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Temporal evidence combination method for multi-sensor target recognition based on DS theory and IFS

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Abstract: In order to effectively deal with the conflict temporal evidences without affecting the sequential and dynamic characteristics in the multi-sensor target recognition (MSTR) system at the decision making level, this paper proposes a Dempster-Shafer (DS) theory and intuitionistic fuzzy set (IFS) based temporal evidence combination method (DSIFS-TECM). To realize the method, the relationship between DS theory and IFS is firstly analyzed. And then the intuitionistic fuzzy possibility degree of intuitionistic fuzzy value (IFPD-IFV) is defined, and a novel ranking method with isotonicity for IFV is proposed. Finally, a calculation method for relative reliability factor (RRF) is designed based on the proposed ranking method. As a proof of the method, numerical analysis and experimental simulation are performed. The results indicate DSIFS-TECM is capable of dealing with the conflict temporal evidences and sensitive to the changing of time. Furthermore, compared with the existing methods, DSIFS-TECM has stronger ability of anti-interference.

Keywords: Dempster-Shafer (DS) theory, intuitionistic fuzzy set (IFS), temporal evidence combination, relative reliability factor.

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1. Introduction

With the increase of competition in the modern information based war, multi-sensor target recognition (MSTR) has more and more significant influence on the war field situation assessment and tactical decision. Compared to single sensor target recognition, MSTR can obtain better recognition performance when the provided redundant and complementary evidences in the spatial and temporal domain are combined in a reasonable way [1]. At present, a relatively effective method in dealing with this problem is Dempster-Shafer (DS) theory, which is also called as DS evidence theory or evidence theory. This theory originates from the model of Dempster and it is improved by Shafer to satisfy the classical principles of probability theory [2]. DS theory has been studied for its ability of dealing with uncertainty and applied in many areas, such as multi-criteria decision making [3], classification [4], and so on.

However, there are still some problems waiting to be solved. For instance, the Dempster combination rule is inapplicable for the completely conflicting evidences and Zadeh paradox [5], and the existing problem of one ticket veto [6,7]. Therefore, a lot of improved methods have been proposed to develop the DS theory from two mainly categories. For one part, some scientists focus on the redistribution of the conflicts, since they doubt the unreasonable results coming from the normalized step in the Dempster combination rule [8]. For the other, some scientists pay their attention on the pretreatment process to the conflict evidences [9]. These two categories stimulate the development of the DS theory.

From the perspective of the research contents, DS theory can be divided into two aspects, namely spatial evidence combination and temporal evidence combination. The spatial evidence combination analyzes the evidences from different sources at the same moment, while temporal evidence combination deals with the case coming from the same source at different moments. Compared to the spatial evidence combination, the temporal evidence combination develops slower and it is more realistic for its consideration of the real-time, sequential and dynamic. Despite the fruitful achievements in the research of the temporal evidence combination [10 – 13], we find that these methods are not effective to handle the conflicts in the temporal domain for their insufficient use for the relationship among temporal evidences.

Aiming at solving these issues, we propose a DS theory and intuitionistic fuzzy set based temporal evidence combination method (DSIFS-TECM) to study the temporal evidence combination at the decision making level according to the thought of pretreatment to the conflict evidences. In the proposed method, in order to conform to the actual

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situation, different discounting factors are adopted according to the evidences at the two adjacent moments. Moreover, the calculation method for the discounting factors are improved. Major contributions of this paper can be summarized as follows.

(i) Different from intuitionistic fuzzy value (IFV)'s ranking methods based on possibility degree, the ranking method of IFV based on Boolean matrix and intuitionistic fuzzy possibility degree of IFV (IFPD-IFV) is proposed, which has isotonicity and can describe not only the probability that one IFV is bigger than the other IFV, but also the probability that one IFV is smaller than the other IFV. Moreover, an approach for relative reliability factor (RRF) calculation based on our proposed ranking method is designed, which needs no priori information.

(ii) Some existing methods assume that the evidences got by sensors become more and more accurate if they do not be disturbed by the outside in the process of combination, which does not conform to the actual situation. Thus in this paper, we develop DSIFS-TECM to make up for the shortcomings of the assumption in some existing methods. As a result, DSIFS-TECM can deal with not only the situation that the accuracy of the evidences at the current moment is bigger than that of at the previous moment, but also the composite situation. More importantly, experiments show that DSIFS-TECM has stronger ability of anti-interference.

The remainder of this paper is organized as follows. In Section 2, the basic concepts of the DS theory and the intuitionistic fuzzy set (IFS) are introduced and the relationship between DS theory and IFS is demonstrated. Then, we propose IFPD-IFV and develop a different IFV's ranking method with isotonicity in Section 3. In Section 4, on the basis of our proposed ranking method, DSIFS-TECM and approach for RRF calculation are intensively studied. To make a demonstration of our method, a numerical simulation is performed in Section 5. Finally, we briefly conclude the whole paper.

2. Basic concept

In the DSIFS-TECM, we deal with the temporal evidence combination in the framework of IFS, so we introduce the concept of the DS theory and IFS briefly.

2.1 DS theory

The frame of discernment Θ, a limited complete set consisting of mutually exclusive propositions, is firstly defined in the DS theory. The power set of Θ is represented as 2^{Θ} that contains all the subsets of Θ.

Definition 1 [2] For any subset A in Θ, $m : 2^{\Theta} \to [0, 1]$ is defined as a basic probability assignment function (BPAF), which satisfies the following three conditions:

(i) $0 \leqslant m(A) \leqslant 1$;

(ii) $m(\varnothing) = 0$;

(iii) $\sum\limits_{A\subseteq\Theta} m(A) = 1$.

where $m(A)$ denotes the basic probability assigned to A, $\varnothing$ is the empty set.

Definition 2 [2] Under conditions of $A \subseteq \Theta$ and $m(A) > 0$, the set A is defined as the focal element.

Definition 3 [2] For any subset A in Θ, belief function Bel and plausibility function Pl are defined as

$$Bel(A) = \sum_{B\subseteq A} m(B) \tag{1}$$

$$Pl(A) = \sum_{B\cap A\neq\varnothing} m(B) = 1 - Bel(\bar{A}) \tag{2}$$

where $\bar{A}$ represents the complementary set of A, $Bel(A)$ represents the degree of justified support given to A by the available evidence, $Pl(A)$ represents the maximum potential support that could be given to A. In other words, $Bel(A)$ and $Pl(A)$ represent the minimum belief degree and maximum belief degree, respectively. The belief interval can be given as $[Bel(A), Pl(A)]$.

Definition 4 [14,15] The procedure of transforming BPAF to probability is called as the probability transformation of BPAF, or Bayesian transformation. Pignistic transformation is the most common method in probability transformation of BPAF, which is defined as

$$BetP(A) = \sum_{B\subseteq\Theta} \frac{|A\cap B|}{|B|} \cdot \frac{m(B)}{1 - m(\varnothing)} \tag{3}$$

where $|B|$ is the cardinality of set B.

Definition 5 [2] For n independent belief functions m_i on Θ, $i = 1, 2, \ldots, n$, the results of the Dempster combination rule is defined as

$$m(A) = m_1 \oplus m_2 \oplus \cdots \oplus m_n = \begin{cases} 0, & A = \varnothing \\ \dfrac{\sum\limits_{\cap A_i = A} \prod\limits_{i=1}^{n} m_i(A_i)}{1 - C}, & A \neq \varnothing \end{cases} \tag{4}$$

where $C = \sum\limits_{\cap A_i=\varnothing} \prod\limits_{i=1}^{n} m_i(A_i)$ represents the degree of conflict between the pieces of evidence. $1 - C$ is the normalization factor that is used to avoid the assigning probability to the empty set in the combination process. It is proved that the Dempster combination rule has the following properties:

(i) Associative law: $(m_1 \oplus m_2) \oplus m_3 = m_1 \oplus (m_2 \oplus m_3)$;

(ii) Commutative law: $m_1 \oplus m_2 = m_2 \oplus m_1$.

It is worth noting that the combination results have no relationship with the order, so the combination of multiple BPAF functions can be instructed as the combination of every two belief functions.

Definition 6 [2] Assuming that the evidence source has a discounting factor ρ $(0 \leqslant \rho \leqslant 1)$, the Shafer evidence discounting rule is defined as the discount of the corresponding BPAF m, represented as

$$m^{\rho}(A) = \begin{cases} (1-\rho)\cdot m(A), & A \subset \Theta \\ (1-\rho)\cdot m(A)+\rho, & A = \Theta \end{cases}. \quad (5)$$

In addition, some special BPAFs with special mathematical form and physical meaning are provided in the DS theory, which are essential in theoretical analysis, algorithm design, and engineering application. The commonly used special BPAFs are shown in the followings.

(i) Vacuous probability assignment (VPA)

$$m(\Theta) = 1 \quad (6)$$

(ii) Simple support function (SSF)

$$m(X) = \begin{cases} \varepsilon, & X = A \\ 1-\varepsilon, & X = \Theta \\ 0, & \text{otherwise} \end{cases} \quad (7)$$

(iii) Dichotomous support function (DSF)

$$m(X) = \begin{cases} \varepsilon, & X = A \\ \sigma, & X = \bar{A} \\ 1-\varepsilon-\sigma, & X = \Theta \end{cases} \quad (8)$$

2.2 IFS

In the classical set theory, any element in universal set can only belong to or not belong to a set. In other words, the values of the characteristic function are only 0 or 1. Then, Zadeh proposed the fuzzy set to extend the characteristic function's range to interval $[0,1]$ [16]. In 1983, professor Atanassove proposed the IFS [17] that added the degree of non-membership to the fuzzy set. IFS is capable of representing the neutral state and making a more flexible, exquisite, comprehensive description for the nature of objective phenomenon. We will show a brief introduction of IFS in the followings.

Definition 7 [17] Assuming that X is a finite universal set. An IFS in X is an object having the form of $\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), v_{\tilde{A}}(x)\rangle | x \in X\}$, where the functions $\mu_{\tilde{A}} : X \to [0,1]$ and $x \in X \to \mu_{\tilde{A}}(x) \in [0,1]$ define the degree of membership for the element $x (x \in X)$ belonging to the set $\tilde{A} (\tilde{A} \subseteq X)$. And the functions $v_{\tilde{A}} : X \to [0,1]$ and $x \in X \to v_{\tilde{A}}(x) \in [0,1]$ define the degree of non-membership for the element $x (x \in X)$ belonging to the set $\tilde{A} (\tilde{A} \subseteq X)$. For every $x \in X$, $0 \leqslant \mu_{\tilde{A}}(x) + v_{\tilde{A}}(x) \leqslant 1$.

In addition, $\pi_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) - v_{\tilde{A}}(x)$ is defined as the hesitation degree of the element x belonging to the set $\tilde{A}$, representing the neutral state. Based on Definition 7, lots of operations about IFS have been defined [17]. For convenience, $\langle x, \mu_{\tilde{A}}(x), v_{\tilde{A}}(x)\rangle$ is represented by $\langle \mu_{\tilde{A}}(x), v_{\tilde{A}}(x)\rangle$, which is also called as the IFV.

It has been demonstrated that IFS is equivalent to the vague set [18]. And IFS can also be represented by the interval $[\mu_{\tilde{A}}(x), 1 - v_{\tilde{A}}(x)]$, where $\mu_{\tilde{A}}(x)$ and $1 - v_{\tilde{A}}(x)$ are the lower bound and upper bound of degree of membership, respectively [19]. In particular, for $\forall x \in X$, when $\pi_{\tilde{A}}(x) = 0$ or $v_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x)$, IFS is equivalent to the fuzzy set. Moreover, for a crisp set $P \subseteq X$ and $P \neq \varnothing$, it also can be represented as $\tilde{P} = \{\langle x, \mu_{\tilde{P}}(x), v_{\tilde{P}}(x)\rangle | x \in X\}$ in the framework of IFS, where $\mu_{\tilde{P}}(x) = 1$ and $v_{\tilde{P}}(x) = 0$ when $x \in P$, $\mu_{\tilde{P}}(x) = 0$ and $v_{\tilde{P}}(x) = 1$ when $x \notin P$. In conclusion, interval value, crisp set, fuzzy set and vague set can be expressed in the framework of IFS, bringing great convenience to researchers.

2.3 Relationship between DS theory and IFS

In recent years, more and more researchers began to pay attention to the relationship between DS theory and IFS. For example, Li et al. analyzed the relationship between DS theory and vague set [20]. Dymova and Sevastjanov made an interpretation about IFS in the framework of DS theory [21,22]. Geng et al and Yager studied DS theory in form of IFS directly [1,23]. The research on the relationship between them is going on, and a brief analysis about it is made in the followings.

Theorem 1 [1] For frame of discernment $\Theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$, based on BPAF, $\forall A \subseteq \Theta$ can be represented by IFS $\tilde{A} = \{\langle \theta_i, \mu_{\tilde{A}}(\theta_i), v_{\tilde{A}}(\theta_i)\rangle | \theta_i \in \Theta\}$, where $\mu_{\tilde{A}}(\theta_i)$ and $v_{\tilde{A}}(\theta_i)$ are denoted the lower bound of the membership degree and non-membership degree for $\theta_i \in \tilde{A}$ respectively.

Proof As $\mu_{\tilde{A}}(\theta_i)$ and $v_{\tilde{A}}(\theta_i)$ are denoted the lower bound of the membership degree and non-membership degree for $\theta_i \in \tilde{A}$ respectively, according to BPAF and Definition 3, they are calculated as

$$\begin{cases} \mu_{\tilde{A}}(\theta_i) = Bel(\{\theta_i\}) \\ v_{\tilde{A}}(\theta_i) = 1 - Pl(\{\theta_i\}) \end{cases}.$$

Then based on Definition 1, it can be obtained that $\mu_{\tilde{A}}(\theta_i) \in [0,1]$ and $v_{\tilde{A}}(\theta_i) \in [0,1]$ as $\varnothing \subseteq \tilde{A} \subseteq \Theta$. Moreover, as

$$\mu_{\tilde{A}}(\theta_i) + v_{\tilde{A}}(\theta_i) = Bel(\{\theta_i\}) + 1 - Pl(\{\theta_i\}) =$$
$$\sum_{B \subseteq \{\theta_i\}} m(B) + 1 - \sum_{B \cap \{\theta_i\} \neq \varnothing} m(B) =$$
$$\sum_{B \cap \{\theta_i\} = B} m(B) + 1 - \sum_{B \cap \{\theta_i\} \neq \varnothing} m(B) =$$
$$1 - \sum_{B \cap \{\theta_i\} = C} m(B), \quad \varnothing \subseteq C \subseteq \Theta,$$

and $1 - \sum_{B\cap\{\theta_i\}=C} m(B) \in [0,1]$, we can get $\tilde{A} = \{\langle\theta_i, \mu_{\tilde{A}}(\theta_i), v_{\tilde{A}}(\theta_i)\rangle | \theta_i \in \Theta\}$ meets the definition of IFS, reflecting that $\forall A \subseteq \Theta$ can be represented in the form of IFS. □

Theorem 2 [21] For a finite universal set $X = \{x_1, x_2, \ldots, x_n\}$, an IFV $\langle\mu_{\tilde{A}}(x_j), v_{\tilde{A}}(x_j)\rangle$ $(1 \leqslant j \leqslant n)$ can be transformed into a BPAF in the DS theory.

Proof An IFV $\langle\mu_{\tilde{A}}(x_j), v_{\tilde{A}}(x_j)\rangle$ can be regarded as the answer to the problem that whether x_j belongs to $\tilde{A}$, so the frame of discernment Θ is $\{\mathrm{Yes}, \mathrm{No}\}$. Then according to the definition about degree of membership and degree of non-membership, based on DSF, we can get that $m(\{\mathrm{Yes}\}) = \mu_{\tilde{A}}(x_j)$, $m(\{\mathrm{No}\}) = v_{\tilde{A}}(x_j)$, and $m(\{\Theta\}) = \pi_{\tilde{A}}(x_j)$. In other words, IFV can be transformed into the form of BPAF in DS theory. □

3. Ranking method of IFV based on Boolean matrix and IFPD-IFV

The problem of ranking IFVs can be seen as ranking interval values since IFV can be represented by $[\mu_{\tilde{A}}(x), 1 - v_{\tilde{A}}(x)]$. Up to now, lots of methods about ranking of interval values have been proposed, which plays an important role in the research of the uncertainty decision. These methods can be classified into two categories in general, namely certainty ranking method and uncertainty ranking method, respectively. The certainty ranking method ranks the interval value by the real number (relative entropy, similarity degree, and correlation degree), which can represent the order structure of interval. The uncertainty ranking method ranks the interval value by the degree of one interval bigger than other intervals, such as the probability degree, reliability degree, and acceptability degree. As we know, the row sum of the possibility degree (RS-PD) matrix based ranking methods [24,25] are widely used in reality and can be formed with three steps. Firstly, establish the possibility degree matrix through comparing any two intervals. Then, calculate each row's sum of the matrix. Finally, rank the intervals based on the order of each row's sum. However, Gao has proved that RS-PD does not have isotonicity that is important to decision making [26]. Moreover, probability degree only describes the probability that one interval is bigger than another interval, and ignores the opposite case. Therefore, we propose a ranking method of IFV based on Boolean matrix and IFPD-IFV to overcome the above two problems.

3.1 IFPD-IFV

Inspired by the idea that an IFV can be represented by an interval value, IFPD-IFV is defined in this paper, which combines the characteristics of IFV with the advantages of IFPD for interval values. Therefore, the proposed IFPD-IFV not only can describe the probability that one IFV is bigger than the other IFV, but also can describe the probability that one IFV is smaller than the other IFV. Moreover, its calculation process only needs some simple operations, such as finding maximum and minimum values. Definition about IFPD for interval values and IFPD-IFV are developed as follows.

Definition 8 [27] IFPD for interval values Assume $a, b \in \Omega$, where $a = [a^-, a^+]$ and $b = [b^-, b^+]$, the IFPD of $a \geqslant b$ is defined as

$$p(a \geqslant b) = \langle\mu_{p(a>b)}, v_{p(a>b)}, \pi_{p(a\geqslant b)}\rangle \tag{9}$$

where

$$\begin{cases} \mu_{p(a\geqslant b)} = \dfrac{|X| + |Y|}{|a| + |b| - |a \cap b|} \\ v_{p(a\geqslant b)} = \dfrac{|\bar{X}| + |\bar{Y}|}{|a| + |b| - |a \cap b|} \\ \pi_{p(a\geqslant b)} = \dfrac{|a \cap b|}{|a| + |b| - |a \cap b|} \end{cases}. \tag{10}$$

$X = \{x | x \in a, \forall y \in b, x \geqslant y\}$, $Y = \{y | y \in b, \forall x \in a, y \leqslant x\}$, $\bar{X}$ and $\bar{Y}$ are logical negations of X and Y, respectively. $\mu_{p(a>b)}$ is defined as the probability that a is bigger than b, $v_{p(a>b)}$ is the probability that a is smaller than b, and $\pi_{p(a\geqslant b)}$ is the defined as the probability that a is equal to b. If $\mu_{p(a\geqslant b)} - v_{p(a\geqslant b)} > 0$, it is defined as $a > b$. If $\mu_{p(a\geqslant b)} - v_{p(a\geqslant b)} < 0$, it is defined as $a < b$. And if $\mu_{p(a\geqslant b)} - v_{p(a\geqslant b)} = 0$, it is defined as $a \sim b$, meaning the probability of $a > b$ is equal to the probability of $a < b$. It has proved that this definition satisfies the following properties:

(i) $\mu_{p(a\geqslant b)}, v_{p(a\geqslant b)}, \pi_{p(a\geqslant b)} \in [0, 1]$, $\mu_{p(a\geqslant b)} + v_{p(a\geqslant b)} + \pi_{p(a\geqslant b)} = 1$;

(ii) $\mu_{p(a\geqslant b)} = v_{p(b\geqslant a)}$, $v_{p(a\geqslant b)} = \mu_{p(b\geqslant a)}$, $\pi_{p(a\geqslant b)} = \pi_{p(b\geqslant a)}$;

(iii) If $a > b$ and $b > c$, then $a > c$.

Definition 9 IFPD-IFV $X = \{x_1, x_2, \ldots, x_n\}$ is a finite universal set, $\tilde{A}$ and $\tilde{B}$ are IFSs in X. For $1 \leqslant i, j \leqslant n$, IFVs $\alpha = \langle\mu_{\tilde{A}}(x_i), v_{\tilde{A}}(x_i)\rangle$ and $\beta = \langle\mu_{\tilde{B}}(x_j), v_{\tilde{B}}(x_j)\rangle$ can be represented by $[\mu_{\tilde{A}}(x_i), 1 - v_{\tilde{A}}(x_i)]$ and $[\mu_{\tilde{B}}(x_j), 1 - v_{\tilde{B}}(x_j)]$, respectively. When $\mu_{\tilde{A}}(x_i) \neq 1 - v_{\tilde{A}}(x_i)$ or $\mu_{\tilde{B}}(x_j) \neq 1 - v_{\tilde{B}}(x_j)$, the IFPD-IFV of $\alpha \geqslant \beta$ is defined as

$$p(\alpha \geqslant \beta) = \langle\mu_{p(\alpha\geqslant\beta)}, v_{p(\alpha\geqslant\beta)}, \pi_{p(\alpha\geqslant\beta)}\rangle \tag{11}$$

where

$$\begin{cases} \mu_{p(\alpha\geqslant\beta)} = \dfrac{|X| + |Y|}{|\alpha| + |\beta| - |\alpha \cap \beta|} \\ v_{p(\alpha\geqslant\beta)} = \dfrac{|\bar{X}| + |\bar{Y}|}{|\alpha| + |\beta| - |\alpha \cap \beta|} \\ \pi_{p(\alpha\geqslant\beta)} = \dfrac{|\alpha \cap \beta|}{|\alpha| + |\beta| - |\alpha \cap \beta|} \end{cases}. \tag{12}$$

$X = \{x|x \in \alpha, \forall y \in \beta, x \geqslant y\}$, $Y = \{y|y \in \beta, \forall x \in \alpha, y \leqslant x\}$, $\bar{X}$ and $\bar{Y}$ are logical negations of X and Y, respectively. $\mu_{p(\alpha\geqslant\beta)}$ is the probability that IFV α is bigger than β, $v_{p(\alpha\geqslant\beta)}$ is the probability that IFV α is smaller than β, and $\pi_{p(\alpha\geqslant\beta)}$ is the probability that IFV α is equal to β. Moreover, when $\mu_{\tilde{A}}(x_i) = 1 - v_{\tilde{A}}(x_i)$ and $\mu_{\tilde{B}}(x_j) = 1 - v_{\tilde{B}}(x_j)$, the IFPD-IFV of $\alpha \geqslant \beta$ is defined as

$$\begin{cases} p(\alpha \geqslant \beta) = \langle 1,0,0\rangle, & \mu_\alpha(x_i) > \mu_\beta(x_j) \\ p(\alpha \geqslant \beta) = \langle 0,1,0\rangle, & \mu_\alpha(x_i) < \mu_\beta(x_j) \\ p(\alpha \geqslant \beta) = \langle 0,0,1\rangle, & \mu_\alpha(x_i) = \mu_\beta(x_j) \end{cases}. \tag{13}$$

If $\mu_{p(\alpha\geqslant\beta)} - v_{p(\alpha\geqslant\beta)} > 0$, it is defined as $\alpha > \beta$. If $\mu_{p(\alpha\geqslant\beta)} - v_{p(\alpha\geqslant\beta)} < 0$, it is defined as $\alpha < \beta$. And if $\mu_{p(\alpha\geqslant\beta)} - v_{p(\alpha\geqslant\beta)} = 0$, it is defined as $\alpha \sim \beta$, meaning the probability of $\alpha > \beta$ is equal to the probability of $\alpha < \beta$. It can be proved that the definition of IFPD-IFV satisfies the following properties according to [27]:

(i) $\mu_{p(\alpha\geqslant\beta)}, v_{p(\alpha\geqslant\beta)}, \pi_{p(\alpha\geqslant\beta)} \in [0,1]$, $\mu_{p(\alpha\geqslant\beta)} + v_{p(\alpha\geqslant\beta)} + \pi_{p(\alpha\geqslant\beta)} = 1$;

(ii) $\mu_{p(\alpha\geqslant\beta)} = v_{p(\alpha\geqslant\beta)}$, $v_{p(\alpha\geqslant\beta)} = \mu_{p(\alpha\geqslant\beta)}$, $\pi_{p(\alpha\geqslant\beta)} = \pi_{p(\alpha\geqslant\beta)}$;

(iii) If IFV $\alpha > \beta$ and $\beta > \gamma$, then $\alpha > \gamma$.

In this paper, based on Definition 9 and the characteristics of IFV, the calculation process of $p(\alpha \geqslant \beta)$ when $\mu_{\tilde{A}}(x_i) \neq 1 - v_{\tilde{A}}(x_i)$ or $\mu_{\tilde{B}}(x_j) \neq 1 - v_{\tilde{B}}(x_j)$ is simplified as

$$\begin{cases} \mu_{p(\alpha\geqslant\beta)} = \dfrac{\max(0, 1 - v_{\tilde{A}}(x_i) - \max(\mu_{\tilde{A}}(x_i), 1 - v_{\tilde{B}}(x_j))) + \max(0, \min(\mu_{\tilde{A}}(x_i), 1 - v_{\tilde{B}}(x_j)) - \mu_{\tilde{B}}(x_j))}{\pi_{\tilde{A}}(x_i) + \pi_{\tilde{B}}(x_j) - \max(0, \min(\pi_{\tilde{A}}(x_i), \pi_{\tilde{B}}(x_j), 1 - v_{\tilde{A}}(x_i) - \mu_{\tilde{B}}(x_j), 1 - v_{\tilde{B}}(x_j) - \mu_{\tilde{A}}(x_i)))} \\ v_{p(\alpha\geqslant\beta)} = \dfrac{\max(0, \min(1 - v_{\tilde{A}}(x_i), \mu_{\tilde{B}}(x_j)) - \mu_{\tilde{A}}(x_i)) + \max(0, 1 - v_{\tilde{B}}(x_j) - \max(1 - v_{\tilde{A}}(x_i), \mu_{\tilde{B}}(x_j)))}{\pi_{\tilde{A}}(x_i) + \pi_{\tilde{B}}(x_j) - \max(0, \min(\pi_{\tilde{A}}(x_i), \pi_{\tilde{B}}(x_j), 1 - v_{\tilde{A}}(x_i) - \mu_{\tilde{B}}(x_j), 1 - v_{\tilde{B}}(x_j) - \mu_{\tilde{A}}(x_i)))} \\ \pi_{p(\alpha\geqslant\beta)} = \dfrac{\max(0, \min(\pi_{\tilde{A}}(x_i), \pi_{\tilde{B}}(x_j), 1 - v_{\tilde{A}}(x_i) - \mu_{\tilde{B}}(x_j), 1 - v_{\tilde{B}}(x_j) - \mu_{\tilde{A}}(x_i)))}{\pi_{\tilde{A}}(x_i) + \pi_{\tilde{B}}(x_j) - \max(0, \min(\pi_{\tilde{A}}(x_i), \pi_{\tilde{B}}(x_j), 1 - v_{\tilde{A}}(x_i) - \mu_{\tilde{B}}(x_j), 1 - v_{\tilde{B}}(x_j) - \mu_{\tilde{A}}(x_i)))} \end{cases}. \tag{14}$$

3.2 Ranking method of IFV based on Boolean matrix and IFPD-IFV

The proposed ranking method of IFV based on Boolean matrix and IFPD-IFV not only has the property of isotonicity, but also has the advantage of IFPD-IFV. Its ranking process for IFVs is provided below.

(i) For N IFVs $A_1, A_2, \dots, A_N$ in the IFS $\tilde{A}$ and $\forall i, j \in [0, N]$, calculate IFPD of $A_i \geqslant A_j$ according to (13) and (14).

(ii) Establish the contrast matrix $\boldsymbol{IFP} = (IFP_{ij})_{N\times N}$ for N IFVs;

$$\boldsymbol{IFP} = \begin{bmatrix} IFP_{11} & IFP_{12} & \cdots & IFP_{1N} \\ IFP_{21} & IFP_{22} & \cdots & IFP_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ IFP_{N1} & IFP_{N2} & \cdots & IFP_{NN} \end{bmatrix} \tag{15}$$

where $IFP_{ij} = p(A_i \geqslant A_j)$, $\mu_{ij} = \mu_{p(A_i\geqslant A_j)}$, $v_{ij} = v_{p(A_i\geqslant A_j)}$, $\pi_{ij} = \pi_{p(A_i\geqslant A_j)}$, and $IFP_{ii} = \langle 0,0,1\rangle$.

(iii) According to $\boldsymbol{IFP}$, establish the Boolean matrix $\boldsymbol{Q} = (q_{ij})_{N\times N}$, which is also called as sort matrix;

$$\boldsymbol{Q} = \begin{bmatrix} q_{11} & q_{12} & \cdots & q_{1N} \\ q_{21} & q_{22} & \cdots & q_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ q_{N1} & q_{N2} & \cdots & q_{NN} \end{bmatrix} \tag{16}$$

where $q_{ij} = \begin{cases} 1, & \mu_{ij} - v_{ij} \geqslant 0 \\ 0, & \mu_{ij} - v_{ij} < 0 \end{cases}$.

(iv) Calculate the sum of each row for $\boldsymbol{Q}$, denoted as

$$\lambda'_i = \sum_{j=1}^{N} q_{ij}. \tag{17}$$

Then we can get $\boldsymbol{\lambda}' = (\lambda'_1, \lambda'_2, \dots, \lambda'_N)$.

(v) Ranking IFVs according to the order of λ'_i.

Then, we will propose the theorem to provide a theoretical basis for the proposed ranking method.

Theorem 3 For given IFVs $A_1, A_2, \dots, A_N$, $\boldsymbol{IFP} = (IFP_{ij})_{N\times N}$ and $\boldsymbol{Q} = (q_{ij})_{N\times N}$ are contrast matrix and sort matrix, respectively. If $\lambda'_i = \sum\limits_{j=1}^{N} q_{ij}, 1 \leqslant i \leqslant N$, then $\lambda'_i \geqslant \lambda'_j \Leftrightarrow \mu_{ij} - v_{ij} \geqslant 0$.

Proof Firstly, we prove the sufficiency. If $\mu_{ij} - v_{ij} < 0$, we can know that $A_i < A_j$. Thus for $\forall \eta \in [0, N]$, if $\mu_{i\eta} - v_{i\eta} \geqslant 0$ that represents $A_i > A_\eta$ or $A_i \sim A_\eta$, we can know that $A_j > A_\eta$ according to property (iii) and get $\mu_{j\eta} - v_{j\eta} > 0$. Then according to (17), we can get that $\lambda'_i < \lambda'_j$, which contradicts with the known condition $\lambda'_i \geqslant \lambda'_j$. Therefore, $\lambda'_i \geqslant \lambda'_j \Rightarrow \mu_{ij} - v_{ij} \geqslant 0$.

Then, we validate the necessity. As $\mu_{ij} - v_{ij} \geqslant 0$, we can know that $A_i > A_j$ or $A_i \sim A_j$. Thus for $\forall \eta \in [0, N]$, if $\mu_{j\eta} - v_{j\eta} \geqslant 0$ that represents $A_j > A_\eta$ or $A_j \sim A_\eta$, we can know that $A_i > A_\eta$ or $A_i \sim A_\eta$ according to property (iii) and get $\mu_{i\eta} - v_{i\eta} \geqslant 0$. Then according to (17), we can get that $\lambda'_i \geqslant \lambda'_j$. Therefore, $\lambda'_i \geqslant \lambda'_j \Leftarrow \mu_{ij} - v_{ij} \geqslant 0$. □

Theorem 4 The ranking result of IFVs based on

$\lambda'_i = \sum_{j=1}^{N} q_{ij}, 1 \leqslant i \leqslant N$ possesses the property of isotonicity.

Proof According to Theorem 3, for given IFVs $A_1, A_2, \ldots, A_N$, if two IFVs satisfy $\mu_{ij} - v_{ij} > 0$ that means $A_i > A_j$, then $\lambda'_i > \lambda'_j$. For any IFV A_{N+1}, if $\mu_{j(N+1)} - v_{j(N+1)} \geqslant 0$ that means $A_j > A_{N+1}$ or $A_j \sim A_{N+1}$, according to property (iii), we can get $A_i > A_{N+1}$ and $\mu_{i(N+1)} - v_{i(N+1)} > 0$, so $\lambda'_i\_\text{new} = \lambda'_i + 1 > \lambda'_j\_\text{new} = \lambda'_j + 1$ that means the order of A_i and A_j is unchanged. In the same way, if $\mu_{j(N+1)} - v_{j(N+1)} < 0$, there are two situations. Firstly, when $\mu_{i(N+1)} - v_{i(N+1)} \geqslant 0$, $\lambda'_i\_\text{new} = \lambda'_i + 1 > \lambda'_j\_\text{new} = \lambda'_j$. Secondly, when $\mu_{i(N+1)} - v_{i(N+1)} < 0$, $\lambda'_i\_\text{new} = \lambda'_i > \lambda'_j\_\text{new} = \lambda'_j$. In both situations, the order of A_i and A_j is also unchanged. Above all, we can know that if $A_i > A_j$, add any IFV into the comparisons, the order of them is unchanged based on λ'_i. In other words, the ranking result of IFVs based on $\lambda'_i = \sum_{j=1}^{N} q_{ij}, 1 \leqslant i \leqslant N$ has isotonicity. □

Example 1 Ranking three IFVs $A_1 = \langle 0.19, 0.41\rangle$, $A_2 = \langle 0.21, 0.46\rangle$, $A_3 = \langle 0.2, 0.59\rangle$.

The contrast matrix $\boldsymbol{IFP}$ for these three IFVs is

$$\boldsymbol{IFP} = \begin{bmatrix} \langle 0,0,1\rangle & \langle 0.125, 0.005, 0.87\rangle & \langle 0.45, 0.025, 0.525\rangle \\ \langle 0.005, 0.125, 0.87\rangle & \langle 0,0,1\rangle & \langle 0.411\,8, 0, 0.588\,2\rangle \\ \langle 0.025, 0.45, 0.525\rangle & \langle 0, 0.411\,8, 0.588\,2\rangle & \langle 0,0,1\rangle \end{bmatrix}.$$

Then we can get

$$\boldsymbol{Q} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

Thus $\boldsymbol{\lambda}' = (3, 2, 1)$, and we can get the ranking of these three IFVs as $A_1 > A_2 > A_3$.

In the same way, comparing with the fourth IFV $\tilde{A}_4 = \langle 0.2, 0.59\rangle$, we can get $\boldsymbol{\lambda'\_\mathrm{new}} = (4, 3, 2, 2)$ with the ranking as $A_1 > A_2 > A_3 \sim A_4$. From the result we can see that the proposed method has isotonicity. However, using the method in [13], we can get the ranking of $A_1 > A_2 > A_3$ and $A_2 > A_1 > A_3 \sim A_4$, respectively. Indicating that the method in [13] has no isotonicity.

4. DS and IFS based temporal evidence combination for MSTR

In this section, we first describe the MSTR system according to recursive distributed non-feedback temporal-spatial evidence combination model. Then, we focus on the TECM. Finally, the calculation method for RRF is designed based on the proposed ranking method of IFV in Section 4.3.

4.1 MSTR system based recursive distributed non-feedback temporal-spatial evidence combination model

Lang and Lynch [10] studied the temporal-spatial evidence combination model based on DS theory. They summarized the characteristics of the recursive centralized structure and the recursive distributed structure. Moreover, the authors proposed the recursive centralized temporal-spatial evidence combination model, recursive distributed non-feedback temporal-spatial evidence combination model and also the recursive distributed feedback temporal-spatial evidence combination model.

A typical application for temporal-spatial evidence combination is MSTR system, in which it is unreasonable to deal with all evidences in the processing center when adopting the recursive centralized temporal-spatial evidence combination model. Meanwhile, it is inconsistent for the intuitive analysis when the sooner evidences have greater influence on the combination results through adopting the recursive distributed feedback temporal-spatial evidence combination model. Therefore, we adopt the recursive distributed non-feedback temporal-spatial evidence combination model to handle the evidences from MSTR system at every moment, as shown in Fig. 1.

In Fig. 1, every sensor i $(i = 1, \ldots, M)$ can be not only a physical sensor like radar and infrared, but also a logical sensor based on different classical methods or different feature subsets. Moreover, $m_k(i)$ represents the evidence from the ith sensor at the moment k, $m_{k-1,k}(i)$ represents the temporal evidence combination result for the ith sensor at the moment k and m_{k_all} is the combination of all the evidences before the moment k. In the MSTR based recursive distributed non-feedback temporal-spatial evidence combination model, both the temporal evidence combination and the spatial evidence combination are needed. However, lots of methods have been proposed to deal with the spatial evidence combination, such as Dempster combination rule and its improved algorithms. Therefore, we focus our point on the temporal evidence combination, shown in the upcoming part. For convenience, sensor i is denoted as S_i, $m_k(i)$ is denoted as m_k and $m_{k-1,k}(i)$ is denoted as $m_{k-1,k}$.

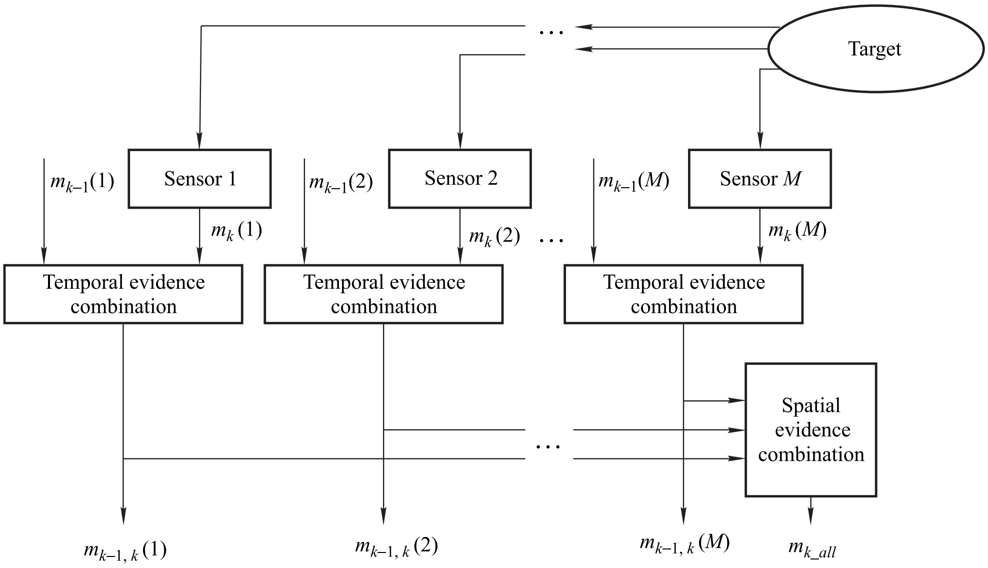

Fig. 1 MSTR system based recursive distributed non-feedback temporal-spatial evidence combination model

4.2 DS theory and IFS based TECM

Temporal evidence combination can be seen as a process of accumulating evidences in the time domain. The evidences in the current moment are the inheritance and renewal for the last moment. To combine the temporal evidences, it is needed to evaluate dynamic reliability for the temporal evidence sequence and handle the conflict evidences between adjacent moments. The method in [13] considers both of them and can deal with the combination of temporal evidences effectively based on the assumption that the evidences got by sensors become more and more accurate if they are disturbed by the outside in the process of combination. It is an outstanding method up to now. However, the assumption of this method does not conform to the actual situation, in which sensors may be broken, and other cases may occur that make the accuracy of the evidences at the current moment lower than that of the evidences at the previous moment. So in this paper, we improve the method in [13] and develop DSIFS-TECM.

DSIFS-TECM: For frame of discernment $\Theta = \{\theta_1, \theta_2, \dots, \theta_n\}$, when the set $\Theta_{\mathrm{MaxSupport}\_m_{k-1}}$ that has maximum support according to m_{k-1} has no intersection with the set $\Theta_{\mathrm{MaxSupport}\_m_k}$ that has maximum support according to m_k, adopt the method in (18). Otherwise, adopt the method in (19).

$$\begin{cases} m_{k-1}^{r_1}(A) = \begin{cases} r_1 \cdot m_{k-1}, & A \subset \Theta \\ r_1 \cdot m_{k-1} + 1 - r_1, & A = \Theta \end{cases} \\ m_k^{r_2}(A) = \begin{cases} r_2 \cdot m_{k-1}, & A \subset \Theta \\ r_2 \cdot m_{k-1} + 1 - r_2, A = \Theta \end{cases} \\ m(A) = m_{k-1}^{r_1}(A) \oplus m_k^{r_2}(A) \end{cases} \tag{18}$$

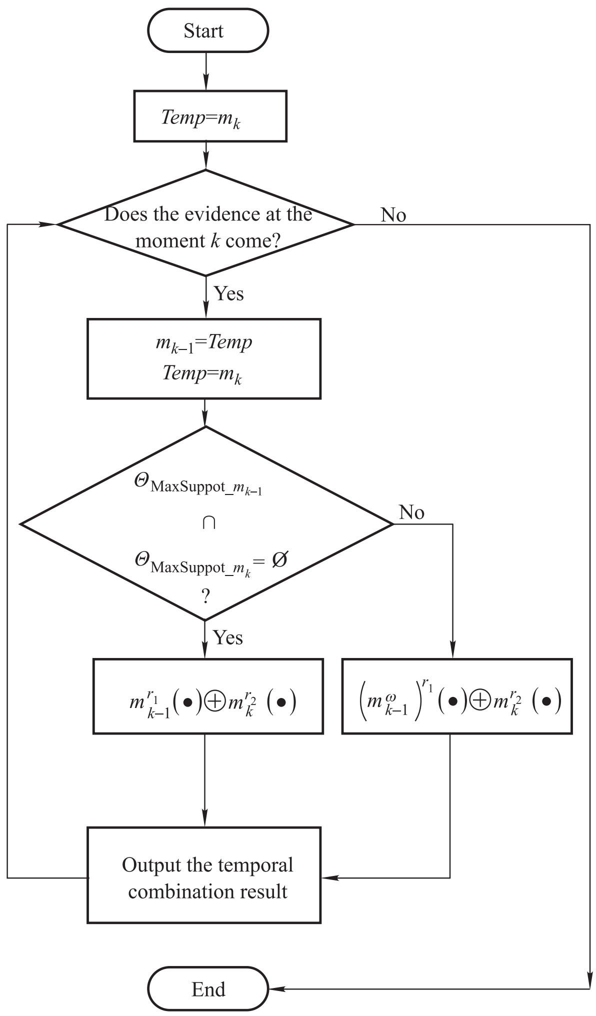

Fig. 2 Algorithm flow for DSIFS-TECM

$$\begin{cases} (m^{\omega}_{k-1})^{r_1}(A) = \begin{cases} r_1 \cdot m^{\omega}_{k-1}, & A \subset \Theta \\ r_1 \cdot m^{\omega}_{k-1} + 1 - r_1, & A = \Theta \end{cases} \\ m^{r_2}_k(A) = \begin{cases} r_2 \cdot m_k, & A \subset \Theta \\ r_2 \cdot m_k + 1 - r_2, & A = \Theta \end{cases} \\ m(A) = (m^{\alpha}_{k-1})^{r_1}(A) \oplus m^{r_2}_k(A) \end{cases} \tag{19}$$

where r_1 and r_2 represent RRF, ω is real-time reliability factor (RTRF) that can be calculated according to [13]. In order to illustrate the method more intuitively, its algorithm flow is described in Fig. 2.

By adopting DSIFS-TECM, we can not only deal with the situation with the evidences accuracy better than that in the previous moment, but also the composite situation. The most important thing for the proposed method is to calculate RRF. Different from the calculation method in [13], we propose a new calculation method for RRF based on the proposed ranking method

4.3 Calculation method for RRF based on the proposed ranking method

There is the idea that the bigger the belief interval is, the greater degree of reliability should be given to its corresponding proposition. However, there are at least n comparison results for the frame of discernment $\Theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$, so the RRF is a compromise for them based on the thought of the group decision making method. In this paper, the calculation method for RRF based on the proposed ranking method is designed. It needs no priori information and can be realized as follows.

(i) For $\forall \theta_i \in \Theta$, represent it in the form of IFV according to m^{ω}_{k-1} (or m_{k-1}) and m_k, and establish a group decision making model based on IFS, denoted by

$$\boldsymbol{F} = \begin{array}{c} \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{array} \begin{array}{c} \begin{array}{cc} m^{\omega}_{k-1} \text{ or } m_{k-1} & m_k \end{array} \\ \begin{pmatrix} Is_{k-1}(\theta_1) & Is_k(\theta_1) \\ Is_{k-1}(\theta_2) & Is_k(\theta_2) \\ \vdots & \vdots \\ Is_{k-1}(\theta_n) & Is_k(\theta_n) \end{pmatrix} \end{array} = \begin{array}{c} \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{array} \begin{array}{c} \begin{array}{cc} m^{\omega}_{k-1} \text{ or } m_{k-1} & m_k \end{array} \\ \begin{pmatrix} \langle Bel_{k-1}(\theta_1), 1 - Pl_{k-1}(\theta_1)\rangle & \langle Bel_k(\theta_1), 1 - Pl_k(\theta_1)\rangle \\ \langle Bel_{k-1}(\theta_2), 1 - Pl_{k-1}(\theta_2)\rangle & \langle Bel_k(\theta_2), 1 - Pl_k(\theta_2)\rangle \\ \vdots & \vdots \\ \langle Bel_{k-1}(\theta_n), 1 - Pl_{k-1}(\theta_n)\rangle & \langle Bel_k(\theta_n), 1 - Pl_k(\theta_n)\rangle \end{pmatrix} \end{array}. \tag{20}$$

(ii) Transform the IFS based group decision making model to the real number based group decision making model according to the proposed ranking method, and its process is shown in the followings.

Firstly, establish the contrast matrix $\boldsymbol{IFP}^{(i)} = (IFP_{gh})_{2\times 2}$ for IFVs $Is_{k-1}(\theta_i)$ and $Is_k(\theta_i)$.

Then, based on the ranking method in Section 3.2, we can get

$$\boldsymbol{\lambda}'_{\boldsymbol{\theta}_i} = (\lambda'_{\theta_i\_1}, \lambda'_{\theta_i\_2})^{\mathrm{T}}. \tag{21}$$

Finally, the real number based group decision making model can be denoted as

$$\boldsymbol{W} = (\lambda'_{\theta_1}, \lambda'_{\theta_2}, \ldots, \lambda'_{\theta_n}). \tag{22}$$

(iii) Calculate RRF by the eigenvalue method, and its process is shown in the followings.

Firstly, calculate the feature vector $\boldsymbol{R}$ corresponding to the maximum eigenvalue of $\boldsymbol{W}$, denoted as

$$\boldsymbol{R}(\boldsymbol{W}\boldsymbol{W}^{\mathrm{T}}) = \lambda_{\max} \cdot \boldsymbol{W}\boldsymbol{W}^{\mathrm{T}}. \tag{23}$$

Then, in order to guarantee that the components in $\boldsymbol{R}$ are positive, we need to normalize it as

$$\boldsymbol{R}' = \boldsymbol{R}/(R_1 + R_2) = (R'_1, R'_2). \tag{24}$$

Finally, the RRF can be calculated as

$$\begin{cases} r_1 = R'_1/\max\{R'_1, R'_2\} \\ r_2 = R'_2/\max\{R'_1, R'_2\} \end{cases}. \tag{25}$$

5. Experimental results

In this section, the performance of DSIFS-TECM is analyzed through numerical analysis and experimental simulation at the decision making level. All algorithms are implemented by using Matlab 2013b and experiments are carried out on a personal computer equipped with an Intel Core i5-4590 dual-core processor and 4GB of RAM, running the 32-bit Microsoft Windows 7 operation system. Experimental results are presented and discussed as follows.

5.1 Analysis for the ability of dealing with conflict evidences

For the frame of discernment $\Theta = \{\theta_1, \theta_2, \theta_3\}$, when the BPAF of a sensor S at $t_1 = 5$ s is m_1 and at $t_2 = 8$ s is m_2, the temporal combination result at $t_2 = 8$ s can be obtained by our proposed method. m_1 and m_2 are completely conflict evidences, denoted as

$$m_1(\{\theta_1\}) = 0.6, m_1(\{\theta_2\}) = 0.1, m_1(\{\theta_3\}) = 0.3$$

$$m_2(\{\theta_1\}) = 0, m_2(\{\theta_2\}) = 0.8, m_2(\{\theta_3\}) = 0.2.$$

The set in Θ that has maximum support according to m_{k-1} is $\{\theta_1\}$ and the set in Θ that has maximum support according to m_2 is $\{\theta_2\}$. As the intersection of $\{\theta_1\}$ and $\{\theta_2\}$ is empty, based on DSIFS-TECM in Section 4.2, there is no need to calculate RTRF. And it is easy to get that $r_1 = 1, r_2 = 0.780\,8$ according to our method in Section 4.3 and new BPAFs are

$$m_1^{r_1}(\{\theta_1\}) = 0.6, m_1^{r_1}(\{\theta_2\}) = 0.1, m_1^{r_1}(\{\theta_3\}) = 0.3$$

$$m_2^{r_2}(\{\theta_1\}) = 0, m_2^{r_2}(\{\theta_2\}) = 0.624\,64$$

$$m_2^{r_2}(\{\theta_3\}) = 0.156\,16, m_2^{r_2}(\Theta) = 0.219\,2.$$

Then, using the Dempster combination rule, we can get the temporal combination result of S_1 at t_2 as

$$m_{12}(\{\theta_1\}) = 0.400\,4,$$

$$m_{12}(\{\theta_2\}) = 0.256\,9, \quad m_{12}(\{\theta_3\}) = 0.342\,8.$$

From the obtained combination result, we can see the support of $\{\theta_1\}$ is the biggest, which conforms to the proposition that is supported by m_1. If we adopt the Dempster combination rule directly, we can get that $m_{12}(\{\theta_1\}) = 0$ and the support of $\{\theta_1\}$ will always be 0. Meanwhile, the Dempster combination rule has the problem of one ticket veto. However, the proposed DSIFS-TECM does not have this problem, showing that DSIFS-TECM has the ability of dealing with conflict evidences and it is conductive to making reasonable decisions.

5.2 Analysis for the ability of anti-interference

Based on the frame of discernment and the information for sensor S in the section 5.1, when the BPAF of S_1 at $t_3 = 16$ s is m_3, we consider the following two situations.

(i) $m_3(\{\theta_1\}) = 0.5, m_3(\{\theta_2\}) = 0.3, m_3(\{\theta_3\}) = 0.2.$

(ii) $m_3(\{\theta_1\}) = 0.1, m_3(\{\theta_2\}) = 0.75, m_3(\{\theta_3\}) = 0.15.$

The combination result at $t_3 = 16$ s for these two situations can be seen in Table 1.

Table 1 Temporal evidence combination result of DSIFS-TECM in the two situations

m_3	BPAF of m_{12}	RRF	Combination result
$m_3(\{\theta_1\}) = 0.5$	$m_{12}(\{\theta_1\}) = 0.400\,4$	$r_1 = 0.5$	$m(\{\theta_1\}) = 0.505\,4$
$m_3(\{\theta_2\}) = 0.3$	$m_{12}(\{\theta_2\}) = 0.256\,9$		$m(\{\theta_2\}) = 0.195\,4$
$m_3(\{\theta_3\}) = 0.2$	$m_{12}(\{\theta_3\}) = 0.342\,8$	$r_2 = 1$	$m(\{\theta_3\}) = 0.299\,2$
$m_3(\{\theta_1\}) = 0.1$	$m_{12}(\{\theta_1\}) = 0.400\,4$	$r_1 = 1$	$m(\{\theta_1\}) = 0.269\,9$
$m_3(\{\theta_2\}) = 0.75$	$m_{12}(\{\theta_2\}) = 0.256\,9$		$m(\{\theta_2\}) = 0.468\,7$
$m_3(\{\theta_3\}) = 0.15$	$m_{12}(\{\theta_3\}) = 0.342\,8$	$r_2 = 0.780\,8$	$m(\{\theta_3\}) = 0.261\,4$

From Table 1 we can see that the combination results are different in different situations. In situation (i), m_3 is close to m_1, and it can be seen as the tested system is interfered at t_2. This interference decreases the support of $\{\theta_1\}$ and leads to a wrong result. At t_3, the tested system recovers to the normal, the support of $\{\theta_1\}$ increases, and the final decision result is $\{\theta_1\}$. In situation (ii), m_3 is close to m_2, and it can be seen as the evidences got at t_1 have large deviations. With the passage of time, the tested system becomes more accurate, the support of $\{\theta_2\}$ increases, and the final decision result is $\{\theta_2\}$. These results show that DSIFS-TECM is sensitive to the change of evidence, and has the ability of anti-interference.

5.3 Experimental simulation

The experimental simulation in this section is conducted on a simulation system of ballistic midcourse target recognition, which is a typical MSTR system.

In the simulation system, continuous observations of targets in the finite time are given and recognition results are provided by adopting the process of spatial temporal evidence combination in Fig. 1. Typically, there are three sensors S_1, S_2 and S_3 at different positions. In order to meet the requirement that the evidence sources are independent in DS theory, the detection information between sensors and different time nodes of the same sensor in the simulation system must not affect each other. Feature extraction and classifier design are not specified here since they do not belong to the scope of this paper. As the possible targets are warhead, balloon and fragment, the frame of discernment for the simulated MSTR is $\Theta = \{\theta_1(\text{warhead}), \theta_2(\text{balloon}), \theta_3(\text{fragment})\}$. Based on the setup of war situation, recognizing the category of the targets depends on the detection information of these three sensors in the continuous time nodes, where the BPAFs of each sensor's detection information for the first five time nodes from the simulation system are shown in Table 2.

5.3.1 Comparison of anti-interference ability

The results of temporal evidence combination by the Dempster combination rule, temporal evidence combination based on composite reliability factor (TEC-CRF) (method in [13]) and DSIFS-TECM are shown in Table 3, where RTRF in TEC-CRF and DSIFS-TECM is set as $e^{-0.05\cdot\Delta t}$ and Δt represents the time difference between the adjacent time nodes.

Table 2 Evidences of three different sensors at five different moments

Time/s	BPAF	S_1	S_2	S_3
$t_1 = 5$	$m(\{\theta_1\})$	0.250	0.230	0.221
	$m(\{\theta_2\})$	0.299	0.306	0.350
	$m(\{\theta_3\})$	0.451	0.464	0.429
$t_2 = 8$	$m(\{\theta_1\})$	0.619	0.598	0.635
	$m(\{\theta_2\})$	0.203	0.251	0.225
	$m(\{\theta_3\})$	0.178	0.151	0.140
$t_3 = 16$	$m(\{\theta_1\})$	0.355	0.364	0.369
	$m(\{\theta_2\})$	0.266	0.246	0.236
	$m(\{\theta_3\})$	0.379	0.390	0.395
$t_4 = 23$	$m(\{\theta_1\})$	0.337	0.318	0.392
	$m(\{\theta_2\})$	0.303	0.269	0.203
	$m(\{\theta_3\})$	0.360	0.413	0.405
$t_5 = 26$	$m(\{\theta_1\})$	0.336	0.346	0.371
	$m(\{\theta_2\})$	0.312	0.305	0.228
	$m(\{\theta_3\})$	0.352	0.349	0.401

Then, using the Dempster combination rule, the results of spatial evidence combination that combines evidences of different sensors at the same moment from Table 3 are shown in Table 4.

From Tables 2 – 4, we can see that the simulation system is disturbed at $t_2 = 8$ s, resulting that the support of $\{\theta_1\}$ is bigger than that of $\{\theta_3\}$. With the addition of subsequent detection information, all these three methods can make the support of $\{\theta_3\}$ increase. However, the Dempster combination rule cannot recognize the correct result until $t_5 = 26$ s. As TEC-CRF and DSIFS-TECM consider attenuation of evidence reliability in the time domain and relative reliability between the evidences of adjacent time nodes, they can recognize the correct result after $t_3 = 16$ s and before $t_3 = 16$ s. The proposed DSIFS-TECM can recover from the disturbed state quicker since it takes the possible situation that the sensors may be disturbed by the outside into account. In other words, DSIFS-TECM has stronger ability of anti-inference, which plays an important role in the real-life applications.

Table 3 Result of temporal evidence combination by Dempster combination rule, TEC-CRF and DSIFS-TECM

Time/s	BPAF	Dempster combination rule			TEC-CRF			DSIFS-TECM		
		S_1	S_2	S_3	S_1	S_2	S_3	S_1	S_2	S_3
$t_1 = 5$	$m(\{\theta_1\})$	0.250	0.300	0.221	0.250	0.300	0.221	0.250	0.300	0.221
	$m(\{\theta_2\})$	0.299	0.256	0.350	0.299	0.256	0.350	0.299	0.256	0.350
	$m(\{\theta_3\})$	0.451	0.444	0.429	0.451	0.444	0.429	0.451	0.444	0.429
$t_2 = 8$	$m(\{\theta_1\})$	0.523 3	0.577 4	0.502 7	0.597 3	0.593 2	0.606 3	0.390 2	0.357 6	0.361 5
	$m(\{\theta_2\})$	0.205 2	0.206 8	0.282 1	0.203 5	0.240 6	0.237 4	0.250 9	0.287 9	0.316 2
	$m(\{\theta_3\})$	0.271 5	0.215 8	0.215 2	0.199 2	0.166 3	0.156 3	0.358 9	0.354 5	0.322 4
$t_3 = 16$	$m(\{\theta_1\})$	0.541 2	0.608 8	0.550 3	0.370 2	0.379 4	0.385 3	0.382 9	0.374 8	0.384 7
	$m(\{\theta_2\})$	0.159 0	0.147 4	0.197 5	0.259 9	0.241 9	0.231 9	0.227 4	0.225 6	0.228 7
	$m(\{\theta_3\})$	0.299 8	0.243 8	0.252 2	0.370 0	0.378 7	0.382 8	0.389 7	0.399 6	0.386 7
$t_4 = 23$	$m(\{\theta_1\})$	0.538 8	0.579 7	0.602 6	0.339 2	0.320 3	0.394 5	0.340 0	0.319 9	0.394 4
	$m(\{\theta_2\})$	0.142 3	0.118 7	0.112 0	0.298 4	0.263 7	0.198 1	0.296 4	0.262 7	0.198 0
	$m(\{\theta_3\})$	0.318 9	0.301 5	0.285 3	0.362 4	0.416 0	0.407 4	0.363 7	0.417 5	0.407 6
$t_5 = 26$	$m(\{\theta_1\})$	0.536 1	0.586 5	0.615 0	0.336 6	0.343 7	0.376 9	0.336 7	0.343 7	0.376 9
	$m(\{\theta_2\})$	0.131 5	0.105 9	0.070 2	0.307 4	0.296 0	0.213 7	0.307 2	0.295 9	0.213 6
	$m(\{\theta_3\})$	0.332 4	0.307 7	0.314 7	0.356 0	0.360 2	0.409 5	0.356 1	0.360 4	0.409 5

Table 4 Combinational results of Dempster combination rule, TEC-CRF and DSIFS-TECM

Time/s	Dempster combination rule			TEC-CRF			DSIFS-TECM		
	$m(\{\theta_1\})$	$m(\{\theta_2\})$	$m(\{\theta_3\})$	$m(\{\theta_1\})$	$m(\{\theta_2\})$	$m(\{\theta_3\})$	$m(\{\theta_1\})$	$m(\{\theta_2\})$	$m(\{\theta_3\})$
$t_1 = 5$	0.128 2	0.207 3	0.664 5	0.128 2	0.207 3	0.664 5	0.128 2	0.207 3	0.664 5
$t_2 = 8$	0.860 7	0.067 9	0.071 4	0.927 5	0.050 2	0.022 4	0.441 3	0.199 8	0.358 9
$t_3 = 16$	0.887 1	0.022 6	0.090 2	0.442 4	0.119 2	0.438 4	0.434 2	0.092 3	0.473 5
$t_4 = 23$	0.865 2	0.008 7	0.126 1	0.357 5	0.130 0	0.512 4	0.356 8	0.128 2	0.514 9
$t_5 = 26$	0.853 6	0.004 3	0.142 1	0.377 3	0.168 3	0.454 4	0.377 4	0.168 0	0.454 6

In order to make the upper analysis more intuitively, Pignistic probability distributions of the three mentioned methods are shown in Figs. 3 – 5.

In Figs. 3 – 5, $BetP(\theta_1)$, $BetP(\theta_2)$ and $BetP(\theta_3)$ represent the support of $\{\theta_1\}$, $\{\theta_2\}$ and $\{\theta_3\}$, respectively. From Fig.3, we can see that $BetP(\theta_1)$ decreases slowly and $BetP(\theta_3)$ increases also slowly after t_2 when adopting the Dempster combination rule. At $t_5 = 26$ s, the value of $BetP(\theta_1)$ is still larger than that of $BetP(\theta_3)$, and a wrong target $\{\theta_1\}$ is recognized. In Fig. 4 and Fig. 5, $BetP(\theta_1)$ decreases quickly and $BetP(\theta_3)$ increases also quickly after t_2 when adopting TEC-CRF and DSIFS-TECM. Furthermore, our proposed DSIFS-TECM recovers to the normal before t_3, while TEC-CRF recovers to the normal after t_3. These figures conform to our analysis before and can strengthen the conclusion that DSIFS-TECM has stronger ability of anti-inference.

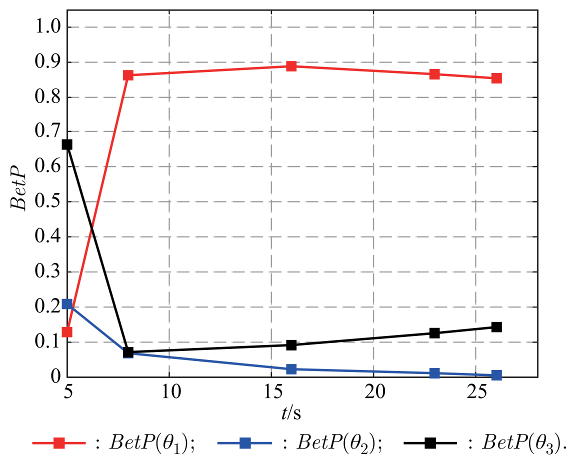

Fig. 3 Pignistic probability distribution of DS theory

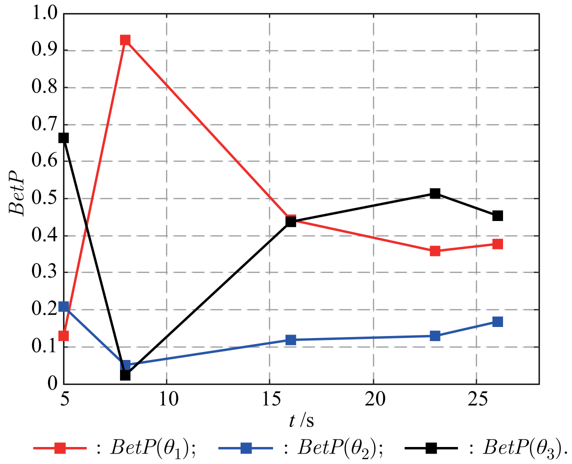

Fig. 4 Pignistic probability distribution of TEC-CRF

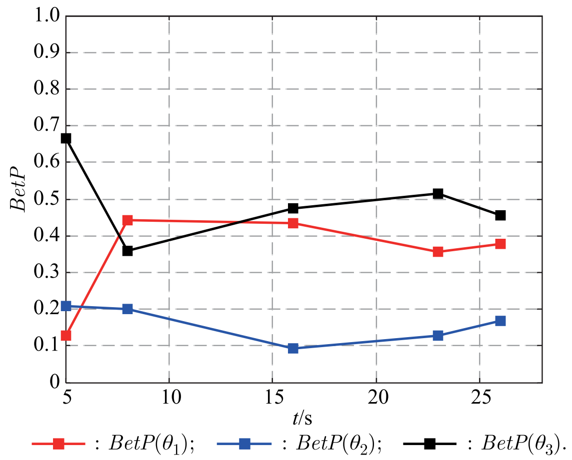

Fig. 5 Pignistic probability distribution of DSIFS-TECM

5.3.2 Characteristic of dynamic

According to the calculation process, the Dempster combination rule does not take time into consideration. However, the proposed DSIFS-TECM considers not only the relationship between the evidences at adjacent moments, but also the reliability over time. In order to show the dynamic characteristic of our proposed method, evidences at t_2 and t_3 in Table 2 are exchanged, with results shown in Tables 5 and 6, respectively.

Table 5 Result of temporal evidence combination when t_2 and t_3 are exchanged

Time/s	BPAF	Dempster combination rule			DSIFS-TECM		
		S_1	S_2	S_3	S_1	S_2	S_3
	$m(\{\theta_1\})$	0.250	0.300	0.221	0.250	0.300	0.221
$t_1=5$	$m(\{\theta_2\})$	0.299	0.256	0.350	0.299	0.256	0.350
	$m(\{\theta_3\})$	0.451	0.444	0.429	0.451	0.444	0.429
	$m(\{\theta_1\})$	0.523 3	0.577 4	0.502 7	0.342 2	0.357 4	0.352 2
$t_2=8$	$m(\{\theta_2\})$	0.205 2	0.206 8	0.282 1	0.261 7	0.237 1	0.237 6
	$m(\{\theta_3\})$	0.271 5	0.215 8	0.215 2	0.396 1	0.405 5	0.410 3
	$m(\{\theta_1\})$	0.541 2	0.608 8	0.550 3	0.499 7	0.620 3	0.524 1
$t_3=16$	$m(\{\theta_2\})$	0.159 0	0.147 4	0.197 5	0.205 5	0.211 3	0.195 3
	$m(\{\theta_3\})$	0.299 8	0.243 8	0.252 2	0.294 9	0.168 4	0.280 6
	$m(\{\theta_1\})$	0.538 8	0.579 7	0.602 6	0.425 9	0.474 1	0.493 4
$t_4=23$	$m(\{\theta_2\})$	0.142 3	0.118 7	0.112 0	0.238 6	0.219 0	0.151 1
	$m(\{\theta_3\})$	0.318 9	0.301 5	0.285 3	0.335 6	0.306 9	0.355 5
	$m(\{\theta_1\})$	0.536 1	0.586 5	0.615 0	0.385 2	0.422 1	0.448 2
$t_5=26$	$m(\{\theta_2\})$	0.131 5	0.105 9	0.070 2	0.262 9	0.246 4	0.153 7
	$m(\{\theta_3\})$	0.332 4	0.307 7	0.314 7	0.352 0	0.331 5	0.398 1

Table 6 Combinational results when t_2 and t_3 are exchanged

Time/s	Dempster combination rule			DSIFS-TECM		
	$m(\{\theta_1\})$	$m(\{\theta_2\})$	$m(\{\theta_3\})$	$m(\{\theta_1\})$	$m(\{\theta_2\})$	$m(\{\theta_3\})$
$t_1=5$	0.128 2	0.207 3	0.664 5	0.128 2	0.207 3	0.664 5
$t_2=8$	0.860 7	0.067 9	0.071 4	0.348 1	0.119 2	0.532 7
$t_3=16$	0.448 2	0.061 5	0.490 3	0.878 8	0.045 9	0.075 4
$t_4=23$	0.865 2	0.008 7	0.126 1	0.691 2	0.054 8	0.254 1
$t_5=26$	0.853 6	0.004 3	0.142 1	0.563 7	0.077 0	0.359 3

From Table 3 to Table 6, it can be seen that the result got by the Dempster combination rule is unchanged when the evidences at t_2 and t_3 are exchanged, and the result got by our proposed method is changed. This phenomenon reflects that the combination result of our proposed method can change along with time, satisfying the dynamic characteristic of MSTR system. Meanwhile, this phenomenon also shows that the Dempster combination rule only has the characteristic of spatial evidence combination.

6. Conclusions

In order to enhance the ability of dealing with conflict temporal evidences for MSTR system, DSIFS-TECM is proposed. To realize the method, the relationship between DS theory and IFS is analyzed, IFPD-IFV is defined, a novel ranking method with isotonicity for IFV is proposed, and a calculation method for RRF based on the proposed ranking method is designed.

Experiments show that DSIFS-TECM can handle conflicts in temporal evidences and has characteristic of dynamic, which reflects that the current combination results in the MSTR system are inheritance and renewal of the moment before. Moreover, when the MSTR system is disturbed, experiments also show that DSIFS-TECM has stronger ability of anti-interference.

Combination of temporal information is a complex engineering problem, and lots of factors need to be considered in the actual situation. Thus combining with other theories is the direction to be studied in the future.

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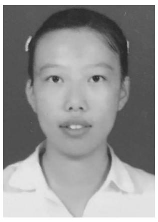
Biographies



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