

Storage reliability assessment model based on competition failure of multi-components in missile

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Abstract: The degradation data of multi-components in missile is derived by periodical testing. How to use these data to assess the storage reliability (SR) of the whole missile is a difficult problem in current research. An SR assessment model based on competition failure of multi-components in missile is proposed. By analyzing the missile life profile and its storage failure feature, the key components in missile are obtained and the characteristics voltage is assumed to be its key performance parameter. When the voltage testing data of key components in missile are available, a state space model (SSM) is applied to obtain the whole missile degradation state, which is defined as the missile degradation degree (DD). A Wiener process with the time-scale model (TSM) is applied to build the degradation failure model with individual variability and nonlinearity. The Weibull distribution and proportional risk model are applied to build an outburst failure model with performance degradation effect. Furthermore, a competition failure model with the correlation between degradation failure and outburst failure is proposed. A numerical example with a set of missiles in storage is analyzed to demonstrate the accuracy and superiority of the proposed model.

Keywords: competition failure model, storage reliability (SR), missile degradation degree (DD), proportional risk model, individual variability.

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1. Introduction

Storage reliability (SR) is a vital tactical and technical indicator, which relates to combat readiness and rapid response capability. Usually, the missile has two types of failure modes, namely, degradation failure mode and outburst failure mode. Missile failure in storage period is usually due to the competition of these two modes. Long-term repair experience shows that the key components failures account for high percent of the whole missile failures. By periodical testing, analyzing and evaluating the performance

parameter data of multi-components, we can obtain the whole missile degradation state.

In order to effectively carry out the condition based maintenance (CBM) in storage period and life extension after storage, it is essential to assess the missile SR in occasion of competition failure. Current researches on reliability modeling with competition of degradation failure and outburst failure mainly focus on the following three aspects.

The first aspect is whether the correlation between degradation failure and outburst failure is taken into account. References [1–3] that viewed the two modes were independent, assumed the reliability mode as the series model and carried out relevant study on the reliability model, which would decrease the accuracy of missile SR assessment. References [4–6] that insisted on the influence of performance degradation on outburst failure, applied the proportional risk model [4], location-scale model [7,8] and degradation threshold shock (DTS) [9] to describe the quantitative influence of performance degradation on outburst failure rate and built a competition failure model with the correlation between degradation failure and outburst failure. Lehmann [9] applied DTS to build the correlation model among outburst failure, degradation process and environment factors, which made the proposed model in accordance with the product competition failure feature and would improve the accuracy of missile SR assessment.

The second aspect is degradation modeling for multivariate parameters, namely, the product degradation state is described by multivariate degradation parameters. There are three approaches to handle this issue.

(i) Integrate multivariate parameters into single parameter and describe the product whole degradation state [10–13]. Bayesian linear model [14], support vector machine (SVM) [15] and state space model (SSM) [16] are usually applied to build the correlation between the product whole degradation state and its multivariate parameters. Wang et

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al. [13] used the Bayesian linear model to build linear correlation between multivariate performance parameters and its accumulative degradation values. However, his model could not suit for nonlinearity. Cong et al. [15] used SVM to integrate two parameters of engine into single parameter, which could reflect the engine whole degradation state.

(ii) Directly build a joint lifetime distribution function with multivariate performance parameters. There are two situations in this case. One is the joint lifetime distribution function with independent parameters [17,18]. Liang et al. [18] firstly applied principal component analysis (PCA) to integrate multivariate parameters into several key independent parameters so as to decrease the amount of parameters, then they used the series model to build the joint lifetime distribution function with the key parameters. The other is the joint lifetime distribution function with correlative parameters. Copula function [19–24] and the multivariate Normal distribution [25,26] are used to describe the correlation among multivariate parameters. However Copula function only suits for the joint lifetime distribution function with two parameters. Zhong et al. [26] used the multivariate Normal distribution to describe the correlation among multivariate parameters. Nevertheless, his research had a certain limitation on that the marginal distribution of each parameter was the Normal distribution and had linear correlation among each other.

(iii) Respectively build the lifetime distribution function with each performance parameters and apply the information integration technology to integrate the estimates of multivariate parameters of the lifetime distribution function into the deriving estimates [27,28]. Wang [28] applied the isometric mapping method to decrease 25 parameters into two parameters, respectively built the lifetime distribution function and obtained the parameters estimates, and then used the D-S evidence synthesis method to integrate the double groups of estimates into one group. However, his research ignored the interaction among multivariate parameters and the using sample information was not sufficient, which would lead to a large deviation from the actual degradation process.

The third aspect is the degradation failure model based on the stochastic process. Because of the good property of calculation and analysis, the Wiener process is applied to model the non-monotonic degradation process. When applying the Wiener process to model the degradation process, there are two problems that need to be considered.

Problem 1 How to deal with nonlinear degradation data? There are two approaches. One is to use the time-scale model (TSM) to directly transform time axis to transform the nonlinearity into linearity and then apply the linear Wiener process to the degradation model [29–31].

This approach has the superiority in reducing the model analysis difficulty and analytical calculation complexity. The other is to directly build a nonlinear Wiener based degradation model [32–35]. Because of the difficulty to obtain the lifetime probability density function (PDF), this approach only derives the approximated PDF with complex calculation, which to some extent limits its application.

Problem 2 How to incorporate the individual variability into the population-based degradation model? Even the products from the same batch are affected by random effects in manufacturing, material, transport and environment, which will lead to difference in the individual degradation rate. The solution is to randomize the Wiener process parameters. Tang et al. [36] and Si et al. [37] only randomized the drift parameter as normal distribution and proposed the lifetime PDF with individual variability, which showed a better model fitting and reduced the calculation complexity.

In order to handle the above issues, the cause of missile storage failure is analyzed so as to determine key components in missile. Then, the characteristics voltage is assumed to be the key performance parameter of the multi-components. The degradation model for multi-components in missile is assumed to be degradation modeling with multivariate parameters. An SSM is applied to integrate multi-components voltage data into the missile degradation degree (DD). The linear Wiener based degradation model with TSM is applied to build the degradation failure model with individual variability and nonlinear data. The Weibull distribution and proportional risk model are applied to build the outburst failure model with the degradation effect. Finally, the competition failure model with the correlation between the degradation failure and outburst failure is proposed.

2. Missile storage failure

Life profile is the temporal description of all the events and environment that the product experiences from birth or delivery to life termination or retirement. The main events of missile in life cycle are warehouse storage, combat duty, depot repair, and executing task or scrap, which are shown in Fig. 1.

Storage failure is the prescriptive function loss caused by the storage reasons in prescriptive storage condition and time. By analyzing the depot repair data of missile, we find that the failure of computer, amplifier, stabilizer, resistance box, power component and flight control component in missile are the main causes of missile failure [38]. Thus we select these six components as the key components for assessing the whole missile degradation state. According

to the electronic product technical characteristic, we select the characteristic voltage as key performance parameter for the above key components. By periodical testing the voltage value of key components in missile, we can assess the whole missile degradation state.

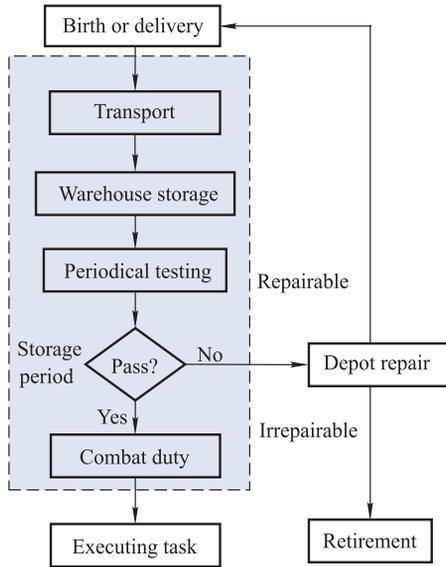


Fig. 1 Missile typical life profile

3. Missile DD

Degradation failure is thought to be the result of degradation failure competition among key components in missile. The characteristic voltage value of multi-component is the external expression of the missile inner degradation state. Then we use SSM to construct a feature matrix of the whole missile health state by these periodical testing data. By calculating the similarity degree between missile normal state and its degradation state, the missile DD is shown in Fig. 2.

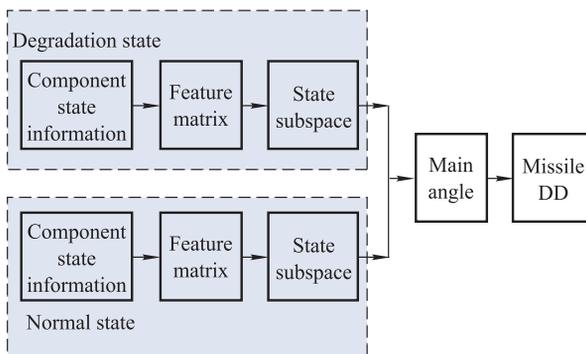


Fig. 2 Missile DD deriving flow

3.1 Constructing state feature matrix

Use the characteristic voltage testing values of multi-component in missile as the whole missile state informa-

tion and its feature matrix in certain state is constructed as

$$\mathbf{X} = (\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_m) = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1m} \\ x_{21} & x_{22} & \dots & x_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \dots & x_{nm} \end{bmatrix} \quad (1)$$

where \mathbf{X} denotes the state feature matrix, $\mathbf{x}_j = (x_{1j}, x_{2j}, \dots, x_{nj})^T$ denotes the eigenvector, x_{ij} is the state data of the i th feature at moment j ($i = 1, 2, \dots, n$; $j = 1, 2, \dots, m$), n is the feature number, m is the number of testing times.

3.2 Building state subspace

The missile state feature matrix contains different state information in the normal state and degradation state. Thus its state subspace in the two states will have some difference. Kernel principal component analysis (KPCA) is applied to build the missile state subspace from its feature matrix.

Nonlinear mapping $\varphi(\cdot)$ is used to map the state feature matrix \mathbf{X} onto a high dimension space F . It is expressed as

$$\mathbf{X} \mapsto \varphi(\cdot) : \varphi(\mathbf{X}) = [\varphi(\mathbf{x}_1), \varphi(\mathbf{x}_2), \dots, \varphi(\mathbf{x}_m)] \quad (2)$$

where $\varphi(\mathbf{X})$ is the state feature matrix in F , $\varphi(\mathbf{x}_j)$ ($j = 1, 2, \dots, m$) is the nonlinear eigenvector corresponding to the eigenvector \mathbf{x}_j .

Feature function of KPCA is written as

$$mh\alpha = k\alpha \quad (3)$$

where α is the weight vector, h is eigenvalue of the covariance matrix C for $\varphi(\mathbf{X})$, k is the kernel matrix and is defined as

$$k_{ij} = \langle \varphi(\mathbf{x}_i), \varphi(\mathbf{x}_j) \rangle = k(\mathbf{x}_i, \mathbf{x}_j) \quad (4)$$

where $i, j = 1, 2, \dots, m$, $k(\cdot)$ denotes the kernel function. We choose a Gaussian kernel function, that is

$$k(\mathbf{x}, \mathbf{y}) = \exp\left(-\frac{\|\mathbf{x} - \mathbf{y}\|^2}{2\sigma^2}\right). \quad (5)$$

The weight vector α can be derived by (3).

By normalizing the eigenvector δ of the covariance matrix C , the orthogonal basic vector ω can be formulated as

$$\omega = \frac{\delta}{\|\delta\|} = \frac{\varphi(\mathbf{X})\alpha}{\sqrt{\alpha^T k \alpha}}. \quad (6)$$

Select the larger eigenvalue h corresponding to ω and build the state subspace S as

$$S = \text{span}(\omega_1, \omega_2, \dots, \omega_r) \quad (7)$$

where r is the dimension of state subspace.

3.3 Deriving main angle

S_0 denotes the missile state subspace in the normal state, S_1 denotes the missile state subspace in the degradation state. Similarity between S_0 and S_1 can be expressed by the main angle of their orthogonal basic vectors. The larger main angle is, the larger similarity of the two state subspaces has. W denotes the inner matrix of basic vectors and is formulated as

$$W = S_0^T S_1. \quad (8)$$

By calculating the singular value of (8), d eigenvalues ($\kappa_1, \kappa_2, \dots, \kappa_d$) can be derived. The main angle θ_i is formulated as

$$\theta_i = \arccos \kappa_i, \quad i = 1, 2, \dots, d. \quad (9)$$

3.4 Calculating missile DD

According to (9), $\theta_i \in [0, \pi/2]$, the larger θ_i is, the less similarity of S_0 and S_1 has. Because the minimum main angle reflects the main similarity information of two subspaces, missile DD is defined as the minimum main angle. ε denotes the missile DD and is formulated as

$$\varepsilon = \sin[\min(\theta_i)]. \quad (10)$$

4. Competition failure model

4.1 Assumptions

Engineering experience indicates that the missile has a degradation failure mode and an outburst failure mode. Its storage failure is mainly due to the competition of these two modes. Thus before modeling competition failure, the following assumptions are given.

(i) Missile degradation state in the storage period can be measured by missile DD. Missile DD can be calculated by the characteristics voltage testing data of the key multi-component in missile.

(ii) Missile DD is a random variable. When missile DD first reaches the prescriptive degradation failure threshold, the missile is judged to be degradation failure.

(iii) When the testing stimulus signal has no output or is beyond the prescribed range by periodical testing the characteristic voltage values of multi-components, the missile is judged to be outburst failure and the latest testing time is assumed to the outburst failure time.

(iv) Missile outburst failure time obeys to the Weibull distribution and its outburst failure rate has positive correlation with its current DD.

4.2 Degradation failure

$\varepsilon(t)$ denotes missile DD. Taking into account of the indi-

vidual degradation variability, $\varepsilon(t)$ is assumed to be a random variable. According to the calculated value of $\varepsilon(t)$, the linear Wiener based degradation model is applied to model the population-based degradation process. There are two aspects of modeling.

(i) Linear degradation data

When the correlation between $\varepsilon(t)$ and time t is linear, the linear Wiener process is applied to describe $\varepsilon(t)$. Without the loss of generality, $\varepsilon(0)$ is thought to be zero, then $\varepsilon(t)$ is formulated as

$$\varepsilon(t) = ut + \sigma B(t) \quad (11)$$

where u is a drift parameter, σ is a diffusion parameter, $B(t)$ is the standard Brownian motion.

Let l ($l > 0$) denote the degradation failure threshold. When $\varepsilon(t)$ first reaches l , the missile is judged to be degradation failure. T_d denotes the degradation failure time and can be expressed as

$$T_d = \inf\{t | \varepsilon(t) \geq l\}. \quad (12)$$

According to the Wiener process property, $\varepsilon(t)$ obeys to the Normal distribution, i.e.

$$\varepsilon(t) \sim N(ut, \sigma^2 t). \quad (13)$$

The PDF of T_d is formulated as

$$g[\varepsilon(t)] = \frac{1}{\sqrt{2\pi\sigma^2 t}} \exp\left[-\frac{(\varepsilon(t) - ut)^2}{2\sigma^2 t}\right]. \quad (14)$$

$R_d(t)$ denotes the reliability function of T_d and is formulated as

$$R_d(t) = p\{T_d > t\} = \Phi\left(\frac{l - ut}{\sigma\sqrt{t}}\right) - \exp\left(\frac{2ul}{\sigma^2}\right) \Phi\left(\frac{-l - ut}{\sigma\sqrt{t}}\right). \quad (15)$$

where $\Phi(\cdot)$ is the cumulative density function (CDF) of the standard Normal distribution.

(ii) Nonlinear degradation data

When the correlation between $\varepsilon(t)$ and time t is nonlinear, TSM is applied to transform nonlinear data into linearity. The typical formula of TSM is written as

$$\tau = \Lambda(t) = t^c \quad (16)$$

where c is an unknown positive parameter. When c is assumed to be one, the correlation turns to be linear.

When the nonlinear data group $[t, \varepsilon(t)]$ is transformed to be a linear data group $[\tau, \varepsilon(\tau)]$ and $\varepsilon(t)$ is equivalent to $\varepsilon(\tau)$, (11) can be reformulated as

$$\varepsilon(\tau) = u\tau + \sigma B(\tau). \quad (17)$$

The transformed PDF of T_d is formulated as

$$g[\varepsilon(\tau)] = \frac{1}{\sqrt{2\pi\sigma^2\tau}} \exp\left[-\frac{(\varepsilon(\tau) - u\tau)^2}{2\sigma^2\tau}\right]. \quad (18)$$

$R_d(t)$ with the linearity transformation is formulated as

$$R_d(t) = \Phi\left(\frac{l - u\tau}{\sigma\sqrt{\tau}}\right) - \exp\left(\frac{2ul}{\sigma^2}\right) \Phi\left(\frac{-l - u\tau}{\sigma\sqrt{\tau}}\right). \quad (19)$$

In order to incorporate the individual degradation variability into the population-based degradation process, the drift parameter u is randomized as the Normal distribution, i.e. $u \sim N(\bar{u}, \sigma_u^2)$. The unknown parameter is derived as

$$\begin{cases} \bar{u} = \frac{1}{M} \sum_{i=1}^M u_i \\ \sigma_u^2 = \frac{1}{M} \sum_{i=1}^M (u_i - \bar{u})^2 \end{cases} \quad (20)$$

where u_i is a drift parameter of the i th missile, and M is the missile degradation failure number.

Thus, the PDF of T_d with individual variability and non-linearity is formulated as

$$g[\varepsilon(\tau)] = \frac{1}{\sqrt{2\pi\sigma^2\tau}} \exp\left[-\frac{(\varepsilon(\tau) - \bar{u}\tau)^2}{2\sigma^2\tau}\right] = \frac{1}{\sqrt{2\pi\sigma^2t^c}} \exp\left[-\frac{(\varepsilon(t) - \bar{u}t^c)^2}{2\sigma^2t^c}\right]. \quad (21)$$

$R_d(t)$ with individual variability and nonlinearity is formulated as

$$R_d(t) = \Phi\left(\frac{l - \bar{u}\tau}{\sqrt{\sigma^2\tau + \sigma_u^2\tau^2}}\right) - \exp\left[\frac{2l}{\sigma^2}\left(\bar{u} + \frac{\sigma_u^2}{\sigma^2}\right)\right] \Phi\left[-\frac{2l\sigma_u^2\tau + \sigma^2(l + \bar{u}\tau)}{\sigma^2\sqrt{\sigma^2\tau + \sigma_u^2\tau^2}}\right] = \Phi\left(\frac{l - \bar{u}t^c}{\sqrt{\sigma^2t^c + \sigma_u^2t^{2c}}}\right) - \exp\left[\frac{2l}{\sigma^2}\left(\bar{u} + \frac{\sigma_u^2}{\sigma^2}\right)\right] \Phi\left[-\frac{2l\sigma_u^2t^c + \sigma^2(l + \bar{u}t^c)}{\sigma^2\sqrt{\sigma^2t^c + \sigma_u^2t^{2c}}}\right]. \quad (22)$$

4.3 Outburst failure

We assume that the missile outburst failure time obeys to the Weibull distribution. T_r denotes the outburst failure time. $f_{r0}(t)$ denotes the lifetime PDF without the influence of degradation process on outburst failure and is formulated as

$$f_{r0}(t) = \frac{m}{\eta} \left(\frac{t}{\eta}\right)^{m-1} \exp\left[-\left(\frac{t}{\eta}\right)^m\right] \quad (23)$$

where m is a shape parameter and η is a scale parameter.

$\lambda_{r0}(t)$ denotes the missile outburst failure rate without the influence of performance degradation and is formulated as

$$\lambda_{r0}(t) = \frac{m}{\eta} \left(\frac{t}{\eta}\right)^{m-1}. \quad (24)$$

4.4 Competition failure

T denotes the missile failure time in storage period. Because the missile failure in storage period is caused by the competition of degradation failure and outburst failure, T can be formulated as

$$T = \min\{T_d, T_r\}. \quad (25)$$

$R(t)$ denotes the missile SR in occasion of competition failure with the correlation between degradation failure and outburst failure and is formulated as

$$R(t) = p\{T > t\} = p\{T_d > t, T_r > t\} = p\{T_r > t | T_d > t\} p\{T_d > t\} = R_{r|d}(t) R_d(t) \quad (26)$$

where $R_{r|d}(t)$ denotes the conditional reliability function (CRF) of the outburst failure time under the condition that the missile does not suffer from degradation failure.

Since the missile outburst failure rate has correlation with current DD, a proportional risk model [39] is applied to describe the positive correlation between the outburst failure rate and missile DD. ε_t denotes the corresponding DD values. $\lambda_r(t, \varepsilon_t)$ denotes the outburst failure rate at time t . We assume that failure does not occur until time t and $\lambda_r(t, \varepsilon_t)$ can be formulated as

$$\lambda_r(t, \varepsilon_t) = \lambda_{r0}(t) q(\varepsilon_t) \quad (27)$$

where $\lambda_{r0}(t)$ is the standard risk function when $q(\varepsilon_t) = 1$, which can be expressed by the missile outburst failure rate without the influence of performance degradation. The typical function form of $q(\varepsilon_t)$ is written as

$$q(\varepsilon_t) = \exp(\beta_0 + \beta_1 \varepsilon_t) \quad (28)$$

where β_0 and β_1 are unknown fixed parameters.

$\varepsilon(t)$ is a random variable. For the guarantee that failure does not occur until time t , (27) can be reformulated as

$$\lambda_r[t, \varepsilon(t)] = \lambda_{r0}(t) \int_0^l q(\varepsilon_t) g(\varepsilon_t) d\varepsilon_t. \quad (29)$$

With the consideration of individual degradation variability and nonlinearity, we substitute (16), (21) and (28) into (29) and derive the formula of $\lambda_r(t, \varepsilon_t)$ as

$$\lambda_r[t, \varepsilon(t)] = \lambda_r[t, \varepsilon(\tau)] = \lambda_{r0}(t) \int_0^l q(\varepsilon_\tau) g(\varepsilon_\tau) d\varepsilon_\tau = \frac{m}{\eta} \left(\frac{t}{\eta}\right)^{m-1}.$$

$$\int_0^l \exp(\beta_0 + \beta_1 \varepsilon_\tau) \frac{1}{\sqrt{2\pi\sigma^2\tau}} \exp\left[-\frac{(\varepsilon_\tau - \bar{u}\tau)^2}{2\sigma^2\tau}\right] d\varepsilon_\tau =$$

$$\frac{m}{\eta} \left(\frac{t}{\eta}\right)^{m-1} \exp(\beta_0 + \beta_1 \bar{u}\tau + \frac{1}{2}\beta_1^2\sigma^2\tau) \cdot$$

$$\Phi\left(\frac{l - \bar{u}\tau - \beta_1\sigma^2\tau}{\sigma\sqrt{\tau}}\right) = \frac{m}{\eta} \left(\frac{t}{\eta}\right)^{m-1} \cdot$$

$$\exp\left(\beta_0 + \beta_1 \bar{u}t^c + \frac{1}{2}\beta_1^2\sigma^2t^c\right) \Phi\left(\frac{l - \bar{u}t^c - \beta_1\sigma^2t^c}{\sigma\sqrt{t^c}}\right). \quad (30)$$

Thus $R_{r|d}(t)$ is formulated as

$$R_{r|d}(t) = \exp\left\{-\int_0^t \lambda_r[\mu, \varepsilon(\mu)]d\mu\right\}. \quad (31)$$

$f_{r|d}(t)$ denotes the condition PDF of the outburst failure time under the condition that missile does not suffer from degradation failure at time t and is formulated as

$$f_{r|d}(t) = \lambda_r[t, \varepsilon(t)] \cdot R_{r|d}(t). \quad (32)$$

Substituting (31) into (26), $R(t)$ is reformulated as

$$R(t) = R_{r|d}(t)R_d(t) =$$

$$\exp\left\{-\int_0^t \lambda_r[\mu, \varepsilon(\mu)]d\mu\right\} R_d(t) = \exp\left\{-\int_0^t \left[\frac{m}{\eta} \left(\frac{t}{\eta}\right)^{m-1} \cdot\right.\right.$$

$$\exp\left(\beta_0 + \beta_1 \bar{u}t^c + \frac{1}{2}\beta_1^2\sigma^2t^c\right) \Phi\left(\frac{l - \bar{u}t^c - \beta_1\sigma^2t^c}{\sigma\sqrt{t^c}}\right) \left.\right\} d\mu \left\{ \right.$$

$$\left. \left\{ \Phi\left(\frac{l - \bar{u}t^c}{\sqrt{\sigma^2t^c + \sigma_u^2t^{2c}}}\right) - \right.\right.$$

$$\left. \left. \exp\left[\frac{2l}{\sigma^2} \left(\bar{u} + \frac{\sigma_u^2}{\sigma^2}\right)\right] \Phi\left[-\frac{2l\sigma_u^2t^c + \sigma^2(l + \bar{u}t^c)}{\sigma^2\sqrt{\sigma^2t^c + \sigma_u^2t^{2c}}}\right] \right\} \right\}. \quad (33)$$

If the missile outburst failure has no correlation with its degradation failure, missile SR $R'(t)$ can be expressed as

$$R'(t) = \exp\left[-\int_0^t \lambda_{r0}(\mu)d\mu\right] R_d(t). \quad (34)$$

ξ denotes the mean time between failure (MTBF) of missile in storage period and is formulated as

$$\xi = \int_0^{+\infty} tR(t)dt. \quad (35)$$

5. Parameters estimation

According to the competition failure modeling above, the unknown parameters are divided into three parts. The first is model parameters of degradation failure and is denoted

as $\theta_1 = (\bar{u}, \sigma, c)$. The second is model parameters of outburst failure and is denoted as $\theta_2 = (m, \eta)$. The third is parameters affecting the outburst failure rate by degradation and is denoted as $\theta_3 = (\beta_0, \beta_1)$.

We assume that a set of missiles with the same type are stored in a prescriptive warehouse. The characteristic voltage of multi-components in missile is periodically tested. t_1, t_2, \dots, t_n denotes the testing time. SSM is applied to calculate missile DD at every testing time. By analyzing the missile DD value, M missiles suffer from the degradation failure in storage period time $(0, t_n]$, N missiles suffer from the outburst failure, and K missiles are still in good condition.

5.1 Estimating θ_1

There are M missiles with degradation failure and the M th missile is tested at time $t_{1M_1}, t_{2M_2}, \dots, t_{MM_M}$. $\varepsilon(t_{ij})$ denotes the DD value of the i th missile at the j th testing time ($i = 1, 2, \dots, M; j = 1, 2, \dots, M_i$). The data group $[t_{ij}, \varepsilon(t_{ij})]$ can be transformed into a linear data group $[\tau_{ij}, \varepsilon(\tau_{ij})]$ by TSM, where $\tau_{ij} = t_{ij}^c$ and $\varepsilon(\tau_{ij}) = \varepsilon(t_{ij})$.

According to the Wiener process property and (21), the likelihood function for θ_1 with degradation failure samples is formulated as

$$L(u_i, \sigma^2) = \prod_{i=1}^M \prod_{j=1}^{M_i} g[\varepsilon(\Delta\tau_{ij})] =$$

$$\prod_{i=1}^M \prod_{j=1}^{M_i} \frac{1}{\sqrt{2\pi\sigma^2\Delta t_{ij}^c}} \exp\left[-\frac{(\varepsilon(\Delta t_{ij}) - u_i\Delta t_{ij}^c)^2}{2\sigma^2\Delta t_{ij}^c}\right] \quad (36)$$

with

$$\begin{cases} \Delta t_{ij} = t_{ij} - t_{i(j-1)} \\ \varepsilon(\Delta t_{ij}) = \varepsilon(t_{ij}) - \varepsilon(t_{i(j-1)}) \end{cases}. \quad (37)$$

The log-likelihood function for θ_1 is reformulated as

$$\ln L(u_i, \sigma^2) =$$

$$-\frac{1}{2} \sum_{i=1}^M \sum_{j=1}^{M_i} \left[\ln(2\pi\Delta t_{ij}^c) + \ln\sigma^2 + \frac{(\varepsilon(\Delta t_{ij}) - u_i\Delta t_{ij}^c)^2}{\sigma^2\Delta t_{ij}^c} \right]. \quad (38)$$

Set the first partial derivatives of $\ln L(u_i, \sigma^2)$ with respect to u_i and σ^2 to zero, then the estimates of (u_i, σ^2) are formulated as

$$\hat{u}_i = \frac{\sum_{i=1}^M \varepsilon(t_{iM_i})}{\sum_{i=1}^M t_{iM_i}^c} \quad (39)$$

$$\hat{\sigma}^2 = \frac{1}{\sum_{i=1}^M M_i} \left[\sum_{i=1}^M \sum_{j=1}^{M_i} \frac{(\Delta \varepsilon(t_{ij}))^2}{\Delta t_{ij}^c} - \frac{\left[\sum_{i=1}^M \varepsilon(t_{iM_i}) \right]^2}{\sum_{i=1}^M t_{iM_i}^c} \right]. \quad (40)$$

Because the solution to (39) and (40) depends on the value of c , the FMINSEARCH function is used to handle this problem. Let c be a variable and $\ln L(u_i, \sigma^2)$ be a optimal function. Firstly, set c_0 be the initial value of c . Then the FMINSEARCH function is used to derive the maximum value of $\ln L(u_i, \sigma^2)$ by searching the value of c . When $\ln L(u_i, \sigma^2)$ reaches its maximum value, the returning value of c is the estimate \hat{c} . \hat{u}_i and $\hat{\sigma}^2$ will be derived by substituting \hat{c} into (20). Finally, \bar{u} and $\hat{\sigma}_u^2$ will be obtained by substituting \hat{u}_i into (21).

5.2 Estimating θ_2 and θ_3

There are N missiles with outburst failure. $t_{1N_1}, t_{2N_2}, \dots, t_{NN_N}$ are the missile outburst failure time. $\varepsilon(t_{1N_1}), \varepsilon(t_{2N_2}), \dots, \varepsilon(t_{NN_N})$ are the corresponding missile DD values. There are K missiles without any failure at the terminal testing time t_n .

According to (32) and (33), the likelihood function for θ_2 and θ_3 with outburst failure samples and no-failure samples is formulated as

$$L(\theta_2, \theta_3) = \prod_{i=1}^N f_{r|d}(t_{iN_i}) \prod_{i=1}^K R(t_n) = \prod_{i=1}^N \lambda_r [t_{iN_i}, \varepsilon(t_{iN_i})] R_{r|d}(t_{iN_i}) \prod_{i=1}^K R(t_n). \quad (41)$$

Substitute (30)–(32) into (41) and the likelihood function for θ_2 and θ_3 with unknown fixed parameters could be built. Due to the calculation complexity of the likelihood function, the Markov chain Monte Carlo (MCMC) is applied to quickly derive the estimates of θ_2 and θ_3 [40].

When substitute $\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3$ into (33) and (35), SR and MTBF of this set of missiles will be obtained.

6. Numerical examples

There are 20 air-to-ground missiles with the same type stored in a warehouse. Computer, amplifier, stabilizer, resistance box, power and flight control are assumed to be the key components of air-to-ground missiles, which can represent for the whole missile health state. The missile storage period is ten years. The characteristic voltage value of multi-components is tested every half a year. The voltage testing values of multi-components in a healthy missile is shown in Fig. 3.

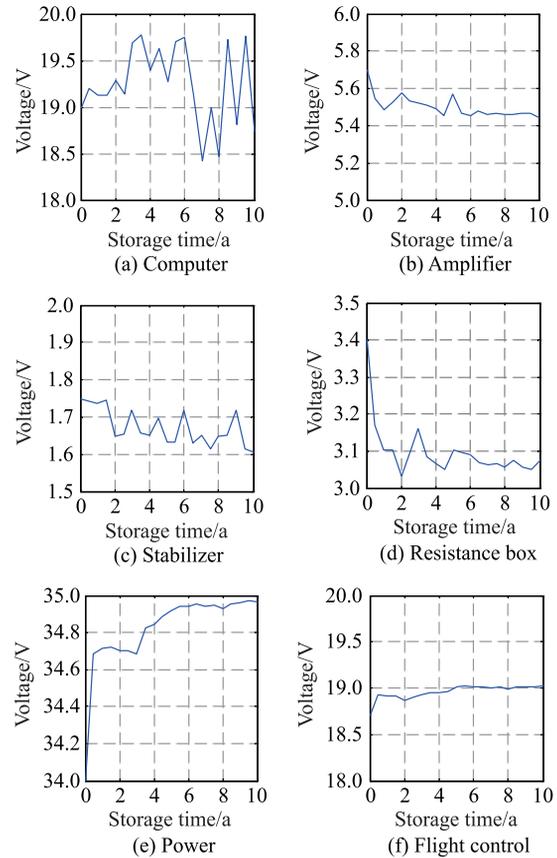


Fig. 3 Voltage testing values of multi-components in a healthy missile

According to the missile design specification, the characteristic voltage standard value of computer, amplifier, stabilizer, resistance box, power component and flight control component are 19 V, 5.7 V, 1.75 V, 3.4 V, 34 V and 18.7 V, respectively. When the characteristic voltage testing value of multi-components exceeds 10% of the standard value, the missile is judged to be degradation failure. The missile DD is calculated to be 0.173 6 at this time, which is assumed to be the degradation threshold l . When the testing value exceeds 100% of the standard value or the testing value is zero, the missile is judged to be outburst failure.

6.1 Missile DD calculation

According to the characteristic voltage testing data of multi-components in missile, SSM is applied to calculate missile DD in each testing time.

According to the above failure judgment, there are eight missiles with degradation failure, seven missiles with outburst failure and five missiles without any failure. The result of missile DD with degradation failure is shown in Fig. 4. Missile degradation failure time and the corresponding DD are shown in Table 1. Missile outburst failure time and the corresponding DD are shown in Table 2.

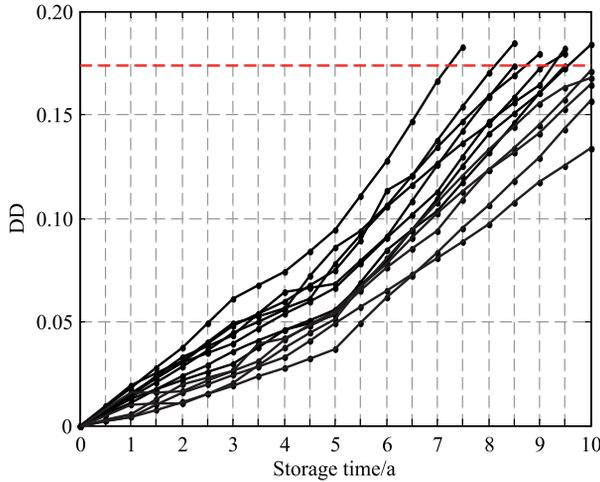


Fig. 4 DD values with degradation failure samples

Table 1 Missile DD with degradation failure time

Missile	t_d	$\varepsilon(t_d)$	Missile	t_d	$\varepsilon(t_d)$
1	7.5	0.182 9	5	9.5	0.173 8
2	8.5	0.173 7	6	9.5	0.181 8
3	8.5	0.184 7	7	9.5	0.179 4
4	9	0.179 3	8	10	0.183 8

Table 2 Missile DD with outburst failure time

Missile	t_r	$\varepsilon(t_r)$	Missile	t_r	$\varepsilon(t_r)$
1	4.5	0.094 8	5	7.5	0.136 0
2	5.5	0.108 3	6	7.5	0.134 4
3	6	0.116 1	7	8.5	0.158 8
4	7	0.120 4			

6.2 Parameters estimation

(i) Estimating θ_1

Set one as the initial value of c . According to the calculated missile DD data with degradation failure, the FMINSEARCH is used to search the maximum value of $\ln L(u_i, \sigma^2)$ for (38). When $\ln L(u_i, \sigma^2)$ reaches its maximum value at 7.419, c is 1.127. Substituting the value of c into (39) and (40), the estimates \hat{u}_i with degradation failure are derived as 0.074 6, 0.081 2, 0.083 9, 0.076 0, 0.072 8, 0.081 0, 0.084 1 and 0.087 4. The estimate $\hat{\sigma}^2$ is 2.881×10^{-6} . Substitute the estimate \hat{u}_i into (20) and the estimate of θ_1 is obtained as

$$\hat{\theta}_1 = (\bar{u}, \hat{\sigma}_2, \hat{c}) = (0.075 3, 2.881 \times 10^{-6}, 1.127).$$

(ii) Estimating θ_2 and θ_3

According to the calculated missile DD data with outburst failure and no-failure, WinBUGS toolbox based on MCMC is applied to solve (40) [40]. The estimates of θ_2 and θ_3 are obtained as

$$\hat{\theta}_2 = (\hat{m}, \hat{\eta}) = (5.644, 6.512)$$

$$\hat{\theta}_3 = (\hat{\beta}_0, \hat{\beta}_1) = (0.073 8, -8.483 7).$$

6.3 Distribution hypothesis testing

(i) DD increment

Let

$$z_i = [\varepsilon(\tau_i) - u\tau_i] / \sigma\sqrt{\tau_i} =$$

$$[\varepsilon(t_i) - ut_i^c] / \hat{\sigma}\sqrt{t_i^c}, \quad i = 1, 2, \dots, 8.$$

According to the estimates $\hat{\theta}_1$ and DD increment of eight missiles with degradation failure, the Anderson-Darling (AD) statistics is applied to test whether z_i obeys to the Normal distribution, i.e. $N(0, 1)$. Fig. 5 shows that DD increment data after transforming into linearity is accepted as obeying to the Normal distribution at 95% confidence level, which means the degradation process of missiles can be described by the Wiener process.

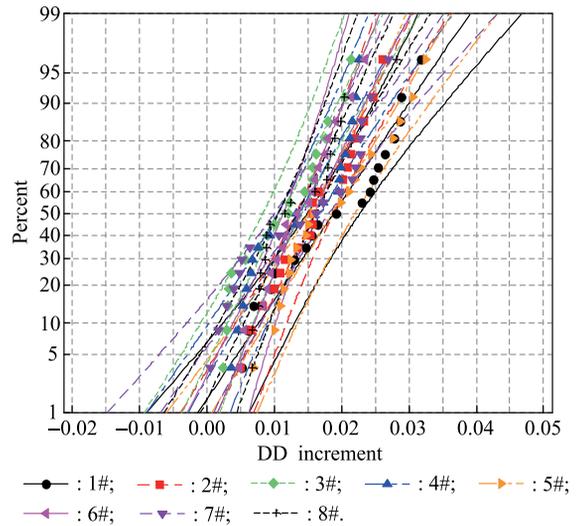


Fig. 5 Goodness of Normal fit test for DD increment

(ii) Outburst failure time

According to the failure time of seven missiles with outburst failure, the AD statistics is applied to test where T_r obeys to Weibull distribution. Fig. 6 shows that T_r is accepted as obeying to the Weibull distribution at 95% confidence level.

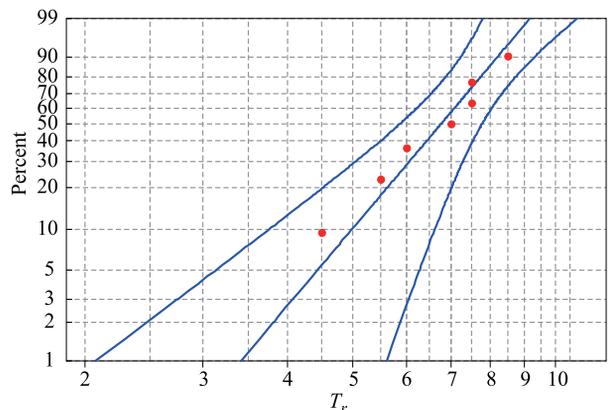


Fig. 6 Goodness of Weibull fit test for outburst failure time

6.4 Assessing SR

Substitute the estimates of $\theta_1, \theta_2, \theta_3$ into (33), then the missile SR $R(t)$ of competition failure with the two correlation failure modes is derived and its corresponding model is denoted as M1. Substitute the estimates of $\theta_1, \theta_2, \theta_3$ into (34), then the missile SR $R'(t)$ of competition failure with the two independence failure modes is derived and its corresponding model is denoted as M2. Substitute the estimates of $\theta_1, \theta_2, \theta_3$ into (22), then the missile SR $R_d(t)$ with only degradation failure information is derived and its corresponding model is denoted as M3. Missile SR curves by M1, M2 and M3 are shown in Fig. 7. Missile storage PDFs by M1, M2 and M3 are shown in Fig. 8.

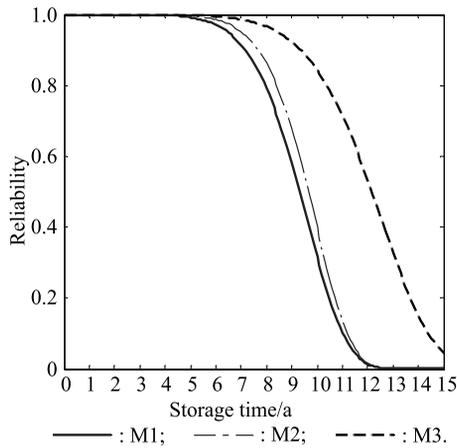


Fig. 7 Missile SR curves by M1, M2 and M3

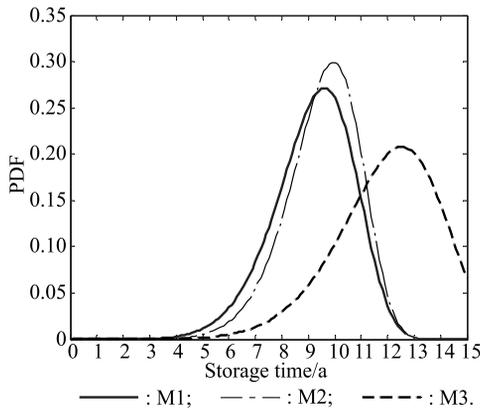


Fig. 8 Missile storage PDFs by M1, M2 and M3

Fig. 7 shows that the value of $R_d(t)$ is largest. This is because that the model with only degradation failure information ignores the information of the outburst failure, which will lead to a high error of missile SR assessment. The value of $R(t)$ is smaller than that of $R'(t)$. This is because that the proposed competition failure model with the correlation failure modes fully uses the population-based information, which is a conservative approach and is in accordance with engineering practice.

Fig. 8 shows that the missile storage PDFs by M1 and M2 are very sharper than that by M3 and they are close to each other, which demonstrates the better model fitting capability of M1 and M2. Compared with M2, M1 has a conservative result of PDF, which leads to a smaller missile storage MTBF and an earlier preparing for the life extension after storage.

6.5 Assessment accuracy

According to (35), the estimates of missile storage MTBF by M1, M2 and M3 are 11.12, 11.83 and 13.41. According to the recorded failure time or terminated testing time of missiles, ξ^* denotes the statistic of missile storage MTBF and is formulated as

$$\xi^* = \frac{T}{r} = \frac{\sum T_r + \sum T_d + \sum t_n}{r_r + r_d} = 11.47 \quad (42)$$

where r_r and r_d are the number of degradation failure and outburst failure, respectively.

In the calculation above, missile testing time was simplified to be its failure time. Actually, the missile failure time is within the interval of the current testing time and the previous testing time, namely, ξ^* is within the interval of [10.97, 11.47].

Thus the estimate of missile storage MTBF by M1 is within the range of [10.97, 11.47], but the estimates of missile storage MTBF by M2 and M3 are not in this range, which proves that the proposed model has a better model fitting capability than that of M2 and M3.

The residual lifetime (RL) is defined as the length of time from the present time to the failure time. According to (35), the storage RL by M1, M2 and M3 from the 6th year to the 10th year is shown in Fig. 9.

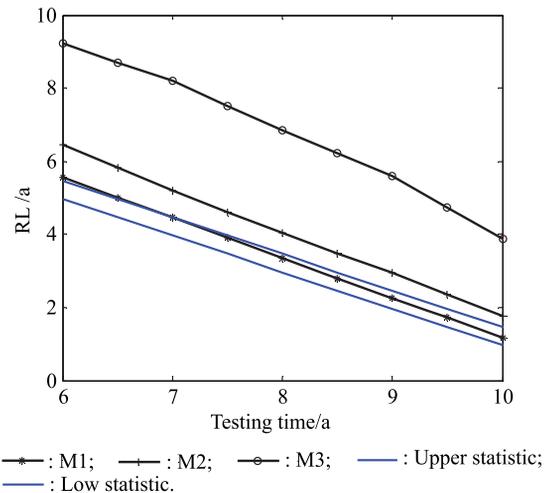


Fig. 9 Missile storage RL by M1, M2 and M3

Fig. 9 shows that the estimates of storage RL by M1 are almost all in the range of its statistics, which proves that

the accuracy of storage RL assessment is improving along with the increasing testing data. However, the estimates of storage RL by M2 and M3 are not in the range of its statistics, which further proves that the proposed model has a higher assessment accuracy and can well reflect the missile RL competition failure feature.

7. Conclusions

This paper has drawn the following conclusions:

(i) An effective approach for degradation competition modeling of multi-components is proposed. According to the characteristic voltage testing data of multi-components in missile, SSM is applied to calculate missile DD, which reflects the whole degradation state.

(ii) The analytical formulas to the missile SR and RL of competition failure with the correlation between degradation failure and outburst failure are obtained with the consideration of individual variability and nonlinear data, which well reflects the missile storage competition failure feature.

(iii) The FMINSEARCH function is applied to derive the optimal estimates of likelihood function with degradation failure information. MCMC is applied to quickly solve the complex likelihood function with outburst failure and no-failure information.

(iv) By comparing the SR, MTBF and RL with the traditional methods, the proposed model shows the better assessment accuracy and model fitting.

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