Multiple UAVs cooperative formation forming control based on back-stepping-like approach

ZHANG Liang*, LU Yi, XU Shida, and FENG Han

Aeronautical Engineering College, Air Force Engineering University, Xi'an 710038, China

Abstract: To ensure multiple unmanned aerial vehicles (UAVs) reach stable formation quickly, a cooperative guidance law based on the back-stepping-like approach is designed in this paper. Adopting the guidance mechanism of virtue leader vehicle, the dynamic equation of tracking errors for each UAV is built. The communication interactive relationships are described based on graph theory, and the guidance law for formation reaching is obtained by the back-stepping-like approach. The formation stability is analyzed by constructing an appropriate Lyapunov function. The simulation results have shown that this guidance and control law can make each UAV converge to the trajectory of the virtue leader ultimately, and has the quicker rate of convergence and lower tracking error.

Keywords: unmanned aerial vehicle (UAV), formation reaching, guidance law, stability.

DOI: 10.21629/JSEE.2018.04.16

1. Introduction

Multi-unmanned aerial vehicle (UAV) formation flight control is a typical application mode for UAV systems [1,2], which has been a hot topic in current academic and military research. Multiple UAVs forming can significantly improve performance such as collaborative reconnaissance, tracking, and identification [3–6]. Therefore, how to make the UAV in different initial positions rendezvous quickly, form a stable formation, and maintain the overall consistency, are the critical issues that the UAV collaborative control researcher need to solve.

In recent years, the relevant scholars have made extensive research from the theoretical and experimental point of view on the UAVs formation control issues. In [7–9], optimal control technique, navigation function and potential field method were used in solving the problem of formation control. In [10], the dynamic coupling characteristics of formation flight system were discussed,

and the formation trajectory tracking controller was designed by the linear quadratic method. These methods usually have a high computational cost and are difficult to implement in a distributed manner. Therefore, Stipanovic et al. used a distributed overlapping control technique to control a set of interconnected UAVs by designing a feedback controller for each UAV in the extended space which can convert it to the original space [11]. Moreover, some scholars introduce predictive control methods to solve the formation control problem. The basic idea is to use the receding optimization method [12]. In [13], the receding horizon control (RHC) method was used to study the high level formation control of UAV. In [14], a UAV formation control method based on hierarchical mechanism and model predictive control (MPC) strategy was proposed. The method is more accurate, but the character of real-time is poor. Therefore, this paper proposes a back-stepping control method to design the law of formation control. Back-stepping technique is a nonlinear control technique proposed by Kokotovic [15] whose basic principle is firstly designing the desired virtual control input and then recursively feedbacking it to the controller to get the actual control input. It can be used to design the controller by synthesizing the appropriate Lyapunov function. Compared with the linear feedback method, it has strong flexibility and does not require the linear closed-loop system.

The leader-follower method was applied in [16,17], where the first UAV is the leader and the others are the followers. It has simplified the multi-UAV control but there still exist some boundary. The West Virginia University proposed the virtual leader method [18,19], where all UAVs are considered into a whole rigid body. Each UAV only has to track the fixed point of the rigid body. Brad et al. studied this problem using the behavior control method and proposed the concept of Formation Geometry Center [20].

This paper takes the virtual lead aircraft guiding UAVs formation as the task scene. Firstly, the dynamic error

Manuscript received June 28, 2017.

*Corresponding author.

model of each UAV's position relative to the virtual leader is established. And the communication between UAV and the virtual leader is established by the graph theory. Secondly, the paper uses the backstepping-like method to design the synergetic guidance law of formation control and analyze its stability. Finally, the validity of the method is verified by constructing the simulation.

2. Problem description and modeling

Considering a scenario in which multiple UAVs form a formation by a virtual leader. Each UAV from different initial positions, tracking virtual leader flight path, quickly forms and maintains the formation. To achieve this goal, firstly a suitable UAV motion model need to be established, which accurately describes the communication interaction between UAVs.

2.1 UVA motion modeling

In general, assume the velocity vector $\mathbf{v}_g$ is perpendicular to the speed of the UAV, the longitudinal axis of the body, the resistance and the thrust on the same straight line, and the lift as shown in Fig. 1. Using the UAV particle model with three degrees of freedom, it is assumed that velocity, track angle and heading angle are first-order dynamic models. According to [21–24], the equations of motion are shown as

$$\begin{cases} \dot{x} = v_g \cos \gamma \cos \chi \\ \dot{y} = v_g \cos \gamma \sin \chi \\ \dot{z} = v_g \sin \gamma \\ \dot{v}_g = c_1(v_g^c - v_g) \\ \dot{\gamma} = c_2(\gamma^c - \gamma) \\ \dot{\chi} = c_3(\chi^c - \chi) \end{cases} \quad (1)$$

where (x, y, z) , v_g , χ and γ represent the position, velocity, heading angle and track angle of UAV in inertial coordinates, respectively. c_1, c_2 and c_3 ($c_1 > 0, c_2 > 0, c_3 > 0$) are time constants, respectively. v_g^c, γ^c and χ^c respectively are the command speed, track angle and heading angle.

Fig. 1 Sketch map of UAV three dimensional motion

For the virtual leader, the five-variable state UAV model in [25] is adopted, but the first-order dynamic model is still

used. At this point, the virtual machine's equations of motion are shown as

$$\begin{cases} \dot{x} = v_g \cos \gamma \cos \chi \\ \dot{y} = v_g \cos \gamma \sin \chi \\ \dot{z} = v_g \sin \gamma \\ \dot{v}_g = c_1(v_g^c - v_g) \\ \dot{\gamma} = \frac{g}{v_g} \phi \eta \\ \dot{\chi} = \frac{g \cos \gamma}{v_g} (\eta - 1) \end{cases} \quad (2)$$

where, χ is heading angle, γ is track angle, g is gravity acceleration, ϕ is roll angle, η is overload factor. Then, through the geometric relationship, the above formula is transformed into

$$\begin{cases} \dot{x} = v_g \cos \gamma \cos \chi \\ \dot{y} = v_g \cos \gamma \sin \chi \\ \dot{z} = v_g \sin \gamma \\ \dot{v}_g = c_1(v_g^c - v_g) \\ \dot{\gamma} = a_v^c / v_g \\ \dot{\chi} = a_h^c / v_g \cos \gamma \end{cases} \quad (3)$$

where a_v^c and a_h^c respectively represent the horizontal component and vertical component of the virtual leader's acceleration.

2.2 UAV communication interaction modeling

Considering a UAVs network system, the local communication relationship between the UAVs is represented by a dynamic graph $G(t)$. Considering a dynamic graph $G(t) = (V, E(t))$, V is a nonempty finite set $V = \{1, 2, \dots, n\}$, $E(t)$ is a set of edge. $E \subseteq V \times V$. Each UAV can be represented as a node, and their interaction is represented by an edge. The adjacency matrix is used to represent the interaction between the UAVs. Then, the adjacency matrix of graph G can be expressed as $\mathbf{A} = [a_{ij}]$, while a_{ij} is the weight of the graph, satisfying

$$a_{ij} = \begin{cases} 1, & (v_i, v_j) \in E \\ 0, & (v_i, v_j) \notin E \end{cases} \quad (4)$$

Here, edge $e_{ij} = (v_i, v_j)$ represents that node v_i can obtain information from node v_j . Thus, node v_i is considered to be the child node, and v_j is its father node. The neighbor set of node v_j can be represented by set $N_j = \{v_i : (v_i, v_j) \in E\}$. At the same time, the diagonal matrix $\mathbf{D}(t) = [d_{ij}]$ is defined, and the diagonal elements are the sum of the row elements of the matrix $\mathbf{A}$, $d_{ii} = \sum_j a_{ij}$, which presents the number of neighbors of the node v_i , which is also called in-degree.

Assume that the virtual leader is embedded in a subset of the network. Here, the reference track information of the

virtual leader at least one node in the network is available. The connectivity of the network is represented by matrix $\mathbf{B}$, defined as $\mathbf{B} = [b_{ij}]$. This is a diagonal matrix whose diagonal elements b_{ii} are traction gain and there is at least one node's $b_{ii} > 0$.

3. Cooperative formation guidance law

For each UAV, the desired dynamic equation is

$$\begin{cases} \dot{x}_i = v_{gi}^d \cos \gamma_i^d \cos \chi_i^d \\ \dot{y}_i = v_{gi}^d \cos \gamma_i^d \sin \chi_i^d \\ \dot{z}_i = v_{gi}^d \sin \gamma_i^d \end{cases} \quad (5)$$

where $i \in \{1, 2, \dots, N\}$, v_{gi}^d , γ_i^d and χ_i^d are generating control commands for the desired speed, track angle and heading angle.

Define the tracking error for each UAV as follows:

$$\begin{cases} e_{xi} = \sum_{j=1}^{N^i} a_{ij}(x_j - x_i) + b_i(x_r - x_i) \\ e_{yi} = \sum_{j=1}^{N^i} a_{ij}(y_j - y_i) + b_i(y_r - y_i) \\ e_{zi} = \sum_{j=1}^{N^i} a_{ij}(z_j - z_i) + b_i(z_r - z_i) \end{cases} \quad (6)$$

where $i \in \{1, 2, \dots, N\}$. Based on the matrix theory, the above formula is transformed into the following form:

$$\begin{cases} \dot{\mathbf{e}}_x = -(\mathbf{L} + \mathbf{B})(\mathbf{x} - \mathbf{x}_r \mathbf{\hat{1}}) \\ \dot{\mathbf{e}}_y = -(\mathbf{L} + \mathbf{B})(\mathbf{y} - \mathbf{y}_r \mathbf{\hat{1}}) \\ \dot{\mathbf{e}}_z = -(\mathbf{L} + \mathbf{B})(\mathbf{z} - \mathbf{z}_r \mathbf{\hat{1}}) \end{cases} \quad (7)$$

where $\mathbf{L} = [l_{ij}]$ is the Laplacian matrix, and $\mathbf{L} = (\mathbf{D} - \mathbf{A})$, $\mathbf{\hat{1}}$ is the unit vector of $N \times 1$.

Then, (7) can be obtained, the following compact form of the error dynamic equation is

$$\begin{cases} \dot{\mathbf{e}}_x = -(\mathbf{L} + \mathbf{B})(\mathbf{x} - \mathbf{x}_r \mathbf{\hat{1}}) \\ \dot{\mathbf{e}}_y = -(\mathbf{L} + \mathbf{B})(\mathbf{y} - \mathbf{y}_r \mathbf{\hat{1}}) \\ \dot{\mathbf{e}}_z = -(\mathbf{L} + \mathbf{B})(\mathbf{z} - \mathbf{z}_r \mathbf{\hat{1}}) \end{cases} \quad (8)$$

Assuming that the error equation is integrated at each position channel as $\dot{\mathbf{x}} = \mathbf{u}_x$, $\dot{\mathbf{y}} = \mathbf{u}_y$ and $\dot{\mathbf{z}} = \mathbf{u}_z$. Based on local feedback, the control law for the following position tracking is available:

$$\begin{cases} \mathbf{u}_x = \alpha_x \mathbf{e}_x + (\mathbf{L} + \mathbf{B}) \dot{\mathbf{x}}_r \mathbf{\hat{1}} \\ \mathbf{u}_y = \alpha_y \mathbf{e}_y + (\mathbf{L} + \mathbf{B}) \dot{\mathbf{y}}_r \mathbf{\hat{1}} \\ \mathbf{u}_z = \alpha_z \mathbf{e}_z + (\mathbf{L} + \mathbf{B}) \dot{\mathbf{z}}_r \mathbf{\hat{1}} \end{cases} \quad (9)$$

where α_x , α_y and α_z are respectively the diagonal positive definite gain matrices of the appropriate dimension.

It can be deduced that if the graph is highly connected, the $\mathbf{L} + \mathbf{B}$ is the positive definite matrix. Here, the expected

value is

$$\begin{cases} v_{gi}^d = \sqrt{(u_{xi})^2 + (u_{yi})^2 + (u_{zi})^2} \\ \gamma_i^d = \tan^{-1} \left(\frac{u_{zi}}{\sqrt{(u_{xi})^2 + (u_{yi})^2}} \right) \\ \chi_i^d = \sin^{-1} \left(\frac{u_{yi}}{u_{xi}} \right) \end{cases} \quad (10)$$

Define the errors of $\mathbf{v}_g$, γ and χ as

$$\begin{cases} \mathbf{e}_{vg} = \mathbf{v}_g - \mathbf{v}_g^d \\ \mathbf{e}_\gamma = \gamma - \gamma^d \\ \mathbf{e}_\chi = \chi - \chi^d \end{cases} \quad (11)$$

where $\mathbf{v}_g = [v_{g1}, v_{g2}, \dots, v_{gN}]^T$, $\gamma = [\gamma_1, \gamma_1, \dots, \gamma_N]^T$, $\chi = [\chi_1, \chi_2, \dots, \chi_N]^T$.

To ensure that the designed control law is convergent, the following error equation is given, according to the exponential convergence function:

$$\begin{cases} \dot{\mathbf{e}}_{vgi} = -\lambda_{vgi} \mathbf{e}_{vgi} \\ \dot{\mathbf{e}}_{\gamma i} = -\lambda_{\gamma i} \mathbf{e}_{\gamma i} \\ \dot{\mathbf{e}}_{\chi i} = -\lambda_{\chi i} \mathbf{e}_{\chi i} \end{cases} \quad (12)$$

where λ_{vgi} , $\lambda_{\gamma i}$ and $\lambda_{\chi i}$ are the specified positive numbers.

The derivatives of v_{gi}^d , λ_i^d and χ_i^d are

$$\begin{cases} \dot{v}_{gi}^d = \frac{\dot{u}_{xi}u_{xi} + \dot{u}_{yi}u_{yi} + \dot{u}_{zi}u_{zi}}{v_{gi}^d} \\ \dot{\gamma}_i^d = \frac{1}{(v_{gi}^d)^2} \frac{\dot{u}_{zi}((u_{xi})^2 + (u_{yi})^2) - u_{zi}(u_{xi}\dot{u}_{xi} + u_{yi}\dot{u}_{yi})}{\sqrt{(u_{xi})^2 + (u_{yi})^2}} \\ \dot{\chi}_i^d = \frac{\dot{u}_{yi} \cos \chi_i^d}{\sqrt{(u_{xi})^2 + (u_{yi})^2}} - \frac{\dot{u}_{xi} \sin \chi_i^d}{\sqrt{(u_{xi})^2 + (u_{yi})^2}} \end{cases} \quad (13)$$

As can be seen from (13) that the derivatives of v_{gi}^d , λ_i^d and χ_i^d need the derivative of the control signal from the position control loop. That is the derivative of u_{xi} , u_{yi} and u_{zi} . Taking the derivative of (9),

$$\begin{cases} \dot{\mathbf{u}}_x = \alpha_x \dot{\mathbf{e}}_x + (\mathbf{L} + \mathbf{B}) \ddot{\mathbf{x}}_r \mathbf{\hat{1}} \\ \dot{\mathbf{u}}_y = \alpha_y \dot{\mathbf{e}}_y + (\mathbf{L} + \mathbf{B}) \ddot{\mathbf{y}}_r \mathbf{\hat{1}} \\ \dot{\mathbf{u}}_z = \alpha_z \dot{\mathbf{e}}_z + (\mathbf{L} + \mathbf{B}) \ddot{\mathbf{z}}_r \mathbf{\hat{1}} \end{cases} \quad (14)$$

It should be noted that the implementation of position tracking control loop requires u_{xi} , u_{yi} and u_{zi} . To achieve speed level tracking requires derivatives of u_{xi} , u_{yi} and u_{zi} . And the tracking scheduled reference trajectory control law can be obtained in accordance with the position variable. If the expected velocity, path angle and heading angle variable are consistent, the expected dynamic equation of (12) can be improved to

$$\begin{cases} \dot{\mathbf{e}}_{vg} = -\lambda_{vg} \mathbf{e}_{vg} + (\mathbf{L} + \mathbf{B}) v_{gr} \mathbf{\hat{1}} \\ \dot{\mathbf{e}}_\gamma = -\lambda_\gamma \mathbf{e}_\gamma + (\mathbf{L} + \mathbf{B}) \gamma_r \mathbf{\hat{1}} \\ \dot{\mathbf{e}}_\chi = -\lambda_\chi \mathbf{e}_\chi + (\mathbf{L} + \mathbf{B}) \chi_r \mathbf{\hat{1}} \end{cases} \quad (15)$$

where λ_{vg} , λ_γ and λ_χ are diagonal positive definite gain matrices. Jointing (1) and (15), the covariance control law is

$$\begin{cases} v_g^{ic} = v_{gi} - \frac{1}{c_{1i}}[-\dot{v}_{gi}^d + \lambda_{vgi}e_{vgi} - b_{ii}v_{gr}] \\ \gamma^{ic} = \gamma_i - \frac{1}{c_{2i}}[-\dot{\gamma}_i^d + \lambda_{\gamma i}e_{\gamma i} - b_{ii}\gamma_r] \\ \chi^{ic} = \chi_i - \frac{1}{c_{3i}}[-\dot{\chi}_i^d + \lambda_{\chi i}e_{\chi i} - b_{ii}\chi_r] \end{cases} \quad (16)$$

4. Stability analysis

To prove the closed-loop stability of the whole system, select the Laypunov function like

$$W = \frac{1}{2}(e_{vg}^T e_{vg} + e_\gamma^T e_\gamma + e_\chi^T e_\chi + e_x^T e_x + e_y^T e_y + e_z^T e_z). \quad (17)$$

Taking the time derivative of W :

$$\dot{W} = e_{vg}^T \dot{e}_{vg} + e_\gamma^T \dot{e}_\gamma + e_\chi^T \dot{e}_\chi + e_x^T \dot{e}_x + e_y^T \dot{e}_y + e_z^T \dot{e}_z. \quad (18)$$

Jointing (15), (8) and (9), the time derivative of the Lyapunov function is given by

$$\begin{aligned} \dot{W} = & e_{vg}^T \dot{e}_{vg} + e_\gamma^T \dot{e}_\gamma + e_\chi^T \dot{e}_\chi + e_x^T \dot{e}_x + e_y^T \dot{e}_y + e_z^T \dot{e}_z = \\ & e_{vg}^T (-\lambda_{vg}e_{vg} + (L+B)v_{gr}\dot{\mathbf{z}}) + \\ & e_\gamma^T (-\lambda_\gamma e_\gamma + (L+B)\gamma_r\dot{\mathbf{z}}) + \\ & e_\chi^T (-\lambda_\chi e_\chi + (L+B)v_\chi\dot{\mathbf{z}}) + \\ & e_x^T (-\alpha_x(L+B)e_x - (L+B)(L+B-I)\dot{x}_r\dot{\mathbf{z}}) + \\ & e_y^T (-\alpha_y(L+B)e_y - (L+B)(L+B-I)\dot{y}_r\dot{\mathbf{z}}) + \\ & e_z^T (-\alpha_z(L+B)e_z - (L+B)(L+B-I)\dot{z}_r\dot{\mathbf{z}}). \end{aligned} \quad (19)$$

According to the related theorem of graph theory, if a graph has a directed spanning tree, the correlation matrix $(L+B)$ is a positive definite matrix. If the minimum and maximum singular values of the matrix $(L+B)$ are represented by $\underline{\sigma}$ and $\bar{\sigma}$ respectively, $\underline{\lambda}_{vg}$, $\underline{\lambda}_\gamma$, $\underline{\lambda}_\chi$ represent the minimum of λ_{vg} , λ_γ , λ_χ , $\underline{\alpha}_x$, $\underline{\alpha}_y$, $\underline{\alpha}_z$ represent the minimum of α_x , α_y , α_z , then

$$\begin{aligned} \dot{W} \leq & -\underline{\lambda}_{vg}\|e_{vg}\|^2 + \bar{\sigma}\|e_{vg}\|\|v_{gr}\dot{\mathbf{z}}\| - \underline{\lambda}_r\|e_\gamma\|^2 + \\ & \bar{\sigma}\|e_\gamma\|\|\gamma_r\dot{\mathbf{z}}\| - \underline{\lambda}_\chi\|e_\chi\|^2 + \bar{\sigma}\|e_\chi\|\|\chi_r\dot{\mathbf{z}}\| - \underline{\alpha}_x\|e_x\|^2 + \\ & \bar{\sigma}\|e_x\|\|\dot{x}_r\dot{\mathbf{z}}\| - \underline{\alpha}_y\|e_y\|^2 + \bar{\sigma}\|e_y\|\|\dot{y}_r\dot{\mathbf{z}}\| - \\ & \underline{\alpha}_z\|e_z\|^2 + \bar{\sigma}\|e_z\|\|\dot{z}_r\dot{\mathbf{z}}\|. \end{aligned} \quad (20)$$

where $\bar{\sigma}_i$ is the maximum singular value of $(L+B-I)$.

Note that the reference trajectory is bounded, which is $\|v_{gr}\dot{\mathbf{z}}\| \leq v_{gr,\max}$, $\|\gamma_r\dot{\mathbf{z}}\| \leq \gamma_{r,\max}$, $\|\chi_r\dot{\mathbf{z}}\| \leq \chi_{r,\max}$, $\|\dot{x}_r\dot{\mathbf{z}}\| \leq x_{dr}$, $\|\dot{y}_r\dot{\mathbf{z}}\| \leq y_{dr}$, $\|\dot{z}_r\dot{\mathbf{z}}\| \leq z_{dr}$:

$$\begin{aligned} \dot{W} \leq & -\underline{\lambda}_{vg}\|e_{vg}\|^2 + \bar{\sigma}\|e_{vg}\| - \underline{\lambda}_r\|e_\gamma\|^2 + \\ & \bar{\sigma}\gamma_{r,\max}\|e_\gamma\| - \underline{\lambda}_\chi\|e_\chi\|^2 + \bar{\sigma}\chi_{r,\max}\|e_\chi\| - \underline{\alpha}_x\|e_x\|^2 + \\ & \bar{\sigma}\|e_x\|x_{dr} - \underline{\alpha}_y\|e_y\|^2 + \bar{\sigma}y_{dr}\|e_y\| - \underline{\alpha}_z\|e_z\|^2 + \\ & \bar{\sigma}\|e_z\|z_{dr} \leq -\beta\|s\|^2 + r\|s\|. \end{aligned} \quad (21)$$

Define $P = \text{diag}(\underline{\lambda}_{vg}, \underline{\lambda}_r, \underline{\lambda}_\chi, \underline{\alpha}_x, \underline{\alpha}_y, \underline{\alpha}_z)$, in which β is the minimum singular value of the matrix P . Meanwhile

$$\begin{aligned} r = & \|\bar{\sigma}v_{gr,\max}, \bar{\sigma}\gamma_{r,\max}, \bar{\sigma}\chi_{r,\max}, \bar{\sigma}_i x_{dr}, \bar{\sigma}_i y_{dr}, \bar{\sigma}_i z_{dr}\| \\ s = & [\|e_{vg}\|, \|e_\gamma\|, \|e_\chi\|, \|e_x\|, \|e_y\|, \|e_z\|]^T. \end{aligned}$$

Obviously, the derivative form of the Lyapunov function is given by

$$\dot{W} \leq -2\beta W + r\sqrt{2W}. \quad (22)$$

Thus,

$$\begin{aligned} \sqrt{W} \leq & \sqrt{W(0)} \exp(-\beta t) + \frac{r}{\beta}(1 - \exp(-\beta t)) \leq \\ & \sqrt{W(0)} + \frac{r}{\sqrt{2}\beta}. \end{aligned} \quad (23)$$

Therefore the conclusions are

(i) If $x_r, y_r, \dots = \text{constant}$, $r = 0$, the graph has a directed spanning tree, and all UAVs in the formation can achieve the same consistency.

(ii) If the desired reference state is constant and there is at least a second derivative, the trajectory tracking error is bounded. Using a large control gain β , the tracking error can be arbitrarily small.

(iii) The system is asymptotically stable if it includes a directed graph with spanning tree, and the state and its second derivative are uniformly bounded at least so that all tracking errors are uniformly bounded.

At this point, according to the above three conclusions, the stability of the guidance system designed in this paper is verified.

5. Simulation analysis

In order to verify the validity of the guidance law of the UAVs coordinated fleet designed in this paper, the simulation environment is built based on Matlab Simulink. All the calculations are operated in a computer, with Intel Core I5 CPU, clocked at 2.8 Ghz, Windows operating system.

Consider a multi-UAV system with three UAVs as shown in Fig. 2. The communication network graph is a strongly connected graph, assuming that the virtual leader

is embedded in the subset of the graph UAV1, then the adjacency matrix A , Laplacian matrix L , traction matrix B and diagonal matrix D are respectively

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \quad L = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ -1 & -1 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$$

Fig. 2 Communication topology of formation control

In the simulation, the initial conditions of each UAV are shown in Table 1, the virtual leader vehicle's other parameters of the initial value of $v_{gr}^c(0) = 12$ m/s, $a_{hr}^c(0) = 0.2$ m/s², $a_{vr}^c(0) = 0.2$ m/s². Meanwhile, the actuator constant and control gain of the three UAVs are: $c_1 = c_2 = c_3 = 10$, $\alpha_1 = \alpha_2 = \alpha_3 = 2$, $\lambda_{vg} = \lambda_\gamma = 2$, $\lambda_\chi = 2$.

Table 1 Initial conditions of each UAV

Number	Status			
	Position/ m	Speed/ (m/s)	Track angle/(°)	Heading angle/(°)
Virtual lead aircraft	(12,100,100)	15	12	12
UAV1	(40,60,70)	8	8	8
UAV2	(50,80,90)	10	13	13
UAV3	(60,100,110)	12	18	18

Simulation results are shown in Figs. 3–5. From the figures, it is clear that from the different initial positions of the three UAVs, under the guidance of the virtual lead aircraft, their relative positions and tracking errors constantly change with time. After about 3 s, UAVs converge to the flight path of the virtual leader vehicle, and eventually reach the consistency of position and speed, thus forming a stable formation.

Fig. 3 Three dimensional trajectory of UAV formation forming

Fig. 4 Change curve of positions tracking error

Fig. 5 Change curve of control inputs tracking error

In order to further verify the advantages of the formation of the guidance system in the convergence rate and tracking error, the optimal control method and the MPC method are used to simulate the simulation under the same initial conditions. It can be observed from Fig. 4 and Fig. 5 that the position tracking error and the control input tracking error of UAV3 change with time while the stable formation is formed. Therefore, in order to make the comparison results more representative, UAV3 tracking virtual is taken leader as an example to verify the three methods in the position tracking error comparison shown in Fig. 6.

Fig. 6 Comparison of three methods on position tracking error

It is clear that, using these three methods, UAV3 can eventually converge to the flight path of the virtual leader, but the time required is not the same. The convergence of the MPC method is the longest while the error fluctuation is less than the optimal control method. The convergence time of the optimal control method is shorter than that of the MPC method, its error fluctuating is larger however. The method introduced in this paper is superior to the optimal control method and the MPC method in terms of convergence rate and tracking error.

5. Conclusions

In this paper, the steering system is constructed based on the back-stepping control method of UAVs, and its convergence is proved by Lyapunov stability theorem. Compared with the optimal control and the MPC method, this method can form a stable multi-machine formation with faster convergence rate and smaller tracking error. It has guiding significance to coordinate multiple UAVs to a rapid combat formation in the battlefield.

References

- [1] FAN Q J, YANG Z, FANG T, et al. Status of multi-UAV cooperative formation flight control research. *Journal of Aeronautical Engineering*, 2009, 30(4): 683–691. (in Chinese)
- [2] KARIMODDINI A, LIN H, CHEN B M, et al. Hybrid formation control of the unmanned aerial vehicles. *Mechatronics*, 2011, 21(5): 886–898.
- [3] RU C J, WEI R X, DAI J, et al. A method of reconstruction of UAV formation based on Nash's bargaining. *Journal of Automation*, 2013, 39(8): 1349–1359. (in Chinese)
- [4] LIAO F, WANG J L. Formation and reconfiguration control for nonholonomic UAVs with velocity constraints. *Proc. of the AIAA Guidance, Navigation, and Control Conference*, 2012: AIAA 2012–4823.
- [5] SONG M, WEI R X, HU L M. Control method of reconnaissance formation of UAV based on virtual leader vehicle. *Systems Engineering and Electronics*, 2010, 32(11): 2412–2415. (in Chinese)
- [6] FU X W, CUI H J, GAO X G. A distributed solution method for multi-UAV assembly problem. *Systems Engineering and Electronics*, 2015, 37(8): 1797–1802. (in Chinese)
- [7] HOW J, KING E, KUWATA Y. Flight demonstrations of cooperative control for UAV teams. *Proc. of the 3rd AIAA Unmanned Unlimited Technical Conference*, 2004: AIAA-2004-6490.
- [8] DE GENNARO M, JADBABAIE A. Formation control for a cooperative multi-agent system using decentralized navigation functions. *Proc. of the American Control Conference*, 2006. DOI: 10.1109/ACC.2006.1656404.
- [9] PAUL T, KROGSTAD T, GRAVDAHL J. Modelling of UAV formation flight using 3D potential field. *Simulation Model Practice Theory*, 2008, 16(9): 1453–1462.
- [10] GIULIETTI F, INNOCENTI M, NAPOLITANO M, et al. Dynamic and control issues of formation flight. *Aerospace Science Technology*, 2005, 9(1): 65–71.
- [11] STIPANOVIC D M, INALHAN G, TEO R, et al. Decentralized overlapping control of a formation of unmanned aerial vehicles. *Automatica*, 2004, 40(8): 1285–1296.
- [12] RU C J, WEI R X, GUO Q. Cognitive game guidance control for UAV autonomous anti-collision. *Control Theory and Applications*, 2014, 31(11): 1555–1560.
- [13] KEVICZKY T, BORRELLI F, FREGENE K, et al. Decentralized receding horizon control and coordination of autonomous vehicle formations. *IEEE Trans. on Control System Technology*, 2008, 16(1): 19–33.
- [14] BEMPORAD A, ROCCHI C. Decentralized hybrid model predictive control of a formation of unmanned aerial vehicles. *Proc. of the 18th IFAC World Congress*, 2011, 44(1): 11900–11906.
- [15] KRSTIC M, KANELAKOPOULOS I, KOKOTOVIC P V. *Nonlinear and adaptive control design*. New York: Wiley-Inter Science, 1995.
- [16] FASANO G, ACCARDO D, MOCCIA A, et al. Multisensor based fully autonomous non-cooperative collision avoidance system for UAVs. *Proc. of the AIAA Infotech@ Aerospace Conference and Exhibit*, 2007: AIAA-2007-2847.
- [17] KUWATA Y, HOW J P. Robust cooperative decentral trajectory optimization using receding horizon MILP. *Proc. of the IEEE American Control Conference*, 2007: 522–527.
- [18] KUWATA Y, HOW J P. Robust cooperative decentralized trajectory optimization using receding horizon MILP. *Proc. of the IEEE American Control Conference*, 2007: 522–527.
- [19] WANG P K C. Navigation strategies for multiple autonomous mobile robots moving in formation. *Journal of Robotic Sys-*

- tem, 1991, 8(2): 177–195. (in Chinese)
- [20] BRAD S, GU Y, MARCELLO R N, et al. 3-aircraft formation flight experiment. Proc. of the 14th IEEE Mediterranean Conference on Control and Automation, 2006: 1–6.
- [21] HOW J, KING E, KUWATA Y. Flight demonstrations of cooperative control for UAV teams. Proc. of the 3rd AIAA “Unmanned Unlimited” Technical Conference, Workshop and Exhibit, 2004: 1–9.
- [22] BEARD R, KINGSTON D, QUIGLEY M, et al. Autonomous vehicle technologies for small fixed-wing UAVs. Journal of Aerospace Computing, Information, and Communication, 2005, 2(3): 92–108.
- [23] WISE R A, RYSDYK R T. UAV coordination for autonomous target tracking. Proc. of the AIAA Guidance, Navigation, and Control Conference and Exhibit, 2006: 1–22.
- [24] REN W, ATKINS E. Nonlinear trajectory tracking for fixed wing UAVs via backstepping and parameter adaptation. Proc. of the AIAA Guidance, Navigation, and Control Conference and Exhibit, 2005: AIAA-2005-6196.
- [25] SUJIT P B, BEARD R. Multiple UAV path planning using anytime algorithms. Proc. of American Control Conference, 2009: 2978–2983.

Biographies

ZHANG Liang was born in 1980. He received his Ph.D. degree from UAVs Faculty, Air Force Engineering University in 2017. His main research interests include more unmanned aircraft collaborative control and combat management.
E-mail: yddsyzl@sohu.com

LU Yi was born in 1974. He received his M.S. degree from Armament Engineering Faculty, Air Force Engineering University. His main research interests include flight path planning and mission planning.
E-mail: afeuluyi@163.com

XU Shida was born in 1995. He received his M.S. degree from Plasma Faculty, Air Force Engineering University. His main research interest includes aerospace plasma dynamics.
E-mail: 1501090478@qq.com

FENG Han was born in 1997. He received his B.S. degree from Aircraft Faculty, Air Force Engineering University. His major is flight vehicle propulsion engineering.
E-mail: 873244253@qq.com