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Remaining lifetime prediction for nonlinear degradation device with random effect

CAI Zhongyi[1,*], CHEN Yunxiang[1], GUO Jiansheng[1], ZHANG Qiang[2], and XIANG Huachun[1]

1. Equipment Management and Unmanned Aerial Vehicle Engineering College, Air Force Engineering University, Xi'an 710051, China;
2. Defense Technology Academy, China Aerospace Science and Industry Corporation Limited, Beijing 100854, China

Abstract: For the large number of nonlinear degradation devices existing in a project, the existing methods have not systematically studied the effects of random effect on the remaining lifetime (RL), the accuracy and efficiency of the parameters estimation are not high, and the current degradation state of the target device is not accurately estimated. In this paper, a nonlinear Wiener degradation model with random effect is proposed and the corresponding probability density function (PDF) of the first hitting time (FHT) is deduced. A parameter estimation method based on modified expectation maximum (EM) algorithm is proposed to obtain the estimated value of fixed coefficient and the priori value of random coefficient in the model. The posterior value of the random coefficient and the current degradation state of target device are updated synchronously by the state space model (SSM) and the Kalman filter algorithm. The PDF of RL with random effect is deduced. A simulation example is analyzed to verify that the proposed method has the obvious advantage over the existing methods in parameter estimation error and RL prediction accuracy.

Keywords: remaining lifetime (RL) prediction, nonlinear degradation model, Wiener process, random coefficient, Kalman filter algorithm.

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1. Introduction

With the rapid development of condition monitoring (CM) technology, to predict the probability of failure in the future for a certain period of time has become a hot issue in the research of prognostics and health management (PHM) technology [1,2]. Stochastic degradation device refers to the device whose performance degrades in operation process due to the combined effects of internal stress and external environment, such as metal fatigue crack increasing [3], lithium battery capacity reduction [4,5], laser electrical performance attenuation [6] and so on, and may further evolve into a failed device. For this type of device, the monitored performance degradation information of device is used to carry out its health assessment, predict its remaining lifetime (RL) and determine the best maintenance time, which is of great significance to ensure its safe operation, reduce the maintenance cost and improve the reliability.

Taking into account the random dynamic of actual operating environment and stress of device, it is generally believed that the RL has a random uncertainty [7,8], so the RL prediction is mainly to obtain the analytical expression of the RL distribution function. The current RL prediction methods are broadly divided into two categories: mechanism model based method and data-driven method [9]. The mechanism model based method is to establish a mathematical model for describing the device's failure mechanism. However, with the increasing complexity of the internal structure of the device, the difficulty of establishing the mechanism model is gradually increasing, which makes this method difficult to apply.

The data-driven method can be further subdivided into the artificial intelligence method and the statistical data-driven method [10]. The artificial intelligence method is to fit the performance degradation trend of the device by machine learning skill with the known degradation data and extrapolate the point estimated RL value, which is hard to reflect the random uncertainty of the RL and makes the application of this method limited. The statistical data-driven method is to model the CM degradation data by establishing the statistical or stochastic model and further derive the probability density function (PDF) of the RL, which makes the research and application of this method important. Common stochastic models are the Wiener process, the Gamma process and the inverse Gaussian process. Among them, the Wiener process has been widely used in reliability analysis and RL prediction because of its good

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computing characteristics and description of random uncertainty [11]. At present, the RL prediction method based on the Wiener process has three main steps.

The first is degradation modeling. From the viewpoint of engineering demand, it is necessary to take the individual degradation differences, the nonlinearity and the time-varying dynamic characteristics into consideration when modeling the performance degradation data of the similar devices. However, most of the existing methods are not considered. The nonlinear degradation modeling with random effects and measurement error has not been systematically studied. Wang et al. [12] studied a linear degradation modeling based on the Wiener process, which is difficult to apply to nonlinear degradation. Kaiser et al. [13] used data conversion technology to convert the nonlinear degradation data into linear data under the assumption that the total degradation path can be linearized, and then studied linear degradation modeling, but this method has a certain limitation. Si et al. [14] and Wang et al. [15] proposed a generalized nonlinear Wiener degradation model and deduced the corresponding PDF, which has a good practicability. Du et al. [16] proposed a degradation modeling method which can deal with linear and nonlinear hybrid systems at the same time.

Random effect refers to the individual degradation differences among similar devices. Variables used to describe random effect are called random coefficients. Peng et al. [17,18] first considered the impact of random effect on the parameter estimation for the linear Wiener degradation model, who treated the random coefficient as a normal or partial normal variable. However, his study has not extended to the nonlinear degradation model. Feng et al. [19], Tang et al. [20] and Zheng et al. [21] proposed a nonlinear Wiener degradation model with measurement error and deduced the approximate analytical PDF of the first hitting time (FHT) through a complex theoretical derivation. However, the influence of the measurement error and the variance term of the random coefficient are still lack of research. Tang et al. [22] also studied the problem of accelerated degradation modeling with random effect.

The second is parameter estimation. In order to obtain the estimates of the fixed coefficient reflecting overall degradation characteristics and the priori values of the random coefficient, the commonly algorithms are the maximum likelihood estimation (MLE), the two-step maximum likelihood estimation (TSMLE), expectation maximum (EM) and so on. MLE is to build the likelihood function and estimate the unknown parameters under the assumption that the difference between the degradation values at the adjacent monitoring time or the difference between the degradation value and the initial degradation value at each monitoring time obeys the normal distribution. MLE has been applied to linear degradation models, nonlinear degradation models [14,15], degradation models with measurement error [23,24] and so on, but the disadvantage is that the variance term of random coefficient may be negative. TSMLE can guarantee that the variance term of the random coefficient is always positive by firstly estimating the random coefficient value of each device and then estimating the mean and variance of random coefficient, but the estimated value of this method is often the local optimal value [20,25]. The EM algorithm considers the drift coefficient of each device in the likelihood function as an implicit variable and the parameter estimates are obtained by the recursive estimation method. This algorithm can obtain the optimal estimate of the parameter and guarantee that the variance term of the random coefficient is positive, but the convergence of the likelihood function is too slow [26].

The third is updating the random coefficient online and obtaining the PDF of the RL. For the RL prediction of the target device, it is necessary to update the random coefficient by its historical CM information and obtain the posterior value of random coefficient to meet its individual degradation characteristics. Gebraeel et al. [27] first used the Bayesian method to update the posterior value of the random coefficient and deduced the relationship between the priori and the posterior values of the random coefficient. Tang et al. [28] extended this method to the RL prediction with measurement error, and deduced the corresponding calculation formula of the random coefficient posterior value. However, the disadvantage of this approach is that it does not accurately estimate the current state of the target device and may lead to a potential prediction error.

In order to synchronously update the random coefficient and the current degradation state, the existing method is usually to apply the state space model (SSM) to describe the degradation trend of the target device [21] and use the Kalman filter to iteratively estimate the random coefficient posterior value [19,29]. Wang et al. [12] and Si et al. [30] added an error term in the drift coefficient of the SSM, which may lead to a negative value of the drift coefficient. Thus the existing SSM needs to be modified.

The study of the RL distribution is mainly based on the PDF of the FHT. Si et al. [31] deduced the RL distribution function based on linear Wiener with random effect proposed by Peng et al. [17]. Then some scholars have deduced the RL distribution function based on nonlinear Wiener [15], nonlinear Wiener with measurement error [32] and multivariate nonlinear Wiener [33].

In summary, this paper proposes a nonlinear degradation model and RL prediction method with random effect for nonlinear stochastic degradation device. Firstly, the nonlinear Wiener degradation model is established, and the PDF of the FHT is deduced with random effect. Then, the parameter estimation method based on the modified EM algorithm is proposed. The estimated value of fixed coefficient and the priori value of the random coefficient are obtained. Finally, the Kalman filter algorithm and the modified SSM are used to update the posterior value of the random coefficient and the current degradation state. The correctness of the proposed method is verified by a simulation example.

2. Nonlinear degradation modeling

Assuming that there is only one key performance parameter for a certain type of device, the failure threshold is w. The nonlinear stochastic degradation process is expressed by the nonlinear Wiener process as follows:

$$X(t) = X(0) + \lambda \Lambda(t;\boldsymbol{\theta}) + \sigma_B B(t) \tag{1}$$

where, λ is the drift coefficient; σ_B is the diffusion coefficient; $B(t)$ is the standard Brownian motion, describing the dynamic characteristics of the degradation process; $\Lambda(t;\boldsymbol{\theta})$ is the time t of the continuous non-decreasing function ($\boldsymbol{\theta}$ is the unknown parameter vector), describing the nonlinear feature; $X(0)$ is the initial degradation value. Without the generality, let $X(0) = 0$.

For a given λ, the PDF of the device FHT is approximately [25] as

$$f_{T|\lambda}(t|\lambda) \approx \frac{1}{\sqrt{2\pi\sigma_B^2 t^3}}\left[w - \lambda\beta(t;\boldsymbol{\theta})\right]\cdot \exp\left[-\frac{(w-\lambda\beta(t;\boldsymbol{\theta}))^2}{2\sigma_B^2 t}\right] \tag{2}$$

with

$$\beta(t;\boldsymbol{\theta}) = \Lambda(t;\boldsymbol{\theta}) - t\Lambda'(t;\boldsymbol{\theta}). \tag{3}$$

In order to reflect the random effects among similar devices, the drift coefficient λ is generally regarded as a normal variable, namely $\lambda \sim \mathrm{N}(\mu_\lambda, \sigma_\lambda^2)$. Firstly, introduce the following lemma [21]:

Lemma 1 If $\lambda \sim \mathrm{N}(\mu, \sigma^2)$, $w_1, w_2, A, B \in \mathbf{R}$ and $C \in \mathbf{R}^+$, then

$$E_a\left[(w_1 - A\lambda)\exp\left(-\frac{(w_2 - B\lambda)^2}{2C}\right)\right] = \sqrt{\frac{C}{B^2\sigma^2 + C}}\left(w_1 - A\frac{Bw_2\sigma^2 + \mu C}{B^2\sigma^2 + C}\right)\cdot \exp\left[-\frac{(w_2 - B\mu)^2}{2(B^2\sigma^2 + C)}\right]. \tag{4}$$

For $\lambda \sim \mathrm{N}(\mu_\lambda, \sigma_\lambda^2)$, the full probability formula is as follows:

$$f_T(t) = \int_\lambda f_{T|\lambda}(t|\lambda)p(\lambda)\mathrm{d}\lambda = \mathrm{E}_\lambda[f_{T|\lambda}(t|\lambda)] \tag{5}$$

where $p(\lambda)$ is the PDF of λ.

According to Lemma 1 and (2), we can deduce that the approximate analytical PDF of the device FHT is deduced as

$$f_T(t) \approx \frac{1}{\displaystyle\int_0^{+\infty} g_T(t)\mathrm{d}t} g_T(t) \tag{6}$$

with

$$g_T(t) \approx \frac{1}{\sqrt{2\pi t^2[\sigma_\lambda^2 \Lambda(t;\boldsymbol{\theta})^2 + \sigma_B^2 t]}}\cdot \exp\left[-\frac{(w - \mu_\lambda\Lambda(t;\boldsymbol{\theta}))^2}{2(\sigma_\lambda^2\Lambda(t;\boldsymbol{\theta})^2 + \sigma_B^2 t)}\right]\cdot \left[w - \mu_\lambda\Lambda(t;\boldsymbol{\theta}) - \frac{w - \mu_\lambda\Lambda(t;\boldsymbol{\theta})}{\sigma_\lambda^2\Lambda(t;\boldsymbol{\theta})^2 + \sigma_B^2 t}\sigma_\lambda^2\Lambda(t;\boldsymbol{\theta})\right]. \tag{7}$$

3. Parameter estimation

Without the generality, let $\Lambda(t;\boldsymbol{\theta}) = t^b$ (b is an unknown parameter). The unknown parameter set in the above nonlinear degradation model is denoted as $\boldsymbol{\Theta} = \{\mu_\lambda, \sigma_\lambda^2, \sigma_B^2, b\}$. σ_B^2, b are the fixed coefficients, describing the overall degradation characteristics of similar devices; $\mu_\lambda, \sigma_\lambda^2$ are the random coefficients, describing the individual degradation characteristics of the target device.

There are n devices with nonlinear stochastic degradation features to carry out performance parameter measurement. The degradation data is denoted as $\boldsymbol{X} = (\boldsymbol{X}_1, \boldsymbol{X}_2, \ldots, \boldsymbol{X}_N)$. The measurement number of the ith devices is m_i. Its performance degradation data vector is $\boldsymbol{X}_i = (X(t_{1,i}), X(t_{2,i}), \ldots, X(t_{m_i,i}))$. The performance degradation data of the ith device at the jth measurement is $X(t_{j,i})$. Its corresponding degradation increment is $\Delta X(t_{j,i})$. Thus

$$\Delta X(t_{j,i}) = X(t_{j,i}) - X(t_{j-1,i}) = \lambda_i(t_{j,i}^b - t_{j-1,i}^b) + \sigma_B B(t_{j,i} - t_{j-1,i}) = \lambda_i \Delta t_{j,i}^b + \sigma_B B(\Delta t_{j,i}) \tag{8}$$

where λ_i is the actual value of the drift coefficient of the ith device.

According to the feature of the Wiener process, the performance degradation increment obeys the normal distribution, namely,

$$\Delta X(t_{j,i}) \sim \mathrm{N}[\lambda_i \Delta t_{j,i}^b, \sigma_B^2 \Delta t_{j,i}]. \tag{9}$$

According to (9), the complete likelihood function is established as follows:

$$L(\boldsymbol{\Theta}|\boldsymbol{X}, \lambda) = \sum_{i=1}^{N}\sum_{j=1}^{m_i} \frac{1}{\sqrt{2\pi\sigma_B^2\Delta t_{j,i}}}\exp\left[-\frac{(\Delta X(t_{j,i}) - \lambda_i\Delta t_{j,i}^b)^2}{2\sigma_B^2\Delta t_{j,i}}\right].$$

$$\sum_{i=1}^{N}\frac{1}{\sqrt{2\pi\sigma_\lambda^2}}\exp\left[-\frac{(\lambda_i-\mu_\lambda)^2}{2\sigma_\lambda^2}\right]. \tag{10}$$

The complete log likelihood function is further calculated as

$$\ln L(\boldsymbol{\Theta}|\boldsymbol{X},\lambda)=-\frac{\sum\limits_{i=1}^{N}m_i}{2}(\ln(2\pi)+\ln\sigma_B^2)-$$
$$\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{m_i}\ln\Delta t_{j,i}-\frac{1}{2\sigma_B^2}\sum_{i=1}^{N}\sum_{j=1}^{m_i}\frac{[\Delta X(t_{j,i})-\lambda_i\Delta t_{j,i}^b]^2}{\Delta t_{j,i}}-$$
$$\frac{N}{2}(\ln(2\pi)+\ln\sigma_\lambda^2)-\frac{1}{2\sigma_\lambda^2}\sum_{i=1}^{N}(\lambda_i-\mu_\lambda)^2. \tag{11}$$

Since λ_i is an implicit value related to $\mu_\lambda,\sigma_\lambda^2$, it is difficult to obtain its estimated value by the traditional MLE. Thus the EM algorithm is applied to solve this problem and divided into two steps: the first step (Step E) is to calculate the expected value and use the current estimate of the implied variable to calculate the maximum likelihood estimate; the second step (Step M) is to maximize the estimate by the MLE in Step E.

Let $\widehat{\boldsymbol{\Theta}}^{(n)}=\{\widehat{\mu}_\lambda^{(n)},\widehat{\sigma}_\lambda^{2(n)},\widehat{\sigma}_B^{2(n)},\widehat{b}^{(n)}\}$ be the parameter estimated values at Step n. The parameter estimation steps are expressed as follows:

Step E Calculate the expectation of the above complete log likelihood function as follows:

$$L(\boldsymbol{\Theta}|\widehat{\boldsymbol{\Theta}}^{(n)})=\mathrm{E}_{\lambda|\boldsymbol{X},\widehat{\boldsymbol{\Theta}}^{(n)}}[\ln L(\boldsymbol{\Theta}|\boldsymbol{X})]=$$
$$-\frac{\sum\limits_{i=1}^{N}m_i}{2}(\ln(2\pi)+\ln\sigma_B^2)-\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{m_i}\ln\Delta t_{j,i}-\frac{1}{2\sigma_B^2}\cdot$$
$$\sum_{i=1}^{N}\sum_{j=1}^{m_i}\frac{1}{\Delta t_{j,i}}\left[\left(\Delta X(t_{j,i})-\mathrm{E}\left(\lambda_i|\boldsymbol{X},\widehat{\boldsymbol{\Theta}}^{(n)}\right)\Delta t_{j,i}^b\right)^2+\right.$$
$$\left.\mathrm{var}\left(\lambda_i|\boldsymbol{X},\widehat{\boldsymbol{\Theta}}^{(n)}\right)\Delta t_{j,i}^{2b}\right]-\frac{N}{2}(\ln(2\pi)+\ln\sigma_\lambda^2)-$$
$$\frac{1}{2\sigma_\lambda^2}\sum_{i=1}^{N}[(\mathrm{E}(\lambda_i|\boldsymbol{X},\widehat{\boldsymbol{\Theta}}^{(n)})-\mu_\lambda)^2+\mathrm{var}(\lambda_i|\boldsymbol{X},\widehat{\boldsymbol{\Theta}}^{(n)})]. \tag{12}$$

According to the Bayesian theory with the given $\boldsymbol{X}$, $\widehat{\boldsymbol{\Theta}}^{(n)}$, it is deduced that $\lambda_i|\boldsymbol{X}_i,\widehat{\boldsymbol{\Theta}}^{(n)}$ obeys the normal distribution, then

$$\lambda_i|\boldsymbol{X}_i,\widehat{\boldsymbol{\Theta}}^{(n)}\sim\mathrm{N}[\mathrm{E}(\lambda_i|\boldsymbol{X},\widehat{\boldsymbol{\Theta}}^{(n)}),\mathrm{var}(\lambda_i|\boldsymbol{X},\widehat{\boldsymbol{\Theta}}^{(n)})] \tag{13}$$

with

$$\begin{cases}\mathrm{E}(\lambda_i|\boldsymbol{X},\widehat{\boldsymbol{\Theta}}^{(n)})=\dfrac{X(t_{j,i})\sigma_\lambda^{2(n)}+\mu_\lambda\sigma_B^{2(n)}}{t_{j,i}^b\sigma_\lambda^{2(n)}+\sigma_B^{2(n)}}\\ \mathrm{var}(\lambda_i|\boldsymbol{X},\widehat{\boldsymbol{\Theta}}^{(n)})=\dfrac{\sigma_\lambda^{2(n)}\sigma_B^{2(n)}}{t_{j,i}^b\sigma_\lambda^{2(n)}+\sigma_B^{2(n)}}\end{cases}. \tag{14}$$

Step M Maximize the calculated complete log likelihood function expectation.

$$\widehat{\boldsymbol{\Theta}}^{(n+1)}=\arg\max_{\boldsymbol{\Theta}}L(\boldsymbol{\Theta}|\widehat{\boldsymbol{\Theta}}^{(n)}) \tag{15}$$

The partial derivative of (12) for $\mu_\lambda,\sigma_\lambda^2,\sigma_B^2$ is calculated as follows:

$$\frac{\partial L(\boldsymbol{\Theta}|\widehat{\boldsymbol{\Theta}}^{(n)})}{\partial\mu_\lambda}=-\frac{1}{\sigma_\lambda^2}\sum_{i=1}^{N}(\mu_\lambda-\mathrm{E}(\lambda_i|\boldsymbol{X},\widehat{\boldsymbol{\Theta}}^{(n)})) \tag{16}$$

$$\frac{\partial L(\boldsymbol{\Theta}|\widehat{\boldsymbol{\Theta}}^{(n)})}{\partial\sigma_\lambda^2}=-\frac{N}{2\sigma_\lambda^2}+\frac{1}{2(\sigma_\lambda^2)^2}\cdot$$
$$\sum_{i=1}^{N}[(\mathrm{E}(\lambda_i|\boldsymbol{X},\widehat{\boldsymbol{\Theta}}^{(n)})-\mu_\lambda)^2+\mathrm{var}(\lambda_i|\boldsymbol{X},\widehat{\boldsymbol{\Theta}}^{(n)})] \tag{17}$$

$$\frac{\partial L(\boldsymbol{\Theta}|\widehat{\boldsymbol{\Theta}}^{(n)})}{\partial\sigma_B^2}=-\frac{\sum\limits_{i=1}^{N}m_i}{2\sigma_B^2}+\frac{1}{2(\sigma_B^2)^2}\cdot$$
$$\sum_{i=1}^{N}\sum_{j=1}^{m_i}\frac{1}{\Delta t_{j,i}}[(\Delta X(t_{j,i})-\mathrm{E}(\lambda_i|\boldsymbol{X},\widehat{\boldsymbol{\Theta}}^{(n)})\Delta t_{j,i}^b)^2+$$
$$\mathrm{var}(\lambda_i|\boldsymbol{X},\widehat{\boldsymbol{\Theta}}^{(n)})\Delta t_{j,i}^{2b}]. \tag{18}$$

Let (16), (17) and (28) be zero and the estimated values of $\mu_\lambda,\sigma_\lambda^2,\sigma_B^2$ at the $n+1$ step limited by b are

$$\widehat{\mu}_\lambda^{(n+1)}(b)=\frac{1}{N}\sum_{i=1}^{N}\mathrm{E}(\lambda_i|\boldsymbol{X},\widehat{\boldsymbol{\Theta}}^{(n)}) \tag{19}$$

$$\widehat{\sigma}_\lambda^{2(n+1)}(b)=$$
$$\frac{1}{N}\sum_{i=1}^{N}[(\mathrm{E}(\lambda_i|\boldsymbol{X},\widehat{\boldsymbol{\Theta}}^{(n)})-\widehat{\mu}_\lambda^{(n+1)})^2+\mathrm{var}(\lambda_i|\boldsymbol{X},\widehat{\boldsymbol{\Theta}}^{(n)})] \tag{20}$$

$$\widehat{\sigma}_B^{2(n+1)}(b)=$$
$$\frac{1}{\sum\limits_{i=1}^{N}m_i}\sum_{i=1}^{N}\sum_{j=1}^{m_i}\frac{1}{\Delta t_{j,i}}[\mathrm{var}(\lambda_i|\boldsymbol{X},\widehat{\boldsymbol{\Theta}}^{(n)})\Delta t_{j,i}^{2b}+$$
$$(\Delta X(t_{j,i})-\mathrm{E}(\lambda_i|\boldsymbol{X},\widehat{\boldsymbol{\Theta}}^{(n)})\Delta t_{j,i}^b)^2]. \tag{21}$$

Because the sample data is not much, the progressive unbiased estimation in (20) will reduce the estimated value of σ_λ^2 as well as the computational efficiency. Therefore, this paper uses the modified unbiased estimation formula, namely

$$\widehat{\sigma}_\lambda^{2(n+1)} = \frac{1}{N-1}\sum_{i=1}^{N}[(\mathrm{E}(\lambda_i|\boldsymbol{X},\widehat{\boldsymbol{\Theta}}^{(n)}) - \widehat{\mu}_\lambda^{(n+1)})^2 + \mathrm{var}(\lambda_i|\boldsymbol{X},\widehat{\boldsymbol{\Theta}}^{(n)})]. \quad (22)$$

Substitute (21), (21) and (22) into (12), and the profile likelihood function of parameter b is obtained as follows:

$$\tilde{L}(b|\widehat{\boldsymbol{\Theta}}^{(n+1)}) = -\frac{\sum_{i=1}^{N} m_i}{2}(\ln(2\pi) + \ln\widehat{\sigma}_B^{2(n+1)} + 1) - \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{m_i}\ln\Delta t_{j,i} - \frac{N}{2}(\ln(2\pi) + \ln\widehat{\sigma}_\lambda^{2(n+1)} + 1). \quad (23)$$

With the help of the Fminsearch function based on the simplex method, the traversal search is performed until $\tilde{L}(b|\widehat{\boldsymbol{\Theta}}^{(n+1)})$ reaches the maximum value, and the returned $\widehat{b}^{(n+1)}$ is the requested value. Let $\widehat{b}^{(n+1)}$ be substituted into (19), (21) and (22) and find the optimal $\widehat{\mu}_\lambda^{(n+1)}, \widehat{\sigma}_\lambda^{2(n+1)}, \widehat{\sigma}_B^{2(n+1)}$.

The above E and M steps are calculated iteratively until the estimated error $\|\widehat{\boldsymbol{\Theta}}^{(n+1)} - \widehat{\boldsymbol{\Theta}}^{(n)}\|$ of the adjacent two steps reaches the specified error requirement.

4. RL prediction model

4.1 Updating the random coefficient and current degradation state

Assume that the history degradation data of the target device from the initial time t_1 to the current time t_h is denoted as $\boldsymbol{X}_{1:h} = (X(t_1), X(t_2), \dots, X(t_h))$. In order to make full use of the target device's history degradation data to update the random coefficient and the current degradation state, the SSM is used to describe its degradation process as follows:

$$\begin{cases} \lambda_h = \lambda_{h-1} + \eta \\ X(t_h) = X(t_{h-1}) + \lambda_{h-1}\Delta t_h^b + \sigma_B\varepsilon_h \end{cases} \quad (24)$$

where, $\eta \sim \mathrm{N}(0, Q)$ is error term and $\varepsilon_h \sim \mathrm{N}(0, \Delta t_h)$.

Since the value η in (24) may be positive or negative, it will affect whether λ_h is positive. At the same time, for the target device, the drift coefficient is a fixed positive value throughout the degradation process [34]. Thus (24) needs a certain transformation as follows:

$$\begin{cases} \lambda_h = \lambda_{h-1} \\ X(t_h) = X(t_{h-1}) + \lambda_{h-1}\Delta t_h^b + \sigma_B\varepsilon_h \end{cases}. \quad (25)$$

The Kalman filter algorithm is used to recursively estimate the implied parameter λ_h in (25). The concrete steps are expressed as follows:

(i) Initialization $\widehat{\lambda}_0 = \mu_\lambda, P_{0|0} = \sigma_\lambda^2, j = 0$.

(ii) Let $j = j + 1$ and calculate the fade factor $\phi(t_j)$:

$$\phi(t_j) = \max\{\phi_0, 1\} \quad (26)$$

with

$$\phi_0 = B(t_j)/C(t_j)$$
$$B(t_j) = V_0(t_j) - \beta\sigma_B^2\Delta t_j$$
$$C(t_j) = P_{j-1|j-1}(\Delta t_j^b)^2$$
$$V_0(t_j) = \begin{cases} (u(t_j))^2, & j = 1 \\ \dfrac{\rho V_0(t_{j-1}) + (u(t_j))^2}{1+\rho}, & j > 1 \end{cases}$$
$$u(t_j) = X(t_j) - X(t_{j-1}) - \widehat{\lambda}_{j-1}\Delta t_j^b$$

where $\beta \geqslant 1$ is a weakening factor and $0 < \rho \leqslant 1$ is a forgetting factor.

(iii) Calculate the predicted value $\widehat{\lambda}_j$:

$$\widehat{\lambda}_j = \widehat{\lambda}_{j-1} + P_{j|j-1}\Delta t_j^b (K_j)^{-1} u(t_j) \quad (27)$$

with

$$P_{j|j-1} = \phi(t_j)P_{j-1|j-1}$$
$$K_j = (\Delta t_j^b)^2 P_{j|j-1} + \sigma_B^2\Delta t_j.$$

(iv) Update the variance $P_{j|j}$:

$$P_{j|j} = P_{j|j-1} - P_{j|j-1}(\Delta t_j^b)^2(K_j)^{-1}P_{j|j-1}. \quad (28)$$

Repeat (ii)–(iv) until $j = h$, then the mean $\widehat{\lambda}_h$ and variance $P_{h|h}$ of λ_h at the current time is obtained, i.e., $\lambda_h \sim \mathrm{N}(\widehat{\lambda}_h, P_{h|h})$. Thus the random coefficient posterior value and the current degradation state of the target device are updated synchronously with the historical degradation information of the target device.

4.2 RL distribution function

The RL of the target device at the current time t_h is denoted as L_h, defined as follows:

$$L_h = \inf\{l_h : X(t_h + l_h) \geqslant w|\boldsymbol{X}_{1:h}\}. \quad (29)$$

Given the current value λ_h of the random coefficient at the current time t_h and the degradation data vector $\boldsymbol{X}_{1:h}$ of the target device, the PDF of the RL can be deduced:

$$f_{L_h|\lambda_h,\boldsymbol{X}_{1:h}}(l_h|\lambda_h,\boldsymbol{X}_{1:h}) \approx \frac{1}{A_{L_h}} g_{L_h|\lambda_h,\boldsymbol{X}_{1:h}}(l_h|\lambda_h,\boldsymbol{X}_{1:h}) \quad (30)$$

with

$$A_{L_h} = \int_0^\infty g_{L_h|\lambda_h,\boldsymbol{X}_{1:h}}(l_h|\lambda_h,\boldsymbol{X}_{1:h})\mathrm{d}l_h \tag{31}$$

$$g_{L_h|\lambda_h,\boldsymbol{X}_{1:h}}(l_h|\lambda_h,\boldsymbol{X}_{1:h}) \approx \frac{1}{\sqrt{2\pi\sigma_B^2 l_h^3}}\cdot$$
$$[w - X(t_h) - \lambda_h(t_h+l_h)^b + \lambda_h l b(t_h+l_h)^{b-1} + \lambda_h t_h^b]\cdot$$
$$\exp\left[-\frac{(w - X(t_h) - \lambda_h(t_h+l_h)^b + \lambda_h t_h^b)^2}{2\sigma_B^2 l_h}\right]. \tag{32}$$

Proof Let $Z(l_h) = X(t_h+l_h) - X(t_h)$ with $l_h = t - t_h$ denoting the RL, and $Z(0) = 0$, then

$$Z(l_h) = X(t_h+l_h) - X(t_h) =$$
$$\lambda(t_h+l_h)^b - \lambda(t_h)^b + \sigma_B B(t_h+l_h) - \sigma_B B(t_h) =$$
$$\lambda[(t_h+l_h)^b - \lambda(t_h)^b] + \sigma_B B(l_h). \tag{33}$$

The RL of the new stochastic process $\{Z(l_h), l_h \geqslant 0\}$ at time t_h is transformed into the FHT reaching the failure threshold $w_h = w - X(t_h)$. From (2), through the mathematical transformation, (30) can be derived. □

For $\lambda_h \sim \mathrm{N}(\widehat{\lambda}_h, P_{h|h})$, based on the full probability formula, given $\boldsymbol{X}_{1:h}$ of the target device, through the mathematical transformation, the PDF of the RL can be derived from (6) as follows:

$$f_{L_h|\boldsymbol{X}_{1:h}}(l_h|\boldsymbol{X}_{1:h}) \approx \frac{1}{A'_{L_h}} g_{L_h|\boldsymbol{X}_{1:h}}(l_h|\boldsymbol{X}_{1:h}) \tag{34}$$

with

$$A'_{L_h} = \int_0^\infty g_{L_h|\boldsymbol{X}_{1:h}}(l_h|\boldsymbol{X}_{1:h})\mathrm{d}l_h \tag{35}$$

$$g_{L_h|\boldsymbol{X}_{1:h}}(l_h|\boldsymbol{X}_{1:h}) \approx \frac{1}{\sqrt{2\pi B^3 l_h^2}}\cdot[B(w - X(t_h)) -$$
$$C(w - X(t_h))((t_h+l_h)^b - t_h^b)P_{h|h} - \widehat{\lambda}_h\sigma_B^2 l_h]\cdot$$
$$\exp\left[-\frac{(w - X(t_h) - \widehat{\lambda}_h(t_h+l_h)^b + \lambda_h t_h^b)^2}{2B}\right] \tag{36}$$

$$B = ((t_h+l_h)^b - t_h^b)^2 P_{h|h} + \sigma_B^2 l_h \tag{37}$$

$$C = l_h(1 - b(t_h+l_h)^{b-1}). \tag{38}$$

According to the PDF of the RL, the expectation of the RL is

$$\mathrm{E}(L_h|\boldsymbol{X}_{1:h}) = \int_0^\infty l_h f_{L_h|\boldsymbol{X}_{1:h}}(l_h|\boldsymbol{X}_{1:h})\mathrm{d}l_h. \tag{39}$$

5. Simulation cases

It is known that the key performance parameter of a high-energy laser is working current. When the laser's working current increases in the amount of time to reach the specified threshold, then the laser fails. According to the six sets of laser degradation data given in [35] shown in Fig. 1, the initial values of parameters $\mu_\lambda, \sigma_\lambda^2, \sigma_B^2, b$ are 1, 0.062 5, 0.04, and 1.5. The Monte Carlo simulation method is used to obtain nine sets of sample performance degradation data shown in Fig. 2.

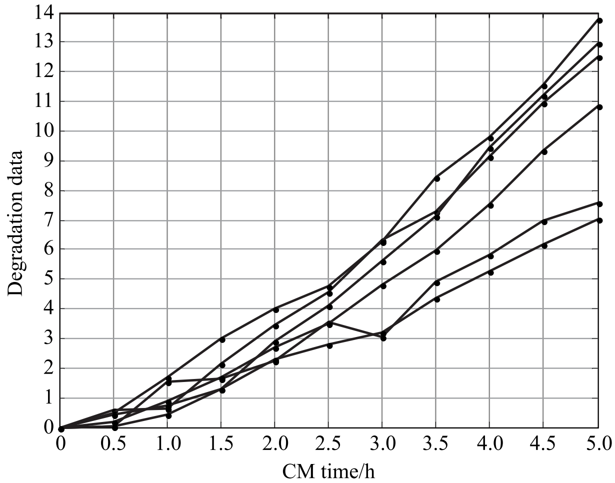

Fig. 1 Six sets of laser degradation data

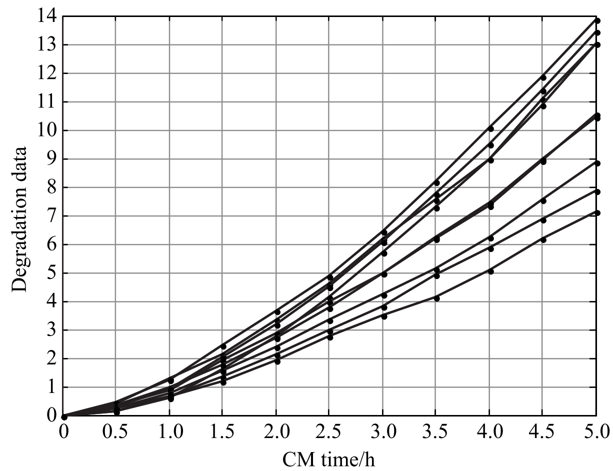

Fig. 2 Nine sets of sample simulated degradation data

In order to verify the correctness and advantage of the nonlinear degradation modeling and RL prediction method proposed in this paper, the Akashi information criterion (AIC) and the mean square error (MSE) are used to judge the current methods. The specific formulas are expressed as follows:

$$\mathrm{AIC} = 2p - 2\ln L(\boldsymbol{\Theta}) \tag{40}$$

where p is the number of unknown parameters; $L(\boldsymbol{\Theta})$ represents the likelihood function value.

$$\mathrm{MSE} = \frac{1}{m}\sum_{j=1}^{m}(\widehat{F}(t_j) - F_0(t_j)) \tag{41}$$

where m is the compared point number; $\widehat{F}(t_j)$ is the estimated cumulative distribution function value at time t_j; $F_0(t_j)$ is the actual cumulative distribution function value at time t_j.

(i) The comparison of degradation model

The nonlinear degradation modeling method proposed in this paper is denoted as M1. The linear degradation modeling method with random effect proposed in [17] is denoted as M2 ($b = 1$). The nonlinear degradation modeling method is denoted as M3 ($\sigma_\lambda^2 = 0$). The modified EM algorithm is used to estimate the parameters. The results are shown in Table 1.

Table 1 Comparison of three degradation modeling methods

Method	μ_λ	σ_λ^2	σ_B^2	b	Lg-LF	AIC	MSE
Real value	1	0.0625	0.04	1.5	–	–	–
M1	0.931	0.068 2	0.088	1.521	−25.641	59.282	0.950
M2	0.951	0.121 0	0.176	1.000	−29.310	64.620	2.321
M3	1.854	0	0.101	1.534	−33.672	73.344	7.319

It can be seen from Table 1 that the Lg-LF value, AIC value and MSE value of M1 are the smallest, which indicates that M1 has the best model fit and the smallest estimation error. Therefore, in order to accurately describe the data characteristics of nonlinear degradation device, the random effect and nonlinear characteristics should be taken into account in the degradation modeling process.

(ii) The parameter estimated results

The modified EM (unbiased estimation), EM (progressive estimation) and TSMLE are used to estimate the unknown parameter set in the nonlinear degradation model. The results are shown in Table 2. It can be seen from Table 2 that the parameter estimated result of the modified EM is near to the real value of the parameter (i.e., the initial value), indicating that the algorithm has a high accuracy and fast convergence speed (Iterating 40 times before convergence, the iterative calculation results shown in Fig. 3).

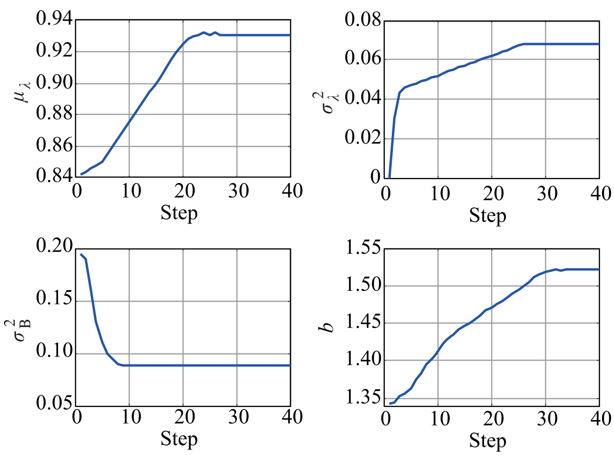

Fig. 3 Results of the modified EM algorithm

Table 2 Parameter estimation results by different algorithms

Algorithm	μ_λ	σ_λ^2	σ_B^2	b
Modified EM	0.931	0.068 2	0.088	1.521
EM	0.931	0.089 1	0.102	1.521
TSMLE	0.976	0.225	0.390	1.580

The estimated result of the traditional EM is also near to the real value but the variance items are larger than that of the modified EM, which indicates that the algorithm has a low estimation accuracy and slow convergence speed (iterating 2 000 times before convergence). The TSMLE is far from the real value, which indicates that the algorithm has the lowest estimation accuracy.

(iii) The RL prediction

The Monte Carlo simulation method is also used to obtain the simulation degradation path of the target device shown in Fig. 4. The performance degradation value of the target device at 5 h is 12.52. In order to verify the correctness of the RL prediction model, assuming that the failure threshold of the device is 12.52, so the target device can be considered to fail at 5h.

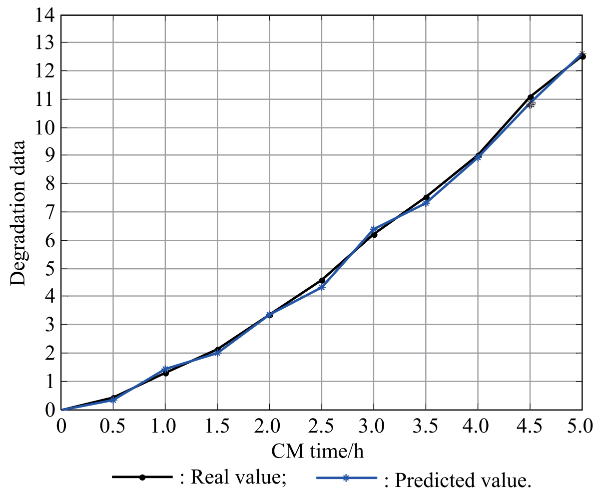

Fig. 4 Simulation path and prediction path of target device

In this paper, the RL prediction model based on the updating of the random coefficient and the current degradation state is denoted as M4. The RL prediction model based on the updating of random coefficient alone proposed in [25] is denoted as M5. The PDF of RL by M4 and M5 is calculated and shown in Fig. 5.

It can be seen from Fig. 4 that the PDF of the RL by M4 and M5 can cover the actual RL value of the target device, and the PDF of the RL by M4 is narrower than that by M5.

This is because M4 uses the history degradation information of target device to update the random coefficient posterior value as well as the current degradation state so that the PDF of the RL by M4 is more in line with the target device's degradation characteristics, which shows that

M4 has a better model fitting than that of M5. This can also be confirmed by the simulation path and prediction path of target device shown in Fig. 4.

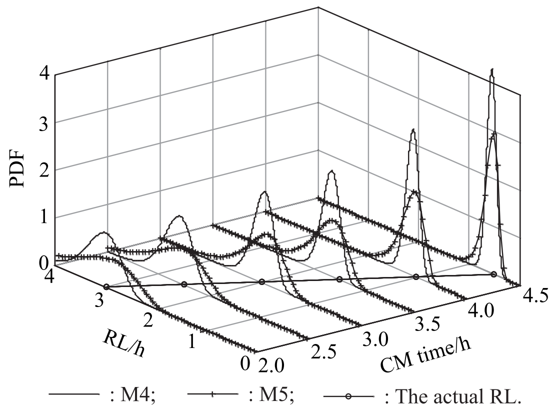

Fig. 5 PDF of the RL by M4 and M5

From (39), the 95% confidence interval of the RL at different CM time are calculated and shown in Fig. 6. It can be seen that the RL confidence interval of M4 and M5 can cover the actual RL value of the target device, and the RL confidence interval of M4 is obviously smaller than that of M5, indicating that the RL prediction accuracy of M4 is higher than that of M5.

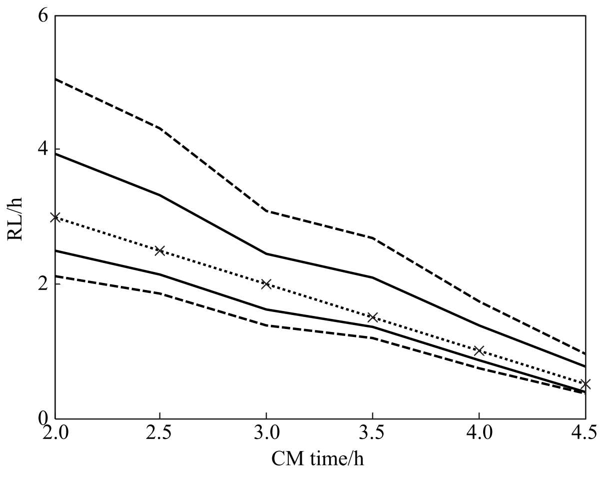

Fig. 6 Confidence interval of RL by M4 and M5

The MSE values of M4 and M5 at different CM time are calculated and shown in Fig. 7. It can be seen that the MSE value of M4 is always lower than that of M5, indicating that the RL prediction accuracy of M4 is higher than that of M5.

At the same time, with the prolonging of the CM time, the confidence interval of the target device RL becomes narrower and the MSE value becomes smaller. This is because the history degradation information of target device is increasing as well as the random coefficient posterior value and the current degradation state is iteratively updated, which gradually improves the RL prediction accuracy.

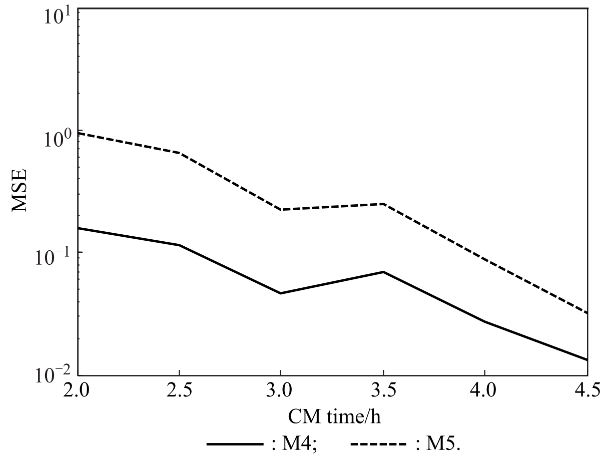

Fig. 7 MSE values of M4 and M5

6. Conclusions

The nonlinear degradation model with random effects is proposed, which can be used to describe the overall degradation characteristics of nonlinear stochastic devices. The proposed model has a better model fitting than the existing degradation model with only random effect or nonlinear features alone.

A parameter estimation method based on the modified EM is proposed with the unbiased estimation formula, which has higher estimation accuracy than the traditional EM algorithm and TSMLE. This approach provides an effective way to solve the problem that the amount of unknown parameters is larger than the amount of likelihood functions.

The modified SSM is used to describe the degradation process of the target device. The Kalman filter algorithm is applied to update the random coefficient and the current degradation state synchronously, so as to derive the RL adaptive prediction of the nonlinear stochastic device.

The simulation case is analyzed to show that the proposed model has better model fitting advantage and prediction precision than the existing model by comparing the parameter estimated results, the PDF and confidence interval of the RL and MSE value, which has a certain engineering application value.

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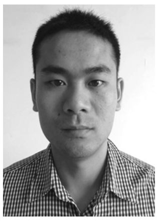
Biographies



CAI Zhongyi was born in 1988. He received his B.S. degree of management engineering in 2010, M.S. degree of management science and engineering in 2012 and Ph.D. degree of management science and engineering in 2016 from Air Force Engineering University. Now he is a lecturer of Device Management & UAV Engineering College, Air Force Engineering University. His research interests are reliability assessment and remaining lifetime prediction. He has published more than 20 research papers.
E-mail: afeuczy@163.com

CHEN Yunxiang was born in 1962. He received his M.S. degree from Air Force Engineering College in 1989 and Ph.D. degree from Northwestern Polytechnical University in 2005. Now he is a professor of Device Management & UAV Engineering College, Air Force Engineering University. His research interests are reliability assessment, materiel maintenance support and materiel development & demonstration. He has published five books and more than 50 research papers. He is an expert of the Air Force in reliability, maintenance and support.
E-mail: cyx87793@163.com

GUO Jiansheng was born in 1965. He received his M.S. degree from Air Force Engineering College in 1991 and Ph.D. degree from Northwestern Polytechnical University in 2009. Now he is a professor of Device Management & UAV Engineering College, Air Force Engineering University. His research interests are system engineering and complex system modeling. He has published three books and more than 20 research papers.
E-mail: amisc@163.com

ZHANG Qiang was born in 1983. He received his B.S. degree of communication and information system from Wuhan University in 2006 and M.S. degree from Air Force Engineering University in 2009. Now he is the industrial development director of Defense Technology Academy, China Aerospace Science and Industry Corporation Limited. His research interests are reliability assessment and project management. He has published five research papers. He is a senior engineer.
E-mail: afzhangq@163.com

XIANG Huachun was born in 1980. He received his M.S. degree of management science and engineering in 2005 and Ph.D. degree of system engineering in 2009 from Air Force Engineering University. Now he is a vice professor and director of Device Management & UAV Engineering College, Air Force Engineering University. His research interests are reliability design, reliability assessment and system engineering. He has published two books and ten research papers.
E-mail: xhc09260926@163.com