

Analysis on decision-making model of plan evaluation based on grey relation projection and combination weight algorithm

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Abstract: In military service joint operations, when there are more operational forces, more multifarious materials are consumed, the support is more complex and fuzzy, the deployment of personnel is more rapid, and the support provided by wartime military material support powers can be more effective. When the principles, requirements, influencing factors and goals of military material support forces are deployed in wartime, an evaluation indicator system is established. Thus, a new combined empowerment method based on an analytic hierarchy process (AHP) is developed to calculate the subjective weights, and the rough entropy method is used to calculate the objective weights. Combination weights can be obtained by calculating the weight preference coefficient error, which is determined by combining the cooperative game method and the minimum deviation into objectives. This approach can determine the grey relation projection coefficient and synthesize the measure scheme superiority to finally optimize the deployment plan using the grey relation projection decision-making method. The results show that the method is feasible and effective; it can provide a more scientific and practical decision-making basis for the military material support power deployment in wartime.

Keywords: method for grey relation projection decision-making, military supply power in war, deployment plan, optimization, analytic hierarchy process (AHP), rough entropy method.

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1. Introduction

The effectiveness of the deployment of material support is related to operational processes in war, and support powers, which are important carriers of material support capability, aggregation and release, have a close relationship with operational processes and integrated support efficiency. To adapt to new material support circumstances, the complexity of deploying material support powers and the fuzzification of staff can be made more effective by choosing an objective and quickly creating an exact

material support force deployment plan. Then, military material support powers can be expeditiously deployed in wartime to provide increased support capacity. Accurate and efficient material support force deployment has important military and realistic significance for establishing and improving regional support systems, ensuring the stability and security of material support, improving the strain and the support capacity of the material support system and ensuring that the support is efficient.

The analytic hierarchy process (AHP), the entropy weighting method and the technique for order preference by similarity to ideal solution (TOPSIS) are typically applied in evaluations of the effectiveness of aerospace experiment to include deployment plan optimization [1], these methods provide techniques for optimizing a plan for deploying military material support forces in wartime. Liang [2] examined the use of the AHP method and the TOPSIS to determine primary and secondary indicator weights, respectively, in the selection of sustainable supplier countries for Iran's steel industry at three levels, and established and verified the feasibility of the fuzzy comprehensive evaluation model. Liu and Jiang [3–5] proposed applying entropy theory to information retrieval, organizational structure optimization and fuzzy comprehensive assessment of network survivability, which can provide a reference for scientific decision making. Jiang and Wu [6–9] proposed a method for decision making by combining the TOPSIS with the grey relational degree, finally, the comprehensive weight of the evaluation indicator can be determined by integrating AHPs and data envelopment analysis (DEA) with the grey relational method.

The grey system, which was founded by Chinese famous professor Deng, is a system of information incomplete [10]. The grey theory is to determine the unknown information system by using the known information and a process of making the system consist of “grey” into “white” [11]. Grey correlation and grey theory combined

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with other methods are applied to the decision. Jahan and Erginel [12,13] proposed a method of using the grey correlation degree of a subjective and objective integrated empowerment. References [14–19] applied grey correlation analysis and TOPSIS to integrated decision-making. References [20] and [21] applied hierarchical analysis and data envelopment analysis and grey correlation method to determine the integrated comprehensive weights of evaluation indexes. Yu and Fang [22,23] used the grey models and grey relational analysis for a methodology for evaluating micro-surfacing treatment on asphalt pavement and clustering for dynamic equivalent modeling of wind farm. Azadi and Dai [24,25] proposed grey relational analysis which can be applied to supplier selection and web service selection. At the same time, it can help to make profitable decisions and investments for applying in natural gas consumption or other aspects by using grey model combining diffusion based on optimization algorithms [26–29].

Based on the above considerations, this paper proposes an algorithm for determining weights using an AHP and a rough entropy method in combination to determine the indicator weights of military material support force deployment plans in wartime. Furthermore, to resolve the information inadequacies and dynamic changes due to inconsistent plan, this paper proposes a decision-making method in combination with the TOPSIS by defining the “grey relation projection coefficient” in a quantitative model of the

deployment plan. Using the weight algorithm based on a combination of an AHP and the rough entropy method, a method of making decisions based on grey relation projection is also proposed, and the effectiveness of the proposed algorithm and the decision-making method is verified by optimizing examples and military material support force deployment plan selections in wartime.

2. Ultimate principle

2.1 Establishment of a system of evaluation indicators for the plan and optimization steps

2.1.1 Establishment of a system of evaluation indicators for plan

When deploying military material support forces in accordance with operational plans and material support determinations, material support should be coordinated to ensure that deployment is timely and reliable; therefore, the plan should maximize support benefits, consider the risks and enhance the defense capability. The plan should successfully deploy support forces so that they complete support tasks reasonably and effectively. The deployment of support powers is influenced by operational deployment, combat power composition, strength, power deployment requirements and natural environmental factors; therefore, the related concept model of material support force deployment decisions shown in Fig. 1 is employed.

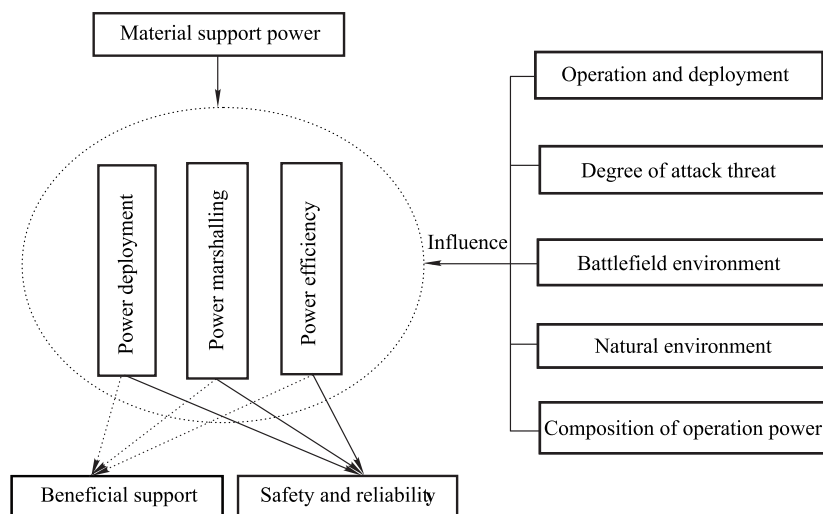


Fig. 1 Decision association diagram for material support power deployment plan

Fig. 1 shows the support power deployment plan, which is a system problem that is influenced by many factors. It is systematic, complex and comprehensive; only a certain indicator (property) is used because it cannot fully satisfy the requirements of evaluating a deployment plan. The power formation, operational environment, and battlefield can influence the deployment of military material supply forces

in wartime; therefore, this paper establishes a system of evaluation indicators for military material supply power deployment plans in wartime based on the principles, objectives and requirements of military material supply force deployment with attention to the objectivity and conciseness of the evaluation indicators and other characteristics, as shown in Fig. 2.

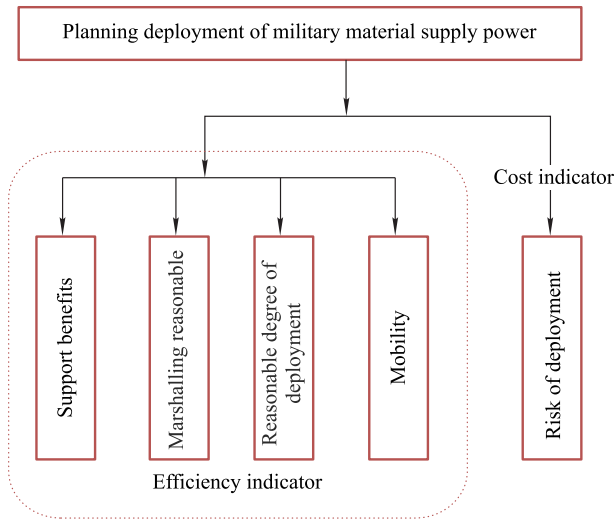


Fig. 2 Evaluation indicator system for a military material supply power deployment plan in wartime

2.1.2 Optimization steps of the evaluation plan

Suppose there are options $P_1, P_2, P_3, \dots, P_m$ for the material support force deployment plan, and each plan has n evaluation indicators. The initial matrix $\mathbf{R} = (x_{ab})_{n \times m}$ can be obtained, and the combination of weights can be determined by applying the AHP and the rough entropy method. The optimal plan can be determined by the grey relation projection-based decision-making method with the following steps:

Step 1 Establish the judgment matrix.

$$\mathbf{V} = (x_{ab})_{n \times m}, \quad a = 1, 2, \dots, n; b = 1, 2, \dots, m \quad (1)$$

The indicators can be classified as efficiency and cost indicators according to their characteristics.

Efficiency indicator:

$$x'_{ab} = x_{ab} / \max x_{ab}. \quad (2)$$

Cost indicator:

$$x'_{ab} = x_{ab} / \min x_{ab}. \quad (3)$$

Step 2 Establish the standardized matrix.

It is important that the established judgment matrix is standardized, and the indicator evaluation matrix is obtained as follows:

$$x'_{ab} = (x_{ab})_{n \times m} \quad (4)$$

$$a = 1, 2, \dots, n; b = 1, 2, \dots, m.$$

Step 3 In this paper, a new weighting algorithm is developed; it uses the AHP method to calculate the subjective weight and the rough entropy method to calculate the objective weight. The combination weights can be obtained by determining the weight preference coefficient error, which is determined by a combination with the co-

operative game method to find the minimum deviation of the objectives.

Step 4 The grey relation projection coefficient can be determined by the grey relation projection-based decision-making method using the numerical ranking of the grey relation projection coefficient to optimize the deployment plan.

2.2 Determination of the indicator weights

Indicator weights are important for comprehensive project evaluations and selecting the optimal military material support deployment plan. Their rationality and accuracy directly affect the reliability of the evaluation results. Presently, there are two methods for determining the indicator weights: a subjective empowerment method and an objective empowerment method. It is credible that combination weights can take both of these methods into consideration.

2.2.1 Determination of the subjective weights

This paper uses an AHP to determine the subjective weights. The main steps shown in Fig. 3 of this process are as follows.

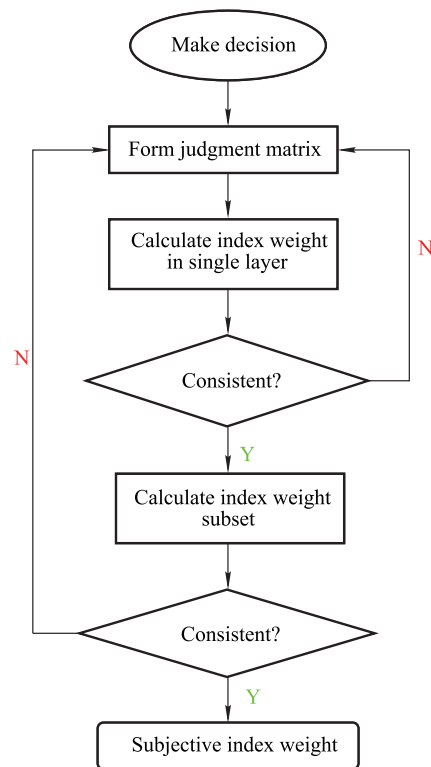


Fig. 3 Flow chart for determining the subjective weights using an AHP

Step 1 Divide the research object into different levels, including an objective layer, an attribute layer and

an indicator layer.

Step 2 Compare the importance of the factors belonging to different levels to form the judgment matrix.

Step 3 Obtain the weight of the judgment matrix and check its consistency. If the matrix passes the check, proceed to Step 4; otherwise, return to Step 2.

Step 4 Determine the subjective weight of each attribute using

$$\mathbf{w}_s = \overline{\mathbf{w}}_s / \sum_{s=1}^n \overline{\mathbf{w}}_s. \quad (5)$$

Step 5 Determine the subjective weights of the n indicators in the indicator layer using the AHP and

$$\mathbf{w}_s = (w_{s_1}, w_{s_2}, w_{s_3}, \dots, w_{s_n}). \quad (6)$$

2.2.2 Determination of the objective weights

Due to the indeterminacy of the indicators, this paper uses the rough entropy method to determine the objective weights. Suppose that the decision matrix for multiple attribute indicators is $\mathbf{B} = (\mathbf{U}, \mathbf{A}, \mathbf{V}, f)$. Then, the rough entropy can be determined with respect to the rough set $\mathbf{R}$ as follows:

$$E_R(x) = \frac{|R^-(x) - R_-(x)|}{|\mathbf{U}|} \ln(1 + |R^-(x) - R_-(x)|) + \sum_{i=1}^n \frac{|X_i|}{|\mathbf{U}|} \ln |x_i| \quad (7)$$

where $\mathbf{U} = (x_1, x_2, x_3, \dots, x_n)$ represents the whole domain, $\mathbf{A}$ represents the set of properties, $\mathbf{V} = \bigcup_{a \in \mathbf{A}} V_a$ represents a collection of attribute values, and $f: \mathbf{U} \times \mathbf{A} \rightarrow \mathbf{V}$ is the information function.

a is the importance of an attribute in $\mathbf{A}$; that is, $Sig(a, \mathbf{A})$ is defined as the change in the rough entropy when $a \notin \mathbf{A}$. The objective weight of the attribute is as follows:

$$\mathbf{w}_o = \frac{Sig(a_j, \mathbf{A})}{\sum_{a \in \mathbf{A}} (E_{A-\{a\}} - E_A(x))}. \quad (8)$$

The subjective weights of the n indicators in the available layer can be obtained using the rough entropy method as follows:

$$\mathbf{w}_o = (w_{o_1}, w_{o_2}, w_{o_3}, \dots, w_{o_n}). \quad (9)$$

2.2.3 Determination of the combination weights

Commonly, an “additive” integration method is used to determine combination weights. Assuming that ϕ and $1 - \phi$ are the subjective and objective weights of the indicator,

respectively, $\mathbf{w}_a = \phi \mathbf{w}_s + (1 - \phi) \mathbf{w}_o$ is considered to integrate the characteristics of the subjective and objective information. Therefore, ϕ and $1 - \phi$ represent the preference coefficient. Therefore, the optimization problem can be expressed as follows:

$$\begin{aligned} \min & \sum_{i=1}^n [\mu_s (\mathbf{w}_i - \mathbf{w}_{s_i})^2 + \mu_o (\mathbf{w}_i - \mathbf{w}_{o_i})^2] \\ \text{s.t.} & \sum_{i=1}^n \mathbf{w}_i = 1, \mathbf{w}_i > 0, \mu_s + \mu_o = 1. \end{aligned} \quad (10)$$

Traditional methods, such as equal distribution, which is used to determine combination weights, include error indicator preference coefficients and subjectivity perspectives. Therefore, this paper applies the cooperative game method to determine the preference coefficient error for combination weights ρ and the specific principle, as seen in the supporting literature [10].

2.3 Grey relation projection-based decision-making method

Several evaluation indicators for military material supply power deployment plans in wartime need to be evaluated to optimize the plan order. Evaluation indicator information presenting qualitative grey is often unknown, and the relationships between various indicators are uncertain; therefore, this paper combines grey system theory, the TOPSIS and vector projection to establish a grey relation projection-based decision-making model to optimize the wartime military material support force deployment plan. The specific steps are as follows:

Step 1 Establish positive and negative ideal decision matrices $\mathbf{x}_{ab}$ by calculating an evaluation indicator, X_{ab} ($a = 1, 2, \dots, n; b = 1, 2, \dots, m$). For the optimal decision matrix, assume $x_0 = (x_0(1), x_0(1), \dots, x_0(m))$ to obtain the best indicator value of the ideal matrix sequence; for the negative decision matrix, assume $x_0 = (x_0(1), x_0(1), \dots, x_0(m))$ for the worst indicator value of the ideal matrix sequence. First, we identify the set of optimal decision plans as follows:

The decision matrix $\mathbf{X}$ is as follows:

$$\mathbf{X} = \begin{bmatrix} C_1 & \cdots & C_m \\ P_1 & x_{11} & \cdots & x_{1m} \\ \vdots & \vdots & \ddots & \vdots \\ P_n & x_{n1} & \cdots & x_{nm} \end{bmatrix}. \quad (11)$$

Step 2 Establish positive and negative ideal plan deci-

sion matrices.

$$U_0^+ = \begin{bmatrix} u_{01}^+ & u_{02}^+ & \cdots & u_{0m}^+ \\ u_{11} & u_{11} & \cdots & u_{1m} \\ u_{21} & u_{22} & \cdots & u_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ u_{n1} & u_{n2} & \cdots & u_{nm} \end{bmatrix} \quad (12)$$

$$U_0^- = \begin{bmatrix} u_{01}^- & u_{02}^- & \cdots & u_{0m}^- \\ u_{11} & u_{11} & \cdots & u_{1m} \\ u_{21} & u_{22} & \cdots & u_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ u_{n1} & u_{n2} & \cdots & u_{nm} \end{bmatrix} \quad (13)$$

Step 3 Calculate the grey relational coefficient.

According to grey relation theory, the j th grey correlation coefficient based on the i th plan and the positive ideal plan U_0^+ can be calculated using

$$g_{ij}^+ = \frac{m + \partial M}{T_{ij}^+ + \partial M}, \quad \partial \in (0, 1). \quad (14)$$

In the equation,

$$\begin{cases} T_{ij}^+ = d(u_{0j}^+, u_{ij}^+) \\ m = \min_i \min_j T_{ij}^+ \\ M = \max_i \max_j T_{ij}^+ \end{cases}. \quad (15)$$

∂ is distinguish coefficient and taken for 0.5.

From the above, we obtain the grey correlation coefficient matrix for every plan, and the matrix for the positive ideal plan is g^+ . In the same way, we obtain the plan of the grey correlation coefficient matrix for every plan, and the matrix for the negative ideal plan is g^- .

$$g^+ = \begin{bmatrix} g_{01}^+ & g_{02}^+ & \cdots & g_{0m}^+ \\ g_{11}^+ & g_{12}^+ & \cdots & g_{1m}^+ \\ g_{21}^+ & g_{22}^+ & \cdots & g_{2m}^+ \\ \vdots & \vdots & \ddots & \vdots \\ g_{nm}^+ & g_{n2}^+ & \cdots & g_{nm}^+ \end{bmatrix} \quad (16)$$

$$g^- = \begin{bmatrix} g_{01}^- & g_{02}^- & \cdots & g_{0m}^- \\ g_{11}^- & g_{12}^- & \cdots & g_{1m}^- \\ g_{21}^- & g_{22}^- & \cdots & g_{2m}^- \\ \vdots & \vdots & \ddots & \vdots \\ g_{nm}^- & g_{n2}^- & \cdots & g_{nm}^- \end{bmatrix} \quad (17)$$

Step 4 Establish weights for the grey positive and negative ideal decision matrices.

Assume $g_{ij}(x_i, x_0)$ are the grey correlation coefficients x_i and x_0 in the data sequence of the ideal grey decision matrix at point j .

$$g_{ij} = \frac{\min_j \min_i |x_{0i} - x_{ij}| + \varepsilon \max_i \max_j |x_{0i} - x_{ij}|}{|x_{0i} - x_{ij}| + \varepsilon \max_i \max_j |x_{0i} - x_{ij}|}$$

$$\theta = \frac{\sum_i \sum_j^m |x_{0i} - x_{ij}|}{(\max_{1 \leq i \leq n} \min_{1 \leq j \leq m} |x_{0i} - x_{ij}|) \cdot mn} \quad (18)$$

Using the combination weight and the grey correlation coefficient, we determine the weighted grey positive and negative ideal incidence matrices, which are

$$g^{+*} = \begin{bmatrix} g_{01}^+ w_1 & g_{02}^+ w_2 & \cdots & g_{0m}^+ w_m \\ g_{11}^+ w_1 & g_{12}^+ w_2 & \cdots & g_{1m}^+ w_m \\ g_{21}^+ w_1 & g_{22}^+ w_2 & \cdots & g_{2m}^+ w_m \\ \vdots & \vdots & \ddots & \vdots \\ g_{nm}^+ w_1 & g_{n2}^+ w_2 & \cdots & g_{nm}^+ w_m \end{bmatrix}$$

$$g^{-*} = \begin{bmatrix} g_{01}^- w_1 & g_{02}^- w_2 & \cdots & g_{0m}^- w_m \\ g_{11}^- w_1 & g_{12}^- w_2 & \cdots & g_{1m}^- w_m \\ g_{21}^- w_1 & g_{22}^- w_2 & \cdots & g_{2m}^- w_m \\ \vdots & \vdots & \ddots & \vdots \\ g_{nm}^- w_1 & g_{n2}^- w_2 & \cdots & g_{nm}^- w_m \end{bmatrix}. \quad (19)$$

In the equation, $g^{+*} = \sum_j^m g_{ij}^+ w_j$ ($i = 1, 2, \dots, n$)

expresses the weighted correlation coefficient of each index in the positive ideal decision-making plan and $g^{-*} = \sum_j^m g_{ij}^- w_j$ ($i = 1, 2, \dots, n$) expresses the weighted correlation coefficient of each index in the negative ideal decision-making plan.

Step 5 Calculate the positive and negative grey relation projection values.

We consider each decision a row vector; therefore, the grey relation projection angle θ_i is the angle of intersection between a decision-making plan S_i ($i = 1, 2, \dots, n$) and the positive ideal plan S_0 . The cosine is as follows:

$$\cos \theta_i = \frac{S_i S_0}{|S_i| |S_0|} = \frac{\sum_j^m w_j g_{ij}^+ \times w_j}{\sqrt{\sum_j^m (w_j g_{ij}^+)^2} \times \sqrt{\sum_j^m w_j^2}}. \quad (20)$$

The projection of decision-making plan S_i ($i = 1, 2, \dots, n$) onto the positive ideal plan S_0 is as follows:

$$G_i^+ = |S_i| \cos \theta_i =$$

$$\frac{S_i S_0}{|S_0|} = \frac{\sum_j^m w_j g_{ij}^+ \times w_j}{\sqrt{\sum_j^m w_j^2}} = \sum_{j=1}^m \frac{w_j^2}{\sqrt{\sum_j^m w_j^2}} \times g_{ij}^+. \quad (21)$$

By normalizing the weights, we obtain the positive grey relation projection weight using

$$f_i = \frac{w_j^2}{\sqrt{\sum_{j=1}^m w_j^2}}, \quad j = 1, 2, \dots, m \quad (22)$$

$$Fd_i^+ = \sum_{j=1}^m g_{ij}^+ f_i, \quad i = 1, 2, \dots, n. \quad (23)$$

In the same way, we obtain the negative grey relation projection weight using

$$f_i = \frac{w_j^2}{\sqrt{\sum_{j=1}^m w_j^2}}, \quad j = 1, 2, \dots, m \quad (24)$$

$$Fd_i^- = \sum_{j=1}^m g_{ij}^- f_i, \quad i = 1, 2, \dots, n. \quad (25)$$

Step 6 Determine the grey relation projection coefficient. Suppose τ is the grey relation projection coefficient. We can determine the objective function using linear programming as follows $\min\{f(\tau)\}$. Using the equation $\frac{\delta f(\tau)}{\delta \tau} = 0$, we obtain τ .

$$\min\{f(\tau)\} = [(\tau Fd_i^+ - Fd_i^-)^2 + [(1-\tau)Fd_i^+ - Fd_i^-]^2],$$

$$\tau = \frac{Fd_i^{+2}}{Fd_i^{+2} + Fd_i^{-2}} \quad (26)$$

Step 7 We can identify the optimal plan by sorting the values of τ , i.e., the numerical size of the grey relation projection coefficient can be determined for a plan that is close to optimal and far from pessimal so that we can choose the optimal plan based on the size of τ .

3. Sample application

To verify the effectiveness of the combination weight algorithm and the grey relation projection-based decision-making method, the example of optimizing a military material support force deployment plan for a certain operation is used with the operational preparation and requirements for material support. Commanders compose four different plans for material support power deployment, which can be written as plans P_1 , P_2 , P_3 , and P_4 . Based on the experts' scores for the five evaluation indicators $C_1 - C_5$, we configure a system of evaluation indicators for each material support force deployment plan and quantify the interval between 0 and 1 on this basis. Therefore, a table

containing the evaluation indicators for the material support force deployment plan evaluation indicator data table can be found in Table 1.

Table 1 Evaluation indices for an operational supply power deployment plan

Plan	C_1	C_2	C_3	C_4	C_5
P_1	0.96	0.85	0.80	0.68	0.84
P_2	0.80	0.60	0.70	0.70	0.55
P_3	0.90	0.80	0.75	0.75	0.73
P_4	0.85	0.75	0.65	0.80	0.68

3.1 Determination of indicator weights

The normalized matrix can be obtained as follows:

$$Y' = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0.33 & 0.17 & 0 \\ 0.63 & 0.8 & 0.67 & 0.58 & 0.62 \\ 0.32 & 0.6 & 0 & 1 & 0.45 \end{bmatrix}.$$

According to the computational method used to determine the indicator weights, we determine the subjective and objective weights using (1)–(9). We do not explain the specific process of computing the weights due to its length and simplicity. The simplified calculation process is as follows:

$$w_s = [0.212 \quad 0.193 \quad 0.175 \quad 0.256 \quad 0.164]$$

$$w_o = [0.280 \quad 0.119 \quad 0.145 \quad 0.184 \quad 0.272].$$

Using the preference error, the combination weight can be determined as follows:

$$w_a = [0.217 \quad 0.188 \quad 0.173 \quad 0.251 \quad 0.172].$$

3.2 Counting in the grey relation projection-based decision-making method

First, we identify the set of optimal decision plans using Table 1 and construct the standard matrix S and the difference matrix S' as follows:

Optimal decision plan set:

$$\{0.96 \quad 0.85 \quad 0.8 \quad 0.8 \quad 0.55\}$$

$$S = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0.333 & 0.1667 & 0 \\ 0.625 & 0.8 & 0.666 & 0.583 & 0.621 \\ 0.313 & 0.6 & 0 & 1 & 0.448 \end{bmatrix}$$

$$S' = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0.667 & 0.833 & 1 \\ 0.375 & 0.2 & 0.334 & 0.417 & 0.379 \\ 0.687 & 0.4 & 1 & 0 & 0.552 \end{bmatrix}.$$

According to the equation in Step 4 of Section 2.3, the resolution ratio μ is 0.813; therefore, the grey correlation judgment matrix Z is as follows:

$$Z = \begin{bmatrix} 1 & 1 & 1 & 0.448 & 0.448 \\ 0.448 & 0.448 & 0.549 & 0.494 & 0.448 \\ 0.684 & 0.803 & 0.709 & 0.661 & 0.682 \\ 0.542 & 0.67 & 0.448 & 1 & 0.596 \end{bmatrix}.$$

The weighted grey positive and negative ideal incidence matrices are as follows:

$$g^{+*} = \begin{bmatrix} 0.217 & 0.188 & 0.173 & 0.251 & 0.172 \\ 0.217 & 0.188 & 0.173 & 0.084 & 0.172 \\ 0.072 & 0.063 & 0.066 & 0.089 & 0.172 \\ 0.101 & 0.110 & 0.084 & 0.113 & 0.093 \\ 0.082 & 0.086 & 0.058 & 0.251 & 0.112 \end{bmatrix}$$

$$g^{-*} = \begin{bmatrix} 0.217 & 0.188 & 0.173 & 0.251 & 0.172 \\ 0.072 & 0.067 & 0.058 & 0.251 & 0.093 \\ 0.217 & 0.188 & 0.127 & 0.215 & 0.093 \\ 0.117 & 0.082 & 0.089 & 0.142 & 0.172 \\ 0.162 & 0.104 & 0.173 & 0.084 & 0.131 \end{bmatrix}.$$

The optimal and pessimal values of the grey relation projection can be obtained from Steps 1–7 in Section 2.3, and the specific data are shown in Table 2.

Table 2 Optimal and pessimal values of the grey relation projection

Evaluation indicator	C_1	C_2	C_3	C_4	C_5
Fd_i^+	0.5	0.397	0.221	0.247	0.308
Fd_i^-	0.5	0.284	0.425	0.296	0.313

From the above steps, we obtain the following weight vector for grey relation projection:

$$W_{tj} = [0.115 \ 0.086 \ 0.073 \ 0.154 \ 0.072].$$

The grey relation projection coefficient can be obtained from the above, i.e.,

$$\tau = [0.662 \ 0.213 \ 0.411 \ 0.492].$$

Therefore, from P_1 , P_2 , P_3 , and P_4 (in numerical order) we select a desirable plan; the results are $P_1 > P_4 > P_3 > P_2$. When a commander chooses a material support power deployment plan, P_1 is the best choice, and P_4 is a better alternative in some operations.

3.3 Plan optimization comparison analysis

In order to further validate the feasibility and superiority of the proposed method, under the same condition of equal weight, the decision method of grey correlation projection analysis method is in contrast to pure TOPSIS or grey relative analysis method.

Only applying the TOPSIS can determine the degree of approximation for the plan and the ranking results (ranking first if the value is high) for $\xi = [0.46, 0.47, 0.50, 0.52]$; the grey correlation degree can be determined by only applying the grey relative analysis method and the ranking results (ranking first if the value is high) for $R = [0.82, 0.72, 0.61, 0.73]$. It can be found that compared with pure TOPSIS or grey relative analysis method, the optimization results' difference of combination weighting and grey relation projection-based decision-making method is more prominent for ranking. Compare the grey relation projection values with the degree of approximation or grey correlation, the better choosing optimized plan by applying the grey correlation projection method has more credibility than by applying the other methods and high degree of distinction among plans, and the methods which are put forward by this paper can more easily eliminate the worst plan.

4. Conclusions

This paper proposes a new algorithm for determining combination weights. It can calculate a combination weight, which consists of a subjective weight determined using an AHP and an objective weight determined using the rough entropy method in a cooperative game. The cooperative game proposed is more scientific and reasonable than determining a course of action using an average weighting method. The method also reduces the influence of subjective and objective factors in an evaluation of the plan indicator weights.

The grey optimization method proposed above is a combination of the weighting algorithm and the grey relation projection-based decision-making method that fully considers the influence of subjective and objective factors in its evaluation of the plan indicator weights, the grey character of the plan and the grey correlation between the influencing factors and the evaluation indicators.

This paper provides a procedure for optimizing military material support power deployment plans and concludes by verifying its practicability and maneuverability with the sample application of optimizing a deployment plan for military material support powers.

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