Calculation and correction of magnetic object positioning error caused by magnetic field gradient tensor measurement

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Abstract: Magnetic field gradient tensor measurement is an important technique to obtain position information of magnetic objects. When using magnetic field sensors to measure magnetic field gradient as the coefficients of tensor, field differentiation is generally approximated by field difference. As a result, magnetic objects positioning by magnetic field gradient tensor measurement always involves an inherent error caused by sensor sizes, leading to a reduction in detectable distance and detectable angle. In this paper, the inherent positioning error caused by magnetic field gradient tensor measurement is calculated and corrected by iterations based on the systematic position error distribution patterns. The results show that, the detectable distance range and the angle range of an ac magnetic object (2.44 Am^2 @1 kHz) can be increased from (0.45 m, 0.75 m), (0° , 25°) to (0.30 m, 0.80 m), (0° , 80°), respectively.

Keywords: magnetic dipole, magnetic gradient tensor, positioning error, error correction.

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1. Introduction

Magnetic dipole is defined as a pair of poles or a closed loop of electric current with constant magnetic moment when their dimensions are close to zero [1]. Accordingly, a magnetic object can be modeled as a dipole at a remote place where the distance is more than 10 times of the object's dimension. Such magnetic objects may include magnetic particle attached tumor [2], magnetized unexploded ordnance [3–6], steel made vehicle [7] and cracks [8]. Thus the problems of detecting, classifying and positioning a magnetic dipole have attracted much attention owing to their wide applications in medical diagnose and treat-

ment, social security, intelligence transportation and non-destructive testing. Starting from 1940s, magnetic anomaly detecting (MAD) is a dominant positioning technique of magnetic objects by analyzing the anomaly signals from magnetic sensor arrays or aircraft routes [9]. However, its detecting distance is strongly limited by the earth field fluctuations, electromagnetic noises and even magnetic fields from carrier. Such problems are greatly solved by the appearance of the magnetic object positioning technique based on magnetic field gradient tensor, which includes nine elements in a 3rd order matrix. It is a more comprehensive description of magnetic field and field gradient, and it is insensitive to the orientation noise as well as the fluctuations of the regional background noise [10–12]. Besides the above advantages of magnetic gradient tensor, the rapid development of magnetic field measurement accuracy also contributes a lot to magnetic object positioning distance and accuracy [10]. In recent years, many other factors are studied to increase the positioning distance and accuracy such as hard and soft iron errors, sensor biases and non-orthogonality of the sensor configuration [13–15]. However, the positioning accuracy and its correction method are seldom reported. We find that the magnetic field gradient cannot be accurately obtained due to the non-ignorable dimension of sensors.

Derived from the basic definition of magnetic field gradient, Section 2 shows how the sensor dimension affects the gradient measurement accuracy and how much it does. Then, the numerical simulations give the inherent positioning error of magnetic dipole based on linear solution of Nara et al. [16] caused by inaccuracies gradient measurement. In Section 3, a correction algorithm is firstly proposed and verified to be feasible by simulation and experiments in Section 4. Finally, some conclusions are drawn in Section 5.

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2. Measurement of gradient tensor and positioning theory

2.1 Magnetic field gradient measurement error caused by sensor dimension

It is difficult to obtain the exact magnetic gradient tensor unless a group of extremely small sensors are applied. When a field $\mathbf{H}$ generated by a certain magnetic source is measured at a test point $\mathbf{r}$, the magnetic gradient tensor may be theoretically defined as

$$\mathbf{G} = \frac{\partial \mathbf{H}}{\partial \mathbf{r}} = \begin{pmatrix} \partial_x H_x & \partial_y H_x & \partial_z H_x \\ \partial_x H_y & \partial_y H_y & \partial_z H_y \\ \partial_x H_z & \partial_y H_z & \partial_z H_z \end{pmatrix} \quad (1)$$

where ∂ is the partial derivative of the magnetic field in the $\mathbf{r}$ direction. $\mathbf{n} = \mathbf{r}/r$, $\mathbf{n}$ is the unit vector along the $\mathbf{r}$ direction, $r = |\mathbf{r}|$ is the distance from the test point to the target. $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are unit vectors along the x , y and z directions, respectively. In technical application, each element of $\mathbf{G}$ is calculated by differencing outputs of two numbered sensors assembled with fixed distance (baseline length δ). Hereby, the estimated tensor $\overline{\mathbf{G}}$, may be rewritten in a limiting form as $\overline{\mathbf{G}} = \lim_{\delta \rightarrow 0} (\mathbf{H}_{1m1} - \mathbf{H}_{1m2})$, where $1m$ refers to x , y , z and suffixes 1, 2 refer to the two sensors.

However, due to the difficulty in satisfying limit condition $\delta \rightarrow 0$, there will be an inherent error between the theoretical gradient tensor $\mathbf{G}$, namely differential gradient, and the estimated gradient tensor $\overline{\mathbf{G}}$, namely differencing gradient. The inherent error is briefly illustrated by calculating results in Fig. 1 that shows the field $\mathbf{H}(\mathbf{m}, \mathbf{r})$, produced by a magnetic dipole placed at origin with its moment $\mathbf{m} = 1 \text{ Am}^2$, parallel to x axis. Then, the magnetic field at position $\mathbf{r}$ from a dipole with moment $\mathbf{m}$ can be formulized as

$$\mathbf{H}(\mathbf{m}, \mathbf{r}) = \frac{3(\mathbf{m} \cdot \mathbf{n})\mathbf{n} - \mathbf{m}}{4\pi r^3}. \quad (2)$$

To make the problem simpler, we suppose that the direction of magnetic moment coincides with the x axis. Take $\partial_x H_x$ for example, supposing that two sensors are assembled at $x = 1 \text{ m}$ and $x = 3 \text{ m}$ to measure the magnetic field gradient at $x = 2 \text{ m}$, the differential and differencing value of $\partial_x H_x$ may be represented by the slope of tangent dashed line l_{AB} , k_{AB} and that of dotted line l_{CD} , k_{CD} , respectively shown in Fig. 1. The line l_{AB} is the tangent to the field intensity curve at a range of 2 m , k_{AB} is the value of the differential gradient at this point. l_{CD} is a line that intersects the field intensity curve at ranges 1 m and 3 m , k_{CD} is the calculated difference gradient at range 2 m , $k_{AB} = 0.0268$ and $k_{CD} = 0.0769$.

Results show an obvious relative inherent error, $(k_{CD} - k_{AB})/k_{AB} \times 100\%$, up to 187% . Although this kind of inherent error may be somewhat alleviated by making δ smaller, sensor dimension effects still reduce the measurement accuracy of the magnetic gradient, leading to significant error of magnetic objects positioning based on magnetic gradient tensor.

Fig. 1 Reasons of magnetic field gradient measurement

2.2 Magnetic object positioning based on gradient tensor

In 2006, Nara et al. [14] proposed a simple linear equation system (3) with a unique solution to the magnetic dipole's position by measuring three components of the field and the magnetic field gradient tensor simultaneously.

$$\mathbf{r} = -3\mathbf{G}^{-1}\mathbf{H}. \quad (3)$$

Once the position is determined, the moment $\mathbf{m}$ can be derived as

$$\mathbf{m} = 2\pi r T \mathbf{H} \quad (4)$$

where T is the transform matrix defined as

$$\mathbf{T} = \begin{pmatrix} 3x^2 - 2r^2 & 3xy & 3xz \\ 3yz & 3y^2 - 2r^2 & 3yz \\ 3zx & 3zy & 3z^2 - 2r^2 \end{pmatrix} \quad (5)$$

Equation (3) indicates an accurate position $\mathbf{r}$ (real distance) of the magnetic dipole as long as the gradient $\mathbf{G}$ and the field $\mathbf{H}$ are precisely measured. However, the determined position $\mathbf{r}^*$ may vary around $\mathbf{r}$ because of the inherent error of $\overline{\mathbf{G}} - \mathbf{G}$ mentioned in Section 2. Thus, it will also lead to an offset on derived moment $\mathbf{m}^*$ to $\mathbf{m}$ according to (4).

In order to evaluate the relative positioning error, $e = \frac{|\mathbf{r}^* - \mathbf{r}|}{r} \times 100\%$, we assume that the three axial sensors are aligned along the same axis, and the centers of the sensors for measuring the components of the gradient tensor are located $\frac{\delta}{2}$ apart from $\mathbf{r}$

in the $\pm i, \pm j, \pm k$ directions. Thus, the three components of the field, H_x, H_y, H_z , and six components of the tensor, $\partial_x H_x, \partial_y H_y, \partial_z H_z, \partial_x H_y, \partial_y H_z, \partial_z H_x$, are measured and other three components of the tensor $\partial_y H_x, \partial_z H_y, \partial_x H_z$ are equal to $\partial_x H_y, \partial_y H_z, \partial_z H_x$ based on the symmetry of the tensor. For unknown $\mathbf{r}$ and $\mathbf{m}$, it will generate a magnetic field and field-gradient tensor following (1). The three field components and the gradient tensor can be measured, and $\mathbf{r}^*, \mathbf{m}^*$ can be obtained with (3) and (4) respectively.

With respects to [14], setting $\delta = 0.014$ m and assuming the angle, $0^\circ \leq \theta < 180^\circ$, between $\mathbf{m}$ and $\mathbf{r}$ is zero, numerical simulations are done when $\mathbf{r}$ is along three typical directions, namely $i, i+j$ and $i+j+k$. Three thin lines in Fig. 2 demonstrate that e caused by inherent error decreases as $\left(\frac{r}{\delta}\right)^{-1}$. The results show that a larger positioning error is caused by a larger baseline δ at constant distance r . This is consistent with the prediction at the end of Section 2.1.

Fig. 2 Systematic error and its corrected results

3. Positioning error correction

We define the positioning error, $e = \frac{|\mathbf{r}^* - \mathbf{r}|}{r} \times 100\%$, $\mathbf{r}^*$ is the positioning result, $\mathbf{r}$ is the true distance between the magnetic dipole and the gradiometer used to measure the magnetic field tensor. If e is less than 10%, we assume the magnetic objects can be positioned, and the distance $\mathbf{r}$ is called detectable distance. When e is equal to 10%, $\mathbf{r}$ is called critical distance. The detectable range is an interval formed by the critical distance. Based on the $\left(\frac{r}{\delta}\right)^{-1}$ regularity in Fig. 2, we guess that $\mathbf{r}^* - \mathbf{r}$ shares a certain distribution pattern and e may be corrected. If $\mathbf{H}$ of an unknown dipole with moment $\mathbf{m}$ is measured at $\mathbf{r}$, the determined position $\mathbf{r}^*$ and derived moment $\mathbf{m}^*$ could be easily obtained by (3) and (4), respectively.

To start the correction, the 1st step is substituting the obtained $\mathbf{r}^*$ and $\mathbf{m}^*$ into (1) to get a virtual field $\mathbf{H}^1$ and field gradient $\mathbf{G}^1$. Then, at the 2nd step, a new position $\mathbf{r}^1$ and moment $\mathbf{m}^1$ may be derived by using $\mathbf{H}^1, \mathbf{G}^1$ with (4) and (5). After that, the 3rd step is repeating the 1st and 2nd steps to get $\mathbf{r}^2$ and $\mathbf{m}^2$ on bias of $\mathbf{H}^2, \mathbf{G}^2$. Thus, in the 4th step, we may estimate the possible $\mathbf{e}^1 = \mathbf{r}^2 - \mathbf{r}^1$ around $\mathbf{r}^*$. Finally, by adjusting $\mathbf{e}^1$ to $\mathbf{r}^*$, a more accurate position $\mathbf{r}_{corrected}^* = \mathbf{r}^* - \mathbf{e}^1$ is obtained. Repeat steps 1–5 until the minus of corrected positions between two cycles is small enough. Fortunately, the correction algorithm is simultaneously verified to be convergent in the typical direction as is shown in thick lines in Fig. 2. Comparing the thin lines and thick lines in Fig. 2, it is obvious to see that the corrected positioning error is less than 100 dB. To make it more comprehensive, $i+j+k$ direction when $\mathbf{r} = 5\delta$ in Fig. 2 is chosen as an example and the iteration results of each step are listed in Table 1.

Table 1 Iteration results of each step

$\mathbf{r}=(4.041\text{E-}02, 4.041\text{E-}02, 4.041\text{E-}02)$		$\mathbf{m}=(5.774\text{E-}01, 5.774\text{E-}01, 5.774\text{E-}01)$	
$\mathbf{r}^*=(4.443\text{E-}02, 4.443\text{E-}02, 4.443\text{E-}02)$		$\mathbf{m}^*=(7.670\text{E-}01, 7.670\text{E-}01, 7.670\text{E-}01)$	
Iteration times		Corrected positions	
1	0.044 43	0.044 43	0.044 43
2	0.040 85	0.040 85	0.040 85
3	0.040 47	0.040 47	0.040 47
4	0.040 42	0.040 42	0.040 42
5	0.040 42	0.040 42	0.040 42
6	0.040 41	0.040 41	0.040 41
7	0.040 41	0.040 41	0.040 41
8	0.040 41	0.040 41	0.040 41

Considering the measurement error, we assume that the three components field $\mathbf{H}$ calculated by (2) contains relative error $\pm(\leq \varepsilon_h = 1\%)$ plus resolution $\pm(\leq \rho_h =$

8×10^{-4} A/m), and the field-gradient $\mathbf{G}$ obtained by using field difference to stand for field differential contains relative error $\pm(\leq \varepsilon_g = 1\%)$ plus resolution $\pm(\leq \rho_g =$

8×10^{-4} A/m), respectively. When such errors are taken into account as the same situation in Fig. 2, the comparison between corrected results plotted by thick lines and uncorrected results plotted by thin lines are shown in Fig. 3. From Fig. 3 the uncorrected results share a U shape as $\frac{r}{\delta}$ increases. Comparing the thick lines in Fig. 2 and the Fig. 3, we may conclude that inherent errors caused by the field difference instead of the field differential is the main error when $\frac{r}{\delta} < 20$ and the measurement errors caused by resolution of sensor is the main error when $\frac{r}{\delta} > 80$. The U shape lines mean that the detecting range is $20 < \frac{r}{\delta} < 80$, in which the positioning errors stay in a relatively low level.

Fig. 3 Comparison between corrected and uncorrected errors as a function of relative detecting distance considering the measurement noise

However, after the correction work is conducted, the left parts of U shape lines are flattened and drop to a level less than 3% as is shown by thick lines in Fig. 3. Thus G measurement is alleviated and the detecting distance is extended to $\frac{r}{\delta} < 80$.

Moreover, the numerical simulations are also conducted for fixed relative distance $\frac{r}{\delta} = 10$ in three typical directions when measurement error is considered and $\theta \in [0^\circ, 180^\circ)$ in order to verify the effectiveness of correction. We find that the positioning error is axial symmetric to $\theta = 90^\circ$. Thus, the results shown in Fig. 4 explain how the positioning error changes as θ grows from 0° to 90° . It is obvious that the corrected and uncorrected errors suddenly rise to a high level as θ approaching 90° . We may attribute this trend to the singularity of G when θ is close to 90° . Supposing that the duration of position error is 10%, we can see from Fig. 4 that the available detecting

range is $\theta < 60^\circ$ for uncorrected results. Nevertheless, the available detecting range is $\theta < 75^\circ$ for corrected results.

Fig. 4 Comparison between corrected and uncorrected errors as a function of dipole angle considering the measurement noise

4. Experiment

A coil (average area $S = 0.2 \text{ m}$, $n = 320$ turns) is set in a $1.2 \text{ m} \times 1.2 \text{ m}$ platform as a magnetic target, which is shown in Fig. 5. The coil is excited by the operational power amplifier (KEPCO-BOP 50–8ML), whose input signal is provided by the reference channel from LIA (locked-in amplifier, signal recover-7265). A sampling resistance, $r_s = 0.1 \text{ Ohm}$, is a series connected between the coil and the operational power amplifier to monitor the coil current. Adjust the Kepco power amplifier to achieve the current is 222.4 mA @ 1 kHz . Thus, the magnetic moment strength of the coil is calculated as 2.44 Am^2 @ 1 kHz according to the equation $m = nI_m S$. The full tensor magnetic gradiometer is configured by nine pairs of counter wound copper coils on print circuit board with half baseline length $\delta = 0.1 \text{ m}$. The induced voltage output of each gradiometer is measured by the input channels of LIA separately. To simplify the experiment, the target coil is moved along the x axis of the platform from $x = 0.3 \text{ m}$ to $x = 0.9 \text{ m}$ keeping its moment m parallel to the i direction.

Fig. 5 The experiment platform

Fig. 6 shows the positioning errors of the target coils based on (3) and its corrected positioning errors based on

the iteration in Section 3. Further investigation in Fig. 7 shows the uncorrected and corrected positioning error as a function of θ at $x = 0.5$ m. It can be seen from Fig. 6 and Fig. 7 that the experiment results show the same trends with the theoretical results in Fig. 3 and Fig. 4, respectively. The positioning error is alleviated from 25% level to 3% level when $\frac{r}{\delta} = 6$. Supposing that the duration of position error is 10%, we can see from Fig. 7 that the available detecting range is $\theta < 25^\circ$ for uncorrected results, nevertheless, the available detecting range is $\theta < 80^\circ$ for corrected results. From the analysis above, we see that the positioning error correction method extends the available detecting distance and the angle ranging from $\frac{r}{\delta} \in (4.5, 7.5)$, $\theta \in (0^\circ, 25^\circ)$ to $\frac{r}{\delta} \in (3, 8)$, $\theta \in (0^\circ, 80^\circ)$, respectively. And that both the available detecting distance and the angle can be extended to more than 20%.

Fig. 6 Comparison between the experimental uncorrected and corrected positioning error as a function of R

Fig. 7 Comparison between the experimental uncorrected and corrected positioning error as a function of θ

5. Conclusions

From the calculation, correction and experiment discussed above, we can conclude that the finite dimension of sen-

sors will cause a significant error of magnetic field gradient measurement and lead to an inherent positioning error in magnetic dipole positioning based on the magnetic field gradient tensor. However, this kind of positioning error can be alleviated according to the $\left(\frac{r}{\delta}\right)^{-1}$ regularity in Fig. 2. The algorithm is effective even though the contribution of random noise is considered. Simulation and experiment results demonstrate that both the detecting range and distance are improved more than 20% by the correction.

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