4, A_5 \rightarrow A_3, A_4$	$A_4, A_5 \rightarrow A_4, A_5$	$A_4, A_5 \rightarrow A_5, A_6$
$A_5, A_6 \rightarrow A_5, A_6$	$A_5, A_6 \rightarrow A_7, A_8$	
$A_6, A_7 \rightarrow A_4, A_5$		
$A_7, A_8 \rightarrow A_7, A_8$	$A_7, A_8 \rightarrow A_6, A_7$	$A_7, A_8 \rightarrow A_{10}, A_{11}$
$A_{10}, A_{11} \rightarrow A_{11}, A_{12}$		
$A_{11}, A_{12} \rightarrow A_{12}, A_{13}$		
$A_{12}, A_{13} \rightarrow A_{11}, A_{12}$		

Set $m = 1$, the IFDSE $d(\mu(m), \gamma(m))$ can be calculated by (10), update $m = m + 1$ and the results are shown in Fig. 5. Then the mean square error $d(m)$ are calculated as follows: $d(1) = 0.447\ 5$, $d(2) = 0.297\ 6$, $d(3) = 0.247\ 7$, $d(4) = 0.357\ 0$.

Fig. 5 Intuitionistic fuzzy distance square error

Finally, an appropriate order value $m = 3$ based on (11) is obtained and the prediction accuracy is improved by choosing this appropriate order seen from Fig. 5. By the way, there exists a phenomenon that numerous identical R_{ij} appears in the row vectors R_I when $m \geq 5$ in experiments. We deem that it is an ineffective intuitionistic fuzzy relationship and draw a conclusion that over high order may invalidate R_I . It is another reason why we need to ascertain an appropriate order.

3.2 Adaptive partition IFTS forecasting

Here, an adaptive IFTS forecasting model is stepwise constructed. Because $m = 3$ is decided in Section 3.1, the former four data as an example of the 3-order model is illustrated by the forecasting procedure.

(i) The universe of discourse initial partition

Firstly, set $p = 6$, $\Delta A = 1\ 000$, and initialize equal partition as follows: $A_1 = [12\ 500, 13\ 500]$,

$A_2 = [13\ 000, 14\ 000]$, $A_3 = [13\ 500, 14\ 500]$, $A_4 = [14\ 000, 15\ 000]$, $A_5 = [14\ 500, 15\ 500]$ and $A_6 = [15\ 000, 16\ 000]$. Two linguistic fuzzification results are calculated by the interval where the enrollments drop and shown in column 3 of Table 4.

Table 4 Enrollments fuzzification

Year	Value	Fuzzification	Membership and non-membership
1971	13 055	A_1, A_2	$< 0.884\ 2, 0.110\ 0 > /u_1 +$ $< 0.063\ 2, 0.890\ 0 > /u_2$
1972	13 563	A_2, A_3	$< 0.867\ 4, 0.126\ 0 > /u_2 +$ $< 0.080\ 0, 0.874\ 0 > /u_3$
1973	13 867	A_2, A_3	$< 0.227\ 4, 0.734\ 0 > /u_2 +$ $< 0.720\ 0, 0.266\ 0 > /u_3$
1974	14 696	A_4, A_5	$< 0.587\ 4, 0.392\ 0 > /u_4 +$ $< 0.360\ 0, 0.608\ 0 > /u_5$

(ii) Historical time series data intuitionistic fuzzification

Secondly, based on Step 2 of the adaptive partition algorithm and (7), set $\lambda = 0.95$, then the former four data of the membership and non-membership functions are calculated and $A_i = < 0, 1 > /u_j$ are omitted, as shown in column 4 in Table 4.

(iii) Intuitionistic fuzzy relationship and vector operator matrix calculation

Then, three intuitionistic fuzzy relationships are obtained, namely, $< A_1, A_2 \rightarrow A_2 A_3 >$, $< A_2 A_3 \rightarrow A_2 A_3 >$ and $< A_2 A_3 \rightarrow A_4 A_5 >$. According to (15) and (13), $r = 1$ and $O(t) = D$ are computed, respectively. The relationship matrix R_μ and R_γ are calculated by (6).

$$R_\mu =$$

$$\begin{bmatrix} 0.884\ 2 & 0.884\ 2 & 0.884\ 2 & 0.884\ 2 & 0.884\ 2 & 0.884\ 2 \\ 0.867\ 4 & 0.867\ 4 & 0.867\ 4 & 0.867\ 4 & 0.867\ 4 & 0.867\ 4 \\ 0.720\ 0 & 0.867\ 4 & 0.720\ 0 & 0.720\ 0 & 0.720\ 0 & 0.720\ 0 \\ 0 & 0.867\ 4 & 0.720\ 0 & 0.587\ 4 & 0.360\ 0 & 0 \\ 0 & 0.867\ 4 & 0.720\ 0 & 0.587\ 4 & 0.360\ 0 & 0 \\ 0 & 0.867\ 4 & 0.720\ 0 & 0.587\ 4 & 0.360\ 0 & 0 \end{bmatrix}$$

$$R_\gamma =$$

$$\begin{bmatrix} 0.110\ 0 & 0.110\ 0 & 0.110\ 0 & 0.110\ 0 & 0.110\ 0 & 0.110\ 0 \\ 0.126\ 0 & 0.126\ 0 & 0.126\ 0 & 0.126\ 0 & 0.126\ 0 & 0.126\ 0 \\ 0.266\ 0 & 0.126\ 0 & 0.266\ 0 & 0.266\ 0 & 0.266\ 0 & 0.266\ 0 \\ 1 & 0.126\ 0 & 0.266\ 0 & 0.392\ 0 & 0.608\ 0 & 1 \\ 1 & 0.126\ 0 & 0.266\ 0 & 0.392\ 0 & 0.608\ 0 & 1 \\ 1 & 0.126\ 0 & 0.266\ 0 & 0.392\ 0 & 0.608\ 0 & 1 \end{bmatrix}.$$

(iv) Forecasting and defuzzification

In succession, $F(t+1)$ is calculated based on (12), forecast fuzzy output $A_i, A_j \rightarrow A_{i+r}, A_{j+r}$. The fuzzification of next enrollments is $< A_5, A_6 >$. The defuzzification value 15 507 is calculated based on (9). Here, choose the membership and non-membership functions of $< A_5, A_6 >$ and $l = 2$.

(v) Universe of discourse partition updating

The scale of interval is unchanged as the forecasting value is not overflow. The new universe of discourse partitions A_i are as follows: $A_1 = [13\ 000, 14\ 000]$, $A_2 = [13\ 500, 14\ 500]$, $A_3 = [14\ 000, 15\ 000]$, $A_4 = [14\ 500, 15\ 500]$, $A_5 = [15\ 000, 16\ 000]$, $A_6 = [15\ 500, 16\ 500]$, $A_7 = [16\ 000, 17\ 000]$. Finally, jump to Step 4 of the adaptive partition algorithm and calculate the rest fuzzification and defuzzification values shown in Table 5, where the numbers in blank denote the update times.

Table 5 Fuzzification forecasting of enrollments

Year	Enrollment	Fuzzification	Forecasting
1975	15 460	(0) A_5, A_6	(0) A_5, A_6
1976	15 311	(1) A_4, A_5	(1) A_5, A_6
1977	15 603	(1) A_5, A_6	(1) A_5, A_6
1978	15 861	(1) A_5, A_6	(1) A_5, A_6
1979	16 807	(2) A_5, A_6	(2) A_4, A_5
1980	16 919	(2) A_5, A_6	(2) A_6, A_7
1981	16 383	(2) A_4, A_5	(2) A_4, A_5
1982	15 433	(2) A_2, A_3	(2) A_3, A_4
1983	15 497	(2) A_2, A_3	(2) A_2, A_3
1984	15 145	(2) A_2, A_3	(2) A_2, A_3
1985	15 163	(2) A_2, A_3	(2) A_2, A_3
1986	15 984	(2) A_3, A_4	(2) A_2, A_3
1987	16 859	(2) A_5, A_6	(2) A_4, A_5
1988	18 150	(3) A_6, A_7	(3) A_6, A_7
1989	18 970	(4) A_5, A_6	(4) A_4, A_5
1990	19 328	(4) A_6, A_7	(4) A_6, A_7
1991	19 337	(4) A_6, A_7	(4) A_6, A_7
1992	18 876	(4) A_5, A_6	(4) A_6, A_7

3.3 Empirical analysis

Here, we compare forecasting results with those models in [1,3,5,37,40], as shown in Fig. 6. The comparison of square error among different models are shown in Fig. 7 and Fig. 8. These figures show that the model proposed in this paper performs better than those models in [1,3,40]. The mean square error (MSE) and average forecasting error rate (AFER) are calculated respectively in Table 6, which indicates the forecasting mean error of our method is lower or almost equal with other methods. It needs to be emphasized that the space complexity may decrease in the adaptive partition model. This experiment will be carried out in next section. Definitions of MSE, the root of mean squared error (RMSE) and AFER are shown as follows, where fv_i is the forecasted value, av_i is the actual value, and fe_i is the forecasted error.

$$\text{MSE} = \frac{\sum_{i=1}^n (fv_i - av_i)^2}{n} \quad (16)$$

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^n (fv_i - av_i)^2}{n}} \quad (17)$$

$$fe = \left| \frac{av - fv}{av} \right| \quad (18)$$

$$\text{AFER} = \frac{\sum fe}{n} \times 100\%. \quad (19)$$

In the same example, the universe was divided into seven parts [3]. The operation matrix was simple, but the forecasting accuracy was unsatisfied. The model in [36] divided the universe into 14 parts, which improved the forecasting accuracy by increasing space costs. However, the partitions of our model are less than eight and updated four times, which improves complexity and accuracy simultaneously.

Fig. 6 Forecasting result and comparison

Fig. 7 Comparison of square error 1

Fig. 8 Comparison of square error 2

Table 6 Comparison of MSE and AFER

Model	Proposed model	Joshi and Kumar [36]	Chen [3]	Huarng [5]	Lee and Chou [40]	Song and Chissom [1]
MSE	103 950	175 559	439 421	239 483	240 047	458 438
AFER/%	1.580 00	2.069 264	2.482 540	3.21 735	2.49 977	3.349 670

4. Forecasting experiments with different patterns

In former section, the proposed IFTS forecasting model is constructed and applied on enrollments of Alabama University and the empirical analysis shows that our model outperforms other models. Here, we demonstrate the generalization ability of the proposed model by different data patterns.

4.1 Forecasting seasonal data pattern

Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) (<http://www.twse.com.tw>) is a typical seasonal pattern, which is widely used in FTS forecasting [6,11,19,29,41,42]. In order to compare with these existing methods, a 4-order adaptive IFTS forecasting model is constructed in training part, where the initial partition is set as $p = 8$, $\Delta A = 100$. In above literature, only a

part of forecasting results are presented, so the comparison results with methods in [6] and [3] from 2000/11/2 to 2000/12/30 are shown in columns 1–5 of Table 7, and the comparison results with methods in [41] and [19] from 2004/11/1 to 2004/12/31 are shown in columns 6–10. As shown in Table 7, the index RMSE of our model is smaller than those methods in [3], [6], [41], and almost equal with [19]. Moreover, we apply the proposed model to forecast

the TAIEX from 1990 to 1999 and compare it with methods proposed in [41,42,29]. The RMSE of the every year are presented in Table 8 and the smallest value obtained by different models is highlighted in bold. Table 8 shows that 10 years average RMSE of the proposed model is lower than that of [41,42,29]. In conclusion, the proposed IFTS model outperforms methods above in such seasonal pattern.

Table 7 Parts of TAIEX forecasting result

Date	Actual model	Hybrid [6]	Chen [3]	Proposed model	Date	Actual model	Chen [41]	Chen [19]	Proposed model
2000/11/2	5 626	5 550	5 300	5 524.34	2004/11/1	5 656.17	5 718.09	-	-
2000/11/3	5 796	5 650	5 750	5 742.54	2004/11/2	5 759.61	5 658.97	-	-
2000/11/4	5 677	5 750	5 450	5 514.26	2004/11/3	5 862.85	5 754.24	5 756.8	-
2000/11/6	5 657	5 650	5 750	5 658.21	2004/11/4	5 860.73	5 863.09	5 865.05	-
2000/11/7	5 877	5 650	5 750	5 865.27	2004/11/5	5 931.31	5 868.04	5 868.44	5 897.32
2000/11/8	6 067	5 850	5 750	5 924.15	2004/11/8	5 937.46	5 927.67	5 935.51	5 932.14
2000/11/9	6 089	5 950	6 075	5 994.35	2004/11/9	5 945.2	5 942.9	5 932.96	5 935.98
2000/11/10	6 088	5 950	6 075	6 035.24	2004/11/10	5 948.49	5 942.85	5 940.7	5 948.36
2000/11/13	5 793	5 950	6 075	5 835.12	2004/11/11	5 874.52	5 952.69	5 945.72	5 936.32
2000/11/14	5 772	5 750	5 450	5 678.65	2004/11/12	5 917.16	5 882.56	5 873.1	5 914.05
2000/11/15	5 737	5 750	5 450	5 724.04	2004/11/15	5 906.69	5 931.63	5 919.36	5 901.52
2000/11/16	5 454	5 750	5 450	5 514.83	2004/11/16	5 910.85	5 908.03	5 905.27	5 896.91
2000/11/17	5 351	5 550	5 300	5 451.29	2004/11/17	6 028.68	5 912.13	5 905.27	5 943.21
2000/11/18	5 167	5 350	5 350	5 198.35	2004/11/18	6 049.49	6 037.08	6 030.88	6 014.24
2000/11/20	4 845	5 150	5 150	4 836.21	2004/11/19	6 026.55	6 042.75	6 051.69	6 054.36
2000/11/21	5 103	4 850	4 850	5 178.36	2004/11/22	5 838.42	6 010.72	6 051.69	5 863.14
2000/11/22	5 130	5 150	5 150	5 145.07	2004/11/23	5 851.1	5 862.23	5 837	5 774.63
2000/11/23	5 146	5 150	5 150	5 136.89	2004/11/24	5 911.31	5 855.49	5 853.3	5 712.24
2000/11/24	5 419	5 150	5 150	5 336.15	2004/11/25	5 855.24	5 911.03	5 913.51	5 741.26
2000/11/27	5 433	5 550	5 300	5 324.84	2004/11/26	5 778.26	5 858.01	5 862.16	5 873.24
2000/11/28	5 362	5 550	5 300	5 382.80	2004/11/29	5 785.26	5 773.75	5 785.57	5 843.84
2000/11/29	5 319	5 350	5 350	5 314.18	2004/11/30	5 844.76	5 783.93	5 780.76	5 815.39
2000/11/30	5 256	5 350	5 250	5 221.92	2004/12/1	5 798.62	5 844.68	5 841.95	5 658.85
2000/12/1	5 342	5 250	5 350	5 336.24	2004/12/2	5 867.95	5 817.26	5 797.2	5 748.36
2000/12/2	5 811	5 350	5 250	5 563.36	2004/12/3	5 893.27	5 861.34	5 865.14	5 896.14
2000/12/4	5 174	5 250	5 250	5 114.66	2004/12/6	5 919.17	5 897.53	5 895.47	5 954.36
2000/12/5	5 199	5 150	5 150	5 214.47	2004/12/7	5 925.28	5 916.83	5 916.36	5 963.17
2000/12/6	5 170	5 150	5 150	5 363.31	2004/12/8	5 892.51	5 910.24	5 920.78	5 847.54
2000/12/7	5 212	5 150	5 150	5 691.01	2004/12/9	5 913.97	5 895.93	5 891.09	5 968.36
2000/12/8	5 252	5 250	5 250	5 332.78	2004/12/10	5 911.63	5 918.24	5 916.17	5 964.78
2000/12/11	5 284	5 250	5 250	5 363.91	2004/12/13	5 878.89	5 915.62	5 910.71	5 912.4
2000/12/12	5 380	5 250	5 250	5 236.31	2004/12/14	5 909.65	5 882.3	5 877.47	5 914.04
2000/12/13	5 384	5 350	5 350	5 342.78	2004/12/15	6 002.58	5 911.86	5 911.85	6 082.16
2000/12/14	5 320	5 350	5 350	5 341.03	2004/12/16	6 019.23	6 002.5	6 004.78	6 024.15
2000/12/15	5 224	5 350	5 350	5 324.0	2004/12/17	6 009.32	5 027.15	6 026.94	6 254.36
2000/12/16	5 134	5 250	5 250	5 436.54	2004/12/20	5 985.94	6 012.86	6 004.82	6 014.25
2000/12/18	5 055	5 150	5 150	5 234.63	2004/12/21	5 987.85	5 999.1	5 984.52	6 114.91
2000/12/19	5 040	5 350	5 450	5 123.14	2004/12/22	6 001.52	6 005.92	5 995.56	6 011.38
2000/12/20	4 947	5 350	5 450	4 721.39	2004/12/23	5 997.67	6 005.86	6 009.23	5 954.1
2000/12/21	4 817	4 950	4 950	4 745.86	2004/12/24	6 019.42	5 997.68	6 005.38	5 987.29
2000/12/22	4 811	4 850	4 850	4 984.25	2004/12/27	5 985.94	6 026.35	6 016.65	6 087.76
2000/12/26	4 721	4 850	4 850	4 832.12	2004/12/28	6 000.57	5 987.87	5 985.02	6 010.47
2000/12/27	4 614	4 750	4 750	4 654.32	2004/12/29	6 088.49	6 018.68	6 008.28	6 000.14
2000/12/28	4 974	4 650	4 650	4 924.24	2004/12/30	6 100.86	6 086.83	6 085.68	6 004.2
2000/12/29	4 743	4 750	4 750	4 785.36	2004/12/31	6 139.69	6 099.45	6 098.05	6 124.85
2000/12/30	4 739	4 750	4 750	4 612.01	-	-	-	-	-
RMSE	-	173.46	200.32	127.88	RMSE	-	155.65	53.3617	77.57

Table 8 Comparison of RMSE for different model

Model	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	Average
Chen [41]	174.62	43.22	42.66	104.17	94.6	54.24	50.5	138.51	117.87	101.33	92.17
Chen and Kao [42]	156.47	56.50	36.45	126.45	62.57	105.52	51.50	125.33	104.12	87.63	91.25
Cai and Zhang [29]	187.10	39.58	39.37	101.80	76.32	56.05	49.45	123.98	118.41	102.34	89.44
Proposed model	148.17	40.14	38.14	98.47	84.69	55.78	51.96	89.87	122.87	76.02	80.61

4.2 Forecasting secular trend data pattern

To show its wide applicability, our method is implemented on another pattern, secular trend model. Monthly total retail sale of social consumer goods (TRSSCG) is a typical

model which is obtained from National Bureau of Statistics of China (www.data.stats.gov.cn) in this section. The specimens include 96 data from January 1994 to December 2011. The parts of data are shown in Table 9.

Table 9 Parts data of TRSSCG

billion

Date	Value	Date	Value	Date	Value	...	Date	Value
Jan 1994	120.85	Jan 1995	160.83	Jan 1996	192.45	...	Jan 2011	1 524.90
Feb 1994	117.81	Feb 1995	150.53	Feb 1996	192.67	...	Feb 2011	1 376.91
Mar 1994	118.30	Mar 1995	154.65	Mar 1996	187.52	...	Mar 2011	1 358.80
Apr 1994	118.59	Apr 1995	154.65	Apr 1996	186.98	...	Apr 2011	1 364.90
May 1994	122.98	May 1995	158.77	May 1996	191.37	...	May 2011	1 469.68
Jun 1994	129.81	Jun 1995	164.96	Jun 1996	198.19	...	Jun 2011	1 456.51
Jul 1994	126.81	Jul 1995	162.90	Jul 1996	190.40	...	Jul 2011	1 440.80
Aug 1994	130.31	Aug 1995	164.96	Aug 1996	193.19	...	Aug 2011	1 470.50
Sept 1994	141.47	Sept 1995	177.33	Sep 1996	210.04	...	Sep 2011	1 586.51
Oct 1994	146.32	Oct 1995	181.46	Oct 1996	216.57	...	Oct 2011	1 654.64
Nov 1994	157.44	Nov 1995	193.83	Nov 1996	230.86	...	Nov 2011	1 612.89
Dec 1994	195.78	Dec 1995	237.13	Dec 1996	287.17	...	Dec 2011	1 773.97

Firstly, set the initial partition as $p = 6$, $\Delta A = 20$, $A_1 = [100, 120]$, $A_2 = [110, 130]$, $A_3 = [120, 140]$, $A_4 = [130, 150]$, $A_5 = [140, 160]$ and $A_6 = [150, 170]$. In the forecasting process, the value of interval updates three times, i.e., $\Delta A_1 = 2 \times 20 = 40$, $\Delta A_2 = 2^2 \times 20 = 80$, $\Delta A_3 = 2^3 \times 20 = 160$. The comparison between actual values and forecast results is shown in Fig. 9, which indicates that the adaptive partition IFTS forecasting model performs excellently on prediction of TRSSCG. The comparisons of square error with other methods are shown in Fig. 10 and Fig. 11. In other methods, set the scale of partition as 10, and the square error distribution figure illustrates that the proposed model makes lower error.

Fig. 9 Forecast result of TRSSCG **Fig. 10 Comparison of square error on TRSSCG (1)** **Fig. 11 Comparison of square error on TRSSCG (2)**

Considering the fact that AFER is influenced by the scale of date sets, we compare the proposed model with others with index MSE only, as shown in Table 10. The index MSE of our model is the minimum among four methods.

Table 10 Comparison of MSE in TRSSCG

Model	Proposed model	Song and Chissom [1]	Lee and Chou [40]	Joshi and Kumar [36]
MSE	7 308	32 452	22 603	11 747

4.3 Empirical analysis

In order to evaluate the computational complexity of different models, we present the time complexity of each method in Table 11. Song's, Lee's and Joshi's models divide every time series into n parts. Chen's, Huarng's and Gangwar's models rely on partition of universe of discourse n and the fuzzy logical relationships groups g , and each FLT needs to be divided. The proposed model needs to calculate the optical order number m , whose computational complexity is equal with other models'.

Based on the particular experiments analyzed above, we can see that the performance of forecasting models presented in [1,36,40] is not bad when applied on the student enrollments data. However, the forecasting MSE is high when applied to the TRSSCG data. Our model obtains effective forecasting results on enrollments dataset and performs better than [3,6,41] on seasonal dataset (e.g. TAIEX). The experiments show that the performance of the same methods may be different when the forecasting objects change. The traditional methods are suitable for smooth pattern only. It is proved that the proposed adaptive IFTS model is appropriate for different situations by adjusting partitions and intervals of universe.

Table 11 Comparison of computational complexity

Model	$T(n)$
Song's model [1]	$O(n^2)$
Chen's model [3]	$O(gn^2)$
Huarng's model [5]	$O(gn^2)$
Gangwar's model [16]	$O(gn^2)$
Joshi's [36]	$O(n^2)$
Lee's [40]	$O(n^2)$
Proposed model	$O(m + n^2)$

5. Conclusions

In this paper, we construct an IFTS forecasting model with order deciding and adaptive partition algorithms. The proposed model can adjust partitions and intervals adaptively by forecasting the vector operator matrix that is validated effectively on enrollments of Alabama. We implement the proposed model on two different dataset patterns.

The TAIEX experimental results show that our model performs better than other relevant methods. The other pattern is the secular trend dataset. We experiment our model on almost eight years' monthly TRSSCG, and both the forecasting accuracy and MSE are the most optimized one. The experimental results illustrate the wide applicability and generality of the proposed model in different patterns.

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