Mixed sensitivity H_∞ control for LTI systems with varying time delays

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Abstract: Designing a robust controller for a system with time-varying delays poses a major challenge. In this paper, we propose a method based on mixed sensitivity H_∞ for the control of linear time invariant (LTI) systems with varying time delays. The time delay is assumed bounded and the upper bound is known. In the technique we propose, the delay affecting the plant to be controlled is treated as an unmodeled uncertainty (in form of multiplicative uncertainty). That uncertainty is approximated and then an H_∞ based controller, for the plant represented by the multiplicative uncertainty and the nominal model, is calculated. The obtained H_∞ controller is used to control the LTI systems with varying time delays. Simulation examples are given to illustrate the effectiveness of the proposed method.

Keywords: linear time invariant (LTI) systems, mixed sensitivity H_∞ , varying time delay, unmodeled uncertainty, H_∞ control.

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1. Introduction

Time delays occur in many physical, industrial and engineering systems. The delay is often a major source of instability of the system and might lead to degradation in control performance. Thus, during the design of a controller for a system with time-delay, the delay shall be properly taken into consideration. In some case the delay can be considered as constant but in most control systems, the delay is varying.

The control of time-delay systems have attracted the attention of many researchers because of their importance and widespread occurrence [1–6]. Further, many other researchers have been interested in the stabilization problem of systems either with constant delay, time-varying delay, constant delay and parameter uncertainty or time-varying delay and parameter uncertainty [7–11]. There are also several research works related to control and stabilization of closed-loop systems with time delay [12,13].

H_∞ control for time-delay systems has been and still is an active field of research [14–19]. Although, there has been many published works on H_∞ control for systems with varying time delay [20–24], there is still room for further investigation of this topic and that will be our goal in this work.

Motivated by the above, a method based on mixed sensitivity H_∞ for the control of linear time invariant (LTI) systems with varying time delays is proposed. In this method, we assume that the time delay is bounded and the upper bound is known. The delay term is treated as unmodeled uncertainty which is approximated. Then a mixed sensitivity H_∞ controller for the nominal system plus the system approximating the delay term is then calculated. The obtained H_∞ controller is used to control the LTI system with varying time delay.

The method we propose is applicable to any LTI system with varying time delay for which the time delay is bounded and the upper bound is known. On the other hand, in this method the delay term is not increased to any value as it is proposed in many existing methods in the literature and which, i.e., increasing the delay term to a specific value, may lead to conservativeness [25]. Thus, we conclude that our method for the control of systems with varying time delays is less conservative than many existing ones.

The remainder of this paper is organized as follows: Section 2 presents the problem statement. In Section 3, a short review of the H_∞ control technique is given. The proposed method for the control of system with varying time delays using H_∞ control is presented in Section 4. Simulation examples illustrate the effectiveness of the proposed method in Section 5. Finally, conclusions are presented in Section 6.

2. Problem statement

Let's consider the closed-loop system shown in Fig. 1.

Fig. 1 Closed loop system with time delay

$G(s) = \frac{B(s)}{A(s)}$ represents the nominal (delay free) system, $B(s)$ and $A(s)$ are linear polynomial functions in Laplace operator s and of degrees m and n respectively. τ is varying-time delay which satisfies $0 < \tau \leq d$ (where d is a positive real number assumed known). $R(s)$ and $Y(s)$ are the reference signal and the output signal, respectively. $C(s)$ is the controller to be designed.

The objective is to find a controller C whose transfer function is a proper rational function of the form $C(s) = \frac{B_C(s)}{A_C(s)}$, where $B_C(s)$, $A_C(s)$ are linear polynomials functions in Laplace operator and of degrees m_C and n_C respectively and which is internally stabilizing the overall system and ensures good tracking of the system output to the reference signal against the delay variation in the time. In this research work a method based on mixed sensitivity H_∞ control is proposed to design such controller. Before presenting this method, a short review of the H_∞ control technique is given in the next two sections.

3. H_∞ control

The H_∞ optimisation approach, introduced by Zames and being developed in the last few decades and still an active research area, has been shown to be an effective and efficient robust design method for linear, time invariant control systems.

The theory of H_∞ control has been well studied and developed in the literature. In this section, we present a brief review of this technique and the mixed sensitivity approach. For more information about the theory of H_∞ control, see [26–28] and references therein.

3.1 H_∞ standard problem

Consider the standard feedback configuration shown in Fig. 2. In this figure, w denotes all the external inputs, such as command signals, disturbances, and sensor noises. z denotes the output signals to be minimized/penalized, which include both performance and robustness measures. y is the vector of measurements available to the controller K and u is the vector of control signals. $P(s)$ is called the generalized plant or interconnected system and K represents a controller.

Fig. 2 Standard feedback configuration

The standard H_∞ optimisation problem consists of finding K to minimize the H_∞ -norm of the transfer matrix from w to z under the constraint that the controller K internally stabilizes the system P .

Partition P as

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}. \quad (1)$$

Then we get the following algebraic equations from Fig. 2:

$$z = P_{11}w + P_{12}u \quad (2)$$

$$y = P_{21}w + P_{22}u \quad (3)$$

$$u = Ky. \quad (4)$$

The transfer matrix from w to z can be obtained directly as

$$z = [P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21}]w := \mathcal{F}_\ell(P, K)w \quad (5)$$

where $\mathcal{F}_\ell(P, K)$ is the lower linear fractional transformation of P and K . The design objective now becomes

$$\min_{K \text{ stabilising } P} \|\mathcal{F}_\ell(P, K)\|_\infty. \quad (6)$$

The H_∞ standard design problem can be also formulated as follows: given a plant $P(s)$ and a positive real number $\gamma > 0$, find a controller $K(s)$ that stabilizes the closed loop system and ensures that $\|\mathcal{F}_\ell(P, K)\|_\infty < \gamma$ [29].

3.2 H_∞ mixed sensitivity problem

It is well known that in control design problems, instead of interesting in a good tracking only, it is reasonable to combine it with other performance. For instance, it makes sense to require a good tracking as well as to limit the control signal energy, as depicted in Fig. 3. In this figure, the transfer function from w to z is

$$T_{zw}(s) = \begin{bmatrix} W_1(s)S(s) \\ W_2(s)T(s) \end{bmatrix} \quad (7)$$

where $S(s)$ and $T(s)$ are given by

$$S(s) = (I + G(s)K(s))^{-1} \quad (8)$$

$$T(s) = K(s)(I + G(s)K(s))^{-1} \quad (9)$$

respectively. $S(s)$ is called the sensitivity function and $T(s)$ is called the control sensitivity function. A small

function S expresses good tracking properties while a small function T expresses small inputs u . Note that there is a trade-off: making S smaller will in general make T larger.

The terms $W_1(s)$ and $W_2(s)$ are weighting functions, which shape the transfer functions $S(s)$ and $T(s)$ respectively.

The objective is: given the configuration depicted in Fig. 3 and a positive real number $\gamma > 0$, find a rational controller $K(s)$ stabilizing the closed-loop and satisfying

$$\|T_{zw}(s)\|_\infty = \left\| \begin{bmatrix} W_S(s) S(s) \\ W_T(s) T(s) \end{bmatrix} \right\|_\infty < \gamma \quad (10)$$

where $W_S(s)$ and $W_T(s)$ are weighting functions. In Fig. 3, r , v and e are the reference signal, the command input and the difference between the reference signal and the output, respectively.

Fig. 3 S/T mixed sensitivity configuration

4. Proposed approach

As we mentioned above, the objective in this research work is to design a controller $C(s)$ whose transfer function is a proper rational function of the form $C(s) = \frac{B_C(s)}{A_C(s)}$ where $B_C(s)$, $A_C(s)$ are linear polynomials functions in Laplace operator and of degrees m_C and n_C respectively (where $m_C < n_C$) and which is internally stabilizing the overall system in Fig. 1 and ensures good tracking of the system output to the reference signal. The approach we propose to meet that objective is as follows.

Step 1 The LTI system to control is represented by the transfer function $G_n(s) = G_0(s)e^{-\tau s}$, the delay term τ is treated as unmodeled uncertainty and the plant G_n is represented by multiplicative uncertainty with a nominal model denoted by G_0 (as given in [30]) and as follows:

$$G_n(s) = G_0(s)(1 + W_m(s)\Delta_m(s)) \quad (11)$$

where $\Delta_m(jw) \leq 1$ and $W_m(jw)$ is approximated as follows.

Since

$$\frac{G_n(s) - G_0(s)}{G_0(s)} = e^{-\tau s} - 1, \quad (12)$$

if we neglect the dynamics of $e^{-\tau s}$, the frequency response magnitude of the relative uncertainty is

$$\frac{|G_n - G_0|}{|G_0|} = |e^{-\tau s} - 1|. \quad (13)$$

From this expression one obtains that

$$|W_m(jw)| = \max \left| \frac{G_n - G_0}{G_0} \right| = \max |e^{-\tau s} - 1|. \quad (14)$$

Step 2 A mixed sensitivity H_∞ controller is designed as explained in subsection 3.2, shown in Fig. 3 and as follows:

- (i) $\Delta_m(s)$ is chosen so that $\Delta_m(jw) \leq 1$;
- (ii) W_m is approximated as explained above;
- (iii) G_n is calculated using (11);
- (iv) Two weighting functions and a positive real number are chosen, let them be denoted by $W_1(s)$, $W_2(s)$ and γ respectively;

(v) A controller is designed based on G_n using the Matlab mixsyn function. Let the controller be denoted by $C(s)$.

Step 3 The obtained controller $C(s)$ is used to control the closed-loop LTI system with time varying delays.

The approach we propose in this paper can be summarized in the following lemma.

Lemma 1 Given a controller $C(s)$ that stabilizes the closed-loop and satisfies

$$\left\| \begin{bmatrix} W_1(s) S(s) \\ W_2(s) T(s) \end{bmatrix} \right\|_\infty < \gamma$$

where

$$S(s) = (I + G_n(s)C(s))^{-1}$$

$$T(s) = C(s)(I + G_n(s)C(s))^{-1}$$

$$G_n(s) = G_0(s)(1 + W_m(s)\Delta_m(s))$$

$$\Delta_m(jw) \leq 1$$

$$|W_m(jw)| = \max \left| \frac{G_n - G_0}{G_0} \right| = \max |e^{-\tau s} - 1|$$

$W_1(s)$ and $W_2(s)$ are weighting functions and γ is a positive. Then, the controller $C(s)$ stabilizes the closed loop considered in Fig. 1 and ensures good tracking of the system output to the reference signal against the delay variation in the time.

5. Simulation examples

This section aims to present some simulation examples in order to show the effectiveness of the proposed approach.

5.1 Examples

5.1.1 Example 1

Consider the plant described by the following transfer function:

$$G_n(s) = G_0(s)e^{-\tau s}$$

where $G_0(s) = \frac{1}{5s+1}$ and $0 \leq \tau \leq 4$.

The objective is to design an H_∞ controller based on the approach proposed.

Firstly, a Matlab simulation is conducted in order to approximate the transfer function of W_m using the following equation:

$$|W_m(j\omega)| = \max |e^{-\tau s} - 1|, \quad 0 \leq \tau \leq 4.$$

The simulation result is

$$W_m(s) = \frac{2.006s + 0.02724}{s + 0.1315}.$$

Fig. 4 shows frequency responses for some random values of τ and the frequency response of $W_m(s)$.

Secondly, the weighting functions are selected as $W_1(s) = \frac{s+0.4}{2s+0.004}$, $W_2(s) = \frac{s+0.1}{0.01s+0.2}$ and the con-

$$C(s) = \frac{4.56s^4 + 93.17s^3 + 39.71s^2 + 4.52s + 0.05343}{0.0001s^5 + 10s^4 + 462.5s^3 + 242.7s^2 + 2.366s + 0.003754}.$$

The outputs of the closed loop system with the PID controller and with H_∞ controller are shown in Fig. 5.

Fig. 5 Comparison between PID controller and H_∞ controller performances

The simulation results show that the designed H_∞ controller achieves good performance in despite of existence of varying time delays in the closed loop. The H_∞ controller ensures better performance than the PID controller; as we can see the H_∞ avoids the overshoot.

5.1.2 Example 2

In this example, we consider the unstable plant with the time-varying delay described by the transfer function

$$G_n(s) = \frac{1}{s^2 - 1.75s + 0.375}e^{-\tau s}, \quad 0 \leq \tau \leq 0.75.$$

troller $C(s)$ is designed based on $G_n(s) = G_0(s)(1 + W_m(s)\Delta_m(s))$ using the Matlab mixsyn function.

Fig. 4 Approximation of uncertain time-delay τ by multiplicative uncertainty where $0 \leq \tau \leq 4$

As a result, we obtain

The objective is to design an H_∞ controller based on the approach proposed.

Firstly, a Matlab simulation is carried out to find the transfer function of W_m using the following equation:

$$|W_m(j\omega)| = \max |e^{-\tau s} - 1|, \quad 0 \leq \tau \leq 0.75.$$

The simulation gives the results

$$W_m(s) = \frac{2.141s + 0.01001}{s + 1.233}.$$

Fig. 6 shows frequency responses for some random values of τ (where $0 \leq \tau \leq 0.75$) and the frequency response of $W_m(s)$.

Fig. 6 Approximation of the uncertain time-delay τ by multiplicative uncertainty where $0 \leq \tau \leq 0.75$

Secondly, The weighting functions are selected as $W_1(s) = \frac{s+9.6}{2s+0.0096}$ and $W_2(s) = \frac{s+2.4}{0.001s+4.8}$ and

another Matlab simulation is carried out to compute the H_∞ controller $C(s)$ based on

$$G_n(s) = \frac{1}{s^2 - 1.75s + 0.375}(1 + W_m(s)\Delta_m(s))$$

$$C(s) = \frac{0.1173s^5 + 563.5s^4 + 1089s^3 + 574.9s^2 + 110.1s + 0.7085}{0.00001s^6 + 0.05017s^5 + 10.63s^4 + 128.9s^3 + 165.7s^2 + 1.839s + 0.005026}.$$

Fig. 7 presents the outputs of the closed loop system with the PID controller and the H_∞ controller.

Fig. 7 Comparison between PID controller and H_∞ controller performances for unstable system

The simulation results in Fig. 7 show that the H_∞ controller stabilizes the closed loop system and ensures good tracking of the output to signal reference while the PID controller fails to stabilize the system.

5.1.3 Example 3

This example deals with the non-minimum phase system whose transfer function is

$$G_n(s) = \frac{-4.906s^2 - 0.5884s + 335.17}{s^4 + 0.5544s^3 + 139.6s^2 + 27.91s} e^{-\tau s}, \quad 0 \leq \tau \leq 1.$$

The nominal system is picked from [31], and a varying-time delay term is added to it.

The objective is to design an H_∞ controller based on the approach proposed.

Firstly, a Matlab simulation is conducted in order to approximate the transfer function of W_m using the following equation:

$$|W_m(j\omega)| = \max |e^{-\tau s} - 1|, \quad 0 \leq \tau \leq 1.$$

The simulation gives the results

$$W_m(s) = \frac{2.136s + 0.02617}{s + 0.6758}.$$

using the Matlab mixsyn function.

The simulation result is

Fig. 8 shows frequency responses for some random values of τ (where $0 \leq \tau \leq 1$) and the frequency response of $W_m(s)$.

Fig. 8 Approximation of uncertain time-delay τ by multiplicative uncertainty where $0 \leq \tau \leq 1$

Secondly, the weighting functions are selected as $W_1(s) = \frac{s + 20}{2s + 0.0222}$ and $W_2(s) = \frac{s + 5}{0.001111s + 10}$, and another Matlab simulation is conducted to compute the H_∞ controller $C(s)$ based on $G_n(s) = \frac{-4.906s^2 - 0.5884s + 335.17}{s^4 + 0.5544s^3 + 139.6s^2 + 27.91s}(1 + W_m(s)\Delta_m(s))$ using the Matlab mixsyn function. Furthermore, a balanced truncation is applied to reduce the controller order. The result of the simulation and the balanced truncation is

$$C(s) = \frac{1.022s + 0.8012}{0.0001s^2 + 0.05235s + 1.625}.$$

Fig. 9 presents the outputs of the closed loop system with the PID controller and the H_∞ controller.

Fig. 9 Comparison between PID controller and H_∞ controller performances for non-minimum phase system

The simulation results show that the H_∞ controller ensures good tracking of the non-minimum phase system output to the reference signal.

5.2 Analysis of the simulation results

All the simulation results show that the method proposed in this work is applicable on stable, unstable and non-minimum phase LTI systems with time varying delays. The delay τ is not increased to d or to $2d$ as it is proposed in many existing methods in the literature and which may lead to conservativeness. This shows that our method is less conservative than many existing ones.

6. Conclusions

In this paper, we propose a method based on mixed sensitivity H_∞ for the control of linear time invariant systems with varying time delays. In this method the time delay is supposed bounded and the upper bound is known. The delay term is considered as unmolded uncertainty. That uncertainty is approximated and a mixed sensitivity H_∞ controller is then calculated. The validity of the method has been illustrated through three simulation examples. The first example deals with a system with time-varying, the second one deals with an unstable system and the third one deals with the non-minimum phase. The performance of the proposed controller has been compared with the conventional PID controller performance. The simulation results have shown the effectiveness of the proposed method.

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