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Marginal optimization method to improve the inconsistent comparison matrix in the analytic hierarchy process

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Abstract: To improve the inconsistency in the analytic hierarchy process (AHP), a new method based on marginal optimization theory is proposed. During the improving process, this paper regards the reduction of consistency ratio (CR) as benefit, and the maximum modification compared to the original pairwise comparison matrix (PCM) as cost, then the improvement of consistency is transformed to a benefit/cost analysis problem. According to the maximal marginal effect principle, the elements of PCM are modified by a fixed increment (or decrement) step by step till the consistency ratio becomes acceptable, which can ensure minimum adjustment to the original PCM so that the decision makers' judgment is preserved as much as possible. The correctness of the proposed method is proved mathematically by theorem. Firstly, the marginal benefit/cost ratio is calculated for each single element of the PCM when it has been modified by a fixed increment (or decrement). Then, modification to the element with the maximum marginal benefit/cost ratio is accepted. Next, the marginal benefit/cost ratio is calculated again upon the revised matrix, and followed by choosing the modification to the element with the maximum marginal benefit/cost ratio. The process of calculating marginal effect and choosing the best modified element is repeated for each revised matrix till acceptable consistency is reached, i.e., CR<0.1. Finally, illustrative examples show the proposed method is more effective and better in preserving the original comparison information than existing methods.

Keywords: analytic hierarchy process (AHP), comparison matrix, consistency, marginal optimization.

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1. Introduction

Analytic hierarchy process (AHP) is a widely used methodology in many real-life decision-making problems [1 – 4]. In AHP, however, the calculated priorities are plausible only if the pairwise comparison matrix (PCM) passed the consistency test [5]. Therefore, the decision makers (DMs) need to adjust the inconsistent PCM before the analytical hierarchy process.

Generally, there are two scenarios concerned with consistency adjustment in AHP: decision-making involving a single decision maker (DM) [5 – 8] (or individual decision making problem) and decision-making realized in presence of several DMs (or group decision-making problem) [9,10]. Pedrycz and Song [11] discussed both scenarios with pairwise matrix constructed by linguistic terms. Some other studies [6,8,12] are concerned with individual decision making models, where the entries of the pairwise matrix are plain numbers. In this paper, we will focus on the individual decision making problem with the inconsistent pairwise matrix constructed by plain numbers. Many studies have presented different ways about such problem. For example, Stan and Conklin [12] proposed the robust estimation method, which identifies the unusual and false observations (they call them UFO) firstly and by modifying the UFO element till the PCM becomes highly consistent. Tseng [5] intended to discover a method of UFO element remediation that is more efficient than Stan and Conklin [12], and they provided a complete transitivity convergence algorithm, which develops a recursive single-stage algorithm to identify the UFO elements and readjust them in any PCM until consistency ratio CR<0.1. However, these studies only pay attention to reducing CR as fast as possible, while the maximum revision element of the PCM usually becomes so large (i.e. change an element from 6 to 3/5) that the original judgement of DMs might have been changed. Some other researchers have noticed this problem, and developed different methods to limit the maximum revision. For example, Xu and Wei [8] proposed an algorithm to derive a positive reciprocal matrix with acceptable consistency (i.e., CR<0.1), and gave the convergence theorem for the algorithm, while two indices were defined to reflect the departure between the modified matrix and the original matrix. And Cao et al. [6] developed a heuristic approach to arrive at a consistent matrix that can retain more information than Xu and Wei [8] by comparing

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the two indices. Cao et al. [6] firstly decomposed the original matrix as the Hadamard product of a consistent matrix and a reciprocal deviation matrix, then constructed the modified matrix via a convex combination of the reciprocal deviation matrix and a zero deviation matrix, and gave the conditions under which the CR value keeps decreasing.

In our previous study (Wu et al. [13]), we think there are two aims for the adjustment of consistency in AHP: one is to make the comparison matrices meet the consistency requirement, and the other is to minimize the deviation between the original PCM and the modified PCM so that the modified matrix preserves the most information of the original PCM. Therefore, we transform the consistency adjustment problem to an optimization problem, where the goal is to minimize the modification of the original comparison matrices (i.e., minimize the two indices of Xu and Wei [8]), and the constraint is to make the modified matrices meet the consistency requirement, i.e., CR<0.1. And the optimization problem is solved by the improved pattern search algorithm.

This research intends to discover a method that derives an acceptable consistency matrix (i.e., CR<0.1), as well as makes the departure between the modified PCM and original PCM even smaller than Xu and Wei [8], Cao et al. [6], Wu et al. [13], etc.

In this paper, we also regard the consistency adjustment problem to be an optimization problem as Wu et al. [13], and we have found that when adjusting the comparison matrices step by step, we hope the elements with greater modification should have less chance to be modified in order to minimize the total modification of the original PCM, on the contrary, the elements with smaller modification should have more chance to be modified.

When the maximum deviation between the modified PCM and the original PCM is regarded as cost, and the total reduction of CR is regarded as benefit, then the consistency improving problem is actually transformed to a benefit/cost analysis problem, in other words, to attain maximum benefit with minimum cost. The benefit/cost analysis problem can be solved easily using the iterative marginal optimization algorithm, which is a basic method in economics studies. The marginal optimization method also has many other successful applications, such as optimal inventory modelling [14,15], reliability optimization problems [16], etc. In this paper, we will use the marginal optimization method to improve the inconsistent PCM in AHP.

The paper is organized as follows. Firstly, we start with notations and definitions of the AHP decision model in Section 2. In Section 3, we present the basic idea and the process of the marginal optimization algorithm, as well as the theorem for applying the algorithm to the consistency adjustment problem. Numerical examples are given in Section 4, followed by conclusions and discussions in Section 5.

2. Notations and definitions

In classical AHP, the PCM $\boldsymbol{A}$ is structured on Saaty's one to nine scales [17], whose element a_{ij} is integer numbers within the interval [1,9], or the reciprocal of those integer numbers. We think the integer value assumption may be a cause that makes the revised PCM against DM's will. For example, the PCM $\boldsymbol{A}$ in Fig. 1 is inconsistent with CR=0.103 7, when we change a_{12} from 3 to 3.1, we obtain a consistently acceptable PCM $\boldsymbol{A}_1$ with CR= 0.096 8, but according to Saaty's one to nine scales, a_{12} should at least be modified to 4 and we obtain a PCM $\boldsymbol{A}_2$ with CR= 0.051 6. Obviously, for the DMs, 3 and 3.1 nearly have no difference, while $a_{12} = 3$ and $a_{12} = 4$ may have deviations in judgement, which may have changed DM's original preference on alternative 1 and alternative 2. Therefore, the definition of continuous PCM is proposed, which allows a_{ij} to be any real number within the interval [1/10,10].

$$\boldsymbol{A}=\begin{bmatrix}1 & 3 & 1\\ 1/3 & 1 & 1/8\\ 1 & 8 & 1\end{bmatrix}\quad \boldsymbol{A}_1=\begin{bmatrix}1 & 3.1 & 1\\ 1/3.1 & 1 & 1/8\\ 1 & 8 & 1\end{bmatrix}\quad \boldsymbol{A}_2=\begin{bmatrix}1 & 4 & 1\\ 1/4 & 1 & 1/8\\ 1 & 8 & 1\end{bmatrix}$$

Fig. 1 An illustration example of continuous PCM

Definition 1 [13] A positive reciprocal matrix $\boldsymbol{A}_R = (a_{ij})_{n\times n}$ is a continuous PCM if, and only if the following conditions hold:

$a_{ij} \in \mathbf{R}, a_{ij}\cdot a_{ji} = 1, \max\{a_{ij}, a_{ji}\} \in [1, 10]$, for $\forall i, j$.

The priority vector $\boldsymbol{\omega} = (\omega_1, \ldots, \omega_n)^{\mathrm{T}}$ can be obtained by eigenvector method (EM),

$$\boldsymbol{A}_R\boldsymbol{\omega} = \lambda_{\max}\boldsymbol{\omega} \tag{1}$$

where $\lambda_{\max}$ is the maximum eigenvalue of $\boldsymbol{A}_R$.

Saaty derives a CR from $\lambda_{\max}$,

$$\mathrm{CR} = \frac{\lambda_{\max} - n}{(n-1)RI} \tag{2}$$

where RI is the random consistency index, and its value is related to the dimension of the matrix $\boldsymbol{A}_R$, see Table 1 [17].

Table 1 Random consistency index $\boldsymbol{RI}$

n	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.52	0.89	1.12	1.26	1.36	1.41	1.46	1.49

A PCM is said to be of acceptable consistency when CR < 0.1.

Definition 2 [13] Let $\boldsymbol{A} = (a_{ij})_{n\times n}$ be a continuous PCM, then the upper half elements of $\boldsymbol{A}$ listed from top down and left to right is called the decision vector, which is denoted as $\boldsymbol{a} = \{a_{12}, \ldots, a_{1n}, a_{23}, \ldots, a_{2n}, \ldots, a_{(n-1)n}\}$, and $\boldsymbol{b} = \{b_{12}, \ldots, b_{1n}, b_{23}, \ldots, b_{2n}, \ldots, b_{(n-1)n}\}$ is called the superior decision vector, which is defined as

$$b_{ij} = \begin{cases} a_{ij}, & a_{ij} \geqslant 1 \\ \dfrac{1}{a_{ij}}, & \text{otherwise} \end{cases}.$$

Obviously, $b_{ij} \geqslant 1$, for all $b_{ij} \in \boldsymbol{b}$.

Then, the original PCM $\boldsymbol{A} = (a_{ij})_{n \times n}$ can be uniquely determined by vector $\boldsymbol{a}$ or $\boldsymbol{b}$.

Let $\boldsymbol{c} = \{c_{12}, \ldots, c_{1n}, c_{23}, \ldots, c_{2n}, \ldots, c_{(n-1)n}\}$ be the superior decision vector of the revised PCM, and the maximum revision σ is defined as the maximum deviation between the revised PCM and original PCM [13]:

$$\sigma = \max\{|c_{ij} - b_{ij}|\}, \quad c_{ij} \in \boldsymbol{c}, b_{ij} \in \boldsymbol{b}. \tag{3}$$

Obviously, the dimension of vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are $(n-1) + (n-2) + \cdots + 1 = n(n-1)/2$. For example, if $\boldsymbol{A}$ is a 4×4 PCM, it can be uniquely determined by a six dimension decision vector.

3. Marginal optimization method

3.1 Tolerance deviations interval

The maximum revision in (3) that is less than 1 is called within the tolerance deviations interval [13]. Accordingly, tolerance deviations intervals are listed in Table 2. For example, when i is strongly important to j, the scale of a_{ij} is 5, and the tolerance deviations interval is [4,6), which means a_{ij} can be substituted by any real number on the interval [4,6). Since we only consider $a_{ij} \geqslant 1$ here, when i is equally important to j ($a_{ij} = 1$), the tolerance deviations interval is set to [1,2). For $a_{ij} < 1$, the tolerance deviations interval can be obtained by $[a_{ij}^L, a_{ij}^U] = [1/a_{ji}^U, 1/a_{ji}^L]$.

Table 2 Linguistic description of scale and its corresponding tolerance deviations interval

Linguistic description	Scale	Tolerance deviations interval
Extreme importance	9	[8,10]
Demonstrated importance	7	[6,8)
Strong importance	5	[4,6)
Moderate importance	3	[2,4)
Equal importance	1	[1,2)

From Table 2, we can see that Saaty's one to nine scales is using integer number ranging from 1 to 9 to quantify the linguistic description of importance, while the tolerance deviations interval actually extends the integer number to an acceptable interval. Also, Fig. 2 gives the corresponding relationship between tolerance deviations intervals and linguistic descriptions of importance scales.

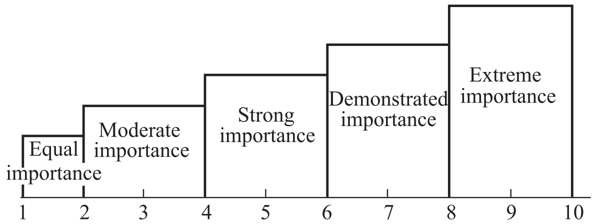

Fig. 2 Relationship between tolerance deviations interval and linguistic description

3.2 Marginal effect analysis

Marginal effect analysis is a basic mathematic method widely used in economy issues. Because many of the variables of interest are interaction terms, to see the overall effect of a condition on growth, one must look at the marginal effect of that variable. Besides economy problems, marginal optimization based on marginal effect analysis has found many applications, such as the optimal spare parts inventory problems [14,15].

The main idea of the marginal optimization method to improve the inconsistent PCM is to increase (or decrease) one element a_{ij} in superior decision vector by a fixed value d for all i, j (d is usually set to be 0.1, 0.01, 0.001, and we will have more discussion on this in Section 3.3 and Section 4), and calculate the marginal effect of each modification $a_{ij} \pm d$, that is, the ratios of the reduction of CR caused by modifying one element to $a_{ij} \pm d$, i.e., we denote as $\mathrm{CR}(a_{ij}) - \mathrm{CR}(a_{ij} \pm d)$, to maximum revision σ derived by (3) for the modified matrix. The modification with the biggest ratio means that it is of greatest marginal effect, or benefit/cost ratio, in other words, it is sensible to revise this element of the superior decision vector to improve CR. Therefore, this element should increase (or decrease) by d, and whether it increases or decreases depends on if it is able to reduce CR to a lower value or not.

Theorem 1 Let $\boldsymbol{A} = [a_{ij}]_{n \times n}$ be a continuous PCM that is inconsistent with $\mathrm{CR}(\boldsymbol{A}) > 0.1$, then there must be one element a_{ij} that can make $\mathrm{CR}(\boldsymbol{A})$ decrease when a_{ij} is modified.

Proof According to Saaty [7], let $r_{ij} = \dfrac{\partial \lambda_{\max}}{\partial a_{ij}}$:

$$r_{ij} = \begin{cases} v_i u_j - u_i v_j a_{ij}^{-2}, & a_{ij} \geqslant 1 \\ v_i u_j a_{ij}^2 - u_i v_j, & a_{ij} < 1 \end{cases} \tag{4}$$

where $\boldsymbol{u} = (u_1, u_2, \ldots, u_n)^{\mathrm{T}}$ and $\boldsymbol{\nu} = (\nu_1, \nu_2, \ldots, \nu_n)^{\mathrm{T}}$ are the principal right eigenvectors of $\boldsymbol{A}$ and $\boldsymbol{A}^{\mathrm{T}}$ respectively. Obviously, when $\boldsymbol{A}$ is inconsistent with $\mathrm{CR}(\boldsymbol{A}) > 0.1$, matrix $\boldsymbol{R} = (r_{ij})_{n \times n}$ holds:

$$r_{ij} = \begin{cases} 0, & i = j \\ -r_{ji}, & i \neq j \end{cases}. \tag{5}$$

$r_{ij} = 0$ holds for all i, j if, and only if $\boldsymbol{A}$ is consistent, i.e., $\mathrm{CR}(\boldsymbol{A}) = 0$. Since $\mathrm{CR}(\boldsymbol{A}) > 0.1$, $\boldsymbol{A}$ is inconsistent, then there must be $r_{pq} \neq 0$ $(p, q \in \{1, 2, \ldots, n\})$ which satisfies the following relationships. If $r_{pq} < 0$, i.e., $\dfrac{\partial \lambda_{\max}}{\partial a_{pq}} < 0$, we can conclude that the bigger a_{pq}, the smaller $\lambda_{\max}$, which means we should increase a_{pq} to reduce $\lambda_{\max}$, thus reduce $\mathrm{CR}(\boldsymbol{A})$ according to (2). On the contrary, if $r_{pq} > 0$, we should decrease a_{pq} to reduce $\lambda_{\max}$ and $\mathrm{CR}(\boldsymbol{A})$. Therefore, by revising a_{pq} when $r_{pq} \neq 0$, the modified matrix would have a reduced $\lambda_{\max}$ so that $\mathrm{CR}(\boldsymbol{A})$ is decreased. □

By Theorem 1, the value of $\mathrm{CR}(\boldsymbol{A})$ $(\mathrm{CR}(\boldsymbol{A}) > 0.1)$ can keep reducing when a proper element in the decision vector of $\boldsymbol{A}$ is modified correctly. This important conclusion is the theoretic basis of the marginal optimization procedure discussed in the following section.

3.3 Marginal optimization procedure

For a PCM $\boldsymbol{A}$, the procedure of marginal optimization is described as below:

Step 1 Calculate $\mathrm{CR}(\boldsymbol{A})$. If $\mathrm{CR}(\boldsymbol{A}) < 0.1$, it means $\boldsymbol{A}$ is of acceptable consistency, thus end the procedure; otherwise, go to Step 2;

Step 2 Initialize parameters. Denote the superior decision vector of the original matrix $\boldsymbol{A}$ as $\boldsymbol{x} = (x_1, x_2, \ldots, x_k)$. Denote the CR value of $\boldsymbol{A}$ as CR_0^*, and the maximum revision as $\sigma_0^* = 0$. Let the increment (or decrement) be a fixed value d $(0 < d < 1)$, where d is usually determined by experience, and we often let $d = 0.1$, 0.01, or 0.001. Since d is the only parameter in our approach, the smaller d is, the better optimization result we can obtain, but more time is required to finish the increased iterations.

Step 3 Calculate the CR value of the modified matrices. For the Ith iteration, (if it is the first iteration, $I = 1$), let

$$\boldsymbol{x}_1 = (x_1 \pm d, x_2, \ldots, x_k)$$
$$\boldsymbol{x}_2 = (x_1, x_2 \pm d, \ldots, x_k)$$
$$\vdots$$
$$\boldsymbol{x}_k = (x_1, x_2, \ldots, x_k \pm d).$$

When $x_l \pm d$ $(l = 1, 2, \ldots, k)$ is beyond the interval [1,10], we set the value to be the nearer boundary of [1,10].

Denote $\mathrm{CR}_I(\boldsymbol{x}_l)$ $(l = 1, 2, \ldots, k)$ as the CR value of the matrix whose superior decision vector is $\boldsymbol{x}_l$. Note that $\boldsymbol{x}_l$ has two values, i.e., $(x_1, \ldots, x_l + d, \ldots, x_k)$ and $(x_1, \ldots, x_l - d, \ldots, x_k)$, we denote the one with lower CR value as $\mathrm{CR}_I(\boldsymbol{x}_l)$.

$$\mathrm{CR}_I(\boldsymbol{x}_l) = \min\{\mathrm{CR}_I([x_1, \ldots, x_l + d, \ldots, x_k]), \mathrm{CR}_I([x_1, \ldots, x_l - d, \ldots, x_k])\}$$

Calculate $\mathrm{CR}_I(\boldsymbol{x}_l)$, for $l = 1, 2, \ldots, k$, and we denote $\mathbf{CR}_I$ as

$$\mathbf{CR}_I = \{\mathrm{CR}_I(\boldsymbol{x}_1), \mathrm{CR}_I(\boldsymbol{x}_2), \ldots, \mathrm{CR}_I(\boldsymbol{x}_k)\}.$$

Step 4 Calculate marginal benefit/cost ratio.

$$F_I(\boldsymbol{x}_l) = \frac{\Delta \mathrm{CR}_I(\boldsymbol{x}_l)}{\sigma_I(\boldsymbol{x}_l)} = \frac{\mathrm{CR}_{I-1}^* - \mathrm{CR}_I(\boldsymbol{x}_l)}{\sigma_I(\boldsymbol{x}_l)} \tag{6}$$

where CR_{I-1}^* is the CR value of the revised PCM determined in the $(I-1)$th iteration; $\sigma_I(\boldsymbol{x}_l)$ is the maximum revision comparing $\boldsymbol{x}_l$ with the superior decision vector of the original PCM $\boldsymbol{A}$ by (3). Note that $\sigma_I(\boldsymbol{x}_l)$ is the deviation between the candidate superior decision vectors $\boldsymbol{x}_l$ and that of the original PCM, as a result, different $\boldsymbol{x}_l$ $(l = 1, 2, \ldots, k)$ may lead to different $\sigma_I(\boldsymbol{x}_l)$.

Denote $\boldsymbol{F}_I = \{F_I(\boldsymbol{x}_1), F_I(\boldsymbol{x}_2), \ldots, F_I(\boldsymbol{x}_k)\}$, and the maximum marginal benefit/cost ratio of the Ith iteration is defined as $F_I(\boldsymbol{x}_l^*) = \max\{\boldsymbol{F}_I\}$, where we assume the lth element, $F_I(\boldsymbol{x}_l)$, to be the maximum value in $\boldsymbol{F}_I$.

Step 5 Choose the final revised superior decision vector of the Ith iteration $\boldsymbol{x}_I^*$. If $\sigma_I(\boldsymbol{x}_l) > \sigma_{I-1}^*$, $\sigma_I(\boldsymbol{x}_p) = \sigma_{I-1}^*$, $\mathrm{CR}_I(\boldsymbol{x}_p) < \mathrm{CR}_{I-1}^*$, and $p \neq l$ $(p = 1, 2, \ldots, k)$ hold, $\boldsymbol{x}_I^* = \boldsymbol{x}_p$. Otherwise, $\boldsymbol{x}_I^* = \boldsymbol{x}_l$. If $\boldsymbol{x}_I^*$ is determined, the corresponding PCM is denoted as A_I^*, the CR value is denoted as CR_I^*, and the maximum revision is denoted as σ_I^*. Then, go to Step 6.

According to the traditional marginal optimization method, we should choose the situation $\boldsymbol{x}_l$ since $F_I(\boldsymbol{x}_l)$ has the maximum marginal benefit/cost ratio. However, it may lead to mistakes when we simply choose $\boldsymbol{x}_l$. Before choosing maximal marginal effect, we should judge whether $\boldsymbol{A}_I^*$ can still be modified to reduce the CR value by revising other elements while the σ_{I-1}^* value is kept unchanged, i.e., $\sigma_I^* = \sigma_{I-1}^*$, and $\mathrm{CR}_I^* < \mathrm{CR}_{I-1}^*$. We will explain this as follows.

For a traditional marginal optimization problem, adding the cost value by one unit will definitely increase the total cost. However, it is different here when we revise one element of the PCM by one unit, the total cost value may keep unchanged due to the definition of the total cost value, the maximum revision σ defined by (3).

For example, $\boldsymbol{A}$ is the original PCM of Example 1 in Section 4.

$$\boldsymbol{A} = \begin{bmatrix} 1 & 3 & 5 \\ 1/3 & 1 & 1/2 \\ 1/5 & 2 & 1 \end{bmatrix}, \quad \mathrm{CR}_0^* = 0.157$$

After the second iteration, we have

$$\boldsymbol{A}_2^* = \begin{bmatrix} 1 & 3.1 & 5 \\ 1/3.1 & 1 & 1/1.9 \\ 1/5 & 1.9 & 1 \end{bmatrix}, \quad \mathrm{CR}_2^* = 0.135\,6,$$

$$\sigma_2^* = 0.1.$$

According to Step 3, we have

$$\boldsymbol{x}_1 = (3.2, 5, 1.9), \quad \mathrm{CR}_3(\boldsymbol{x}_1) = 0.127\,9,$$
$$\sigma_3(\boldsymbol{x}_1) = 0.2$$
$$\boldsymbol{x}_2 = (3.1, 4.9, 1.9), \quad \mathrm{CR}_3(\boldsymbol{x}_2) = 0.130\,7,$$
$$\sigma_3(\boldsymbol{x}_2) = 0.1$$
$$\boldsymbol{x}_3 = (3.1, 5, 1.8), \quad \mathrm{CR}_3(\boldsymbol{x}_3) = 0.122\,7,$$
$$\sigma_3(\boldsymbol{x}_3) = 0.2$$
$$F_3(\boldsymbol{x}_1) = \frac{0.135\,6 - 0.127\,9}{0.2} = 0.038\,5,$$
$$F_3(\boldsymbol{x}_2) = \frac{0.135\,6 - 0.130\,7}{0.1} = 0.049,$$
$$F_3(\boldsymbol{x}_3) = \frac{0.135\,6 - 0.122\,7}{0.2} = 0.064\,5,$$
$$\boldsymbol{F}_3 = \{0.038\,5, 0.049, 0.064\,5\}.$$

Obviously, $\boldsymbol{x}_3$ has the maximal marginal effect since $F_3(\boldsymbol{x}_3) = \max\{\boldsymbol{F}_3\}$. However, we find that, $\boldsymbol{x}_2$ can reduce the CR value but keep the σ_{I-1}^* value unchanged, i.e., $\sigma_3(\boldsymbol{x}_2) = \sigma_2^* = 0.1$. The conditions $\sigma_3(\boldsymbol{x}_3) > \sigma_2^*$, and $\sigma_3(\boldsymbol{x}_2) = \sigma_2^*$, and $\mathrm{CR}_3(\boldsymbol{x}_2) = 0.130\,7 < \mathrm{CR}_2^*$ are satisfied, thus we should choose $\boldsymbol{x}_2$ instead of $\boldsymbol{x}_3$, since $\boldsymbol{x}_2$ has reduced the CR value while the maximum revision is unchanged. In other words, $\boldsymbol{x}_2$ has gained benefits (reduce CR value from 0.135 6 to 0.130 7) without changing the cost (the maximum revision $\sigma_3(\boldsymbol{x}_2) = 0.1$), while $\boldsymbol{x}_3$ has gained more benefits than $\boldsymbol{x}_2$ (reduce CR value from 0.135 6 to 0.122 7), but increased the cost (the maximum revision is increased from 0.1 to 0.2) at the same time. Obviously, it is better to choose $\boldsymbol{x}_2$ than $\boldsymbol{x}_3$ so that the CR value can be reduced while more original comparison information can be preserved.

Step 6 Judge if CR_I^* is acceptable. If $\mathrm{CR}_I^* < 0.1$, end the procedure and go to Step 7; otherwise, continue the iteration, let $\boldsymbol{x} = \boldsymbol{x}_I^*$, then, let $I = I + 1$, and go to Step 3 to continue revising $\boldsymbol{A}_I^*$.

Step 7 Judge whether $\boldsymbol{A}_I^*$ is acceptable. If $\sigma_I(\boldsymbol{x}_I^*) \leqslant 1$, $\boldsymbol{A}_I^*$ is said to be acceptable since the maximum revised elements are within the tolerance deviations interval according to Section 3.1, then we calculate $\lambda_{\max}$, CR, the priority vector $\boldsymbol{\omega}$, the maximum revision σ of $\boldsymbol{A}_I^*$, and end the procedure; otherwise, $\boldsymbol{A}_I^*$ is said to be unacceptable, and go to Step 8.

Step 8 The revised matrix $\boldsymbol{A}_I^*$ will be fed back to human reconsideration. If DMs accept the revised matrix, we will end the procedure and calculate $\lambda_{\max}$, CR, the priority vector $\boldsymbol{\omega}$, the maximum revision σ of the revised matrix. If DMs do not accept $\boldsymbol{A}_I^*$, we will ask the DMs to reconsider the elements in $\boldsymbol{A}_I^*$ that have the greatest modification comparing with the original matrix $\boldsymbol{A}$, and give new judgement to these elements so that a new original matrix $\boldsymbol{A}'$ is obtained. Let $\boldsymbol{A} = \boldsymbol{A}'$, and go back to Step 1 to repeat the optimize-feedback-optimize process till the acceptable revised matrix is found.

4. Illustrative examples

In order to test and compare the consistency improving method illustrated above with other methods, this paper applies the proposed method to some examples from published papers.

Example 1 The 3×3 PCM $\boldsymbol{A}$ and its $\lambda_{\max}$, CR, and $\boldsymbol{\omega}$ [13,18] are

	1	2	3	$\boldsymbol{\omega}$
1	1	3	5	0.657 1
2	1/3	1	1/2	0.146 6
3	1/5	2	1	0.196 3

$$\lambda_{\max} = 3.163\,2, \quad \mathrm{CR} = 0.157 > 0.1.$$

According to Definition 2, we can derive the decision vector $\boldsymbol{a} = \{3, 5, 1/2\}$, and the superior decision vector $\boldsymbol{b} = \{3, 5, 2\}$. Set $d = 0.1$, and use the marginal optimization procedure in Section 3.3. After seven iterations, we get the acceptable modified matrix and its $\lambda_{\max}$, CR, $\boldsymbol{\omega}$, and σ as follows.

	1	2	3	$\boldsymbol{\omega}$
1	1	3.2	4.8	0.660 8
2	0.312 5	1	0.588 2	0.151 1
3	0.208 3	1.7	1	0.188 1

$$\lambda_{\max} = 3.098\,2, \quad \mathrm{CR} = 0.094\,4 < 0.1, \quad \sigma = 0.3 < 1.$$

Besides, in order to demonstrate the marginal optimization procedure step by step, we have presented the modified matrices for each iteration in Table 3, in which the asterisk symbol "$*$" denotes the chosen element to be modified in the superior decision vector during the Ith iteration. Fig. 3 shows how CR and σ value change for each iteration. We can see that as the iteration number increases, CR value keeps decreasing, and the maximum revision σ keeps unchanged or increasing by d.

Table 3 Modified matrices for each iteration during marginal optimization procedure

Iteration I	a_{12}	a_{13}	$1/a_{23}$	CR_I^*	σ_I^*
0	3	5	2.0*	0.157	0
1	3*	5	1.9	0.143 7	0.1
2	3.1	5*	1.9	0.135 6	0.1
3	3.1	4.9	1.9*	0.130 7	0.1
4	3.1*	4.9	1.8	0.118	0.2
5	3.2	4.9*	1.8	0.110 9	0.2
6	3.2	4.8	1.8*	0.106 4	0.2
7	3.2	4.8	1.7	0.094 4	0.3

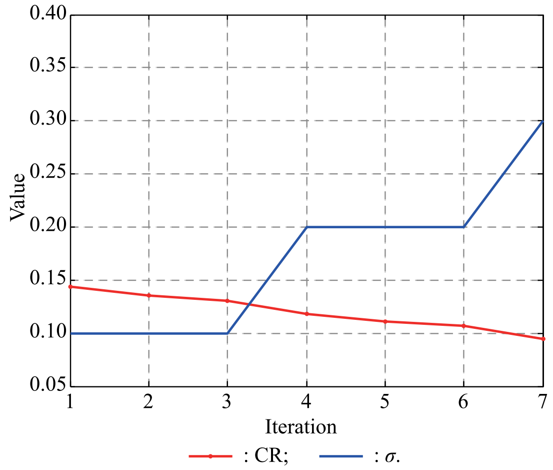

Fig. 3 CR and σ value for each iteration in Example 1

Example 2 The 4×4 PCM $\boldsymbol{A}$ and its $\lambda_{\max}$, CR, and $\boldsymbol{\omega}$ [13,19] are

	1	2	3	4	$\boldsymbol{\omega}$
1	1	2	5/2	5	0.444 3
2	1/2	1	9	2	0.350 2
3	2/5	1/9	1	1/2	0.081 2
4	1/5	1/2	2	1	0.124 3

$$\lambda_{\max} = 4.431\,7, \quad \mathrm{CR} = 0.161\,7 > 0.1.$$

For $d = 0.1, 0.01, 0.001$, the final results are presented in Table 4.

(i) $d = 0.1$. The revised matrix is obtained after 24 iterations in 0.625 s, and its $\lambda_{\max}$, CR, $\boldsymbol{\omega}$, and σ are

	1	2	3	4	$\boldsymbol{\omega}$
1	1	1.6	2.9	4.6	0.422 9
2	0.625	1	8.6	2.4	0.38
3	0.344 8	0.116 3	1	0.625	0.080 7
4	0.217 4	0.416 7	1.6	1	0.116 5

$$\lambda_{\max} = 4.257\,4, \quad \mathrm{CR} = 0.096\,39 < 0.1, \quad \sigma = 0.4 < 1.$$

(ii) $d = 0.01$. The revised matrix is obtained after 224 iterations in 4.6 s, and its $\lambda_{\max}$, CR, $\boldsymbol{\omega}$, and σ are

	1	2	3	4	$\boldsymbol{\omega}$
1	1	1.62	2.88	4.63	0.424 4
2	0.617 3	1	8.63	2.37	0.377 9
3	0.347 2	0.115 9	1	0.613 5	0.080 5
4	0.216	0.421 9	1.63	1	0.117 1

$$\lambda_{\max} = 4.266\,1, \quad \mathrm{CR} = 0.099\,66 < 0.1, \quad \sigma = 0.38 < 1.$$

(iii) $d = 0.001$. The revised matrix is obtained after 2 244 iterations in 46.8 s, and its $\lambda_{\max}$, CR, $\boldsymbol{\omega}$, and σ are

	1	2	3	4	$\boldsymbol{\omega}$
1	1	1.626	2.874	4.626	0.424 6
2	0.615	1	8.626	2.374	0.377 8
3	0.347 9	0.115 9	1	0.615	0.080 6
4	0.216 2	0.421 2	1.626	1	0.116 9

$$\lambda_{\max} = 4.267, \quad \mathrm{CR} = 0.099\,995 < 0.1, \quad \sigma = 0.374 < 1.$$

Fig. 4 shows how CR and σ values change for each iteration with respect to the three cases: $d = 0.1, 0.01, 0.001$. We can see that CR and σ values change similarly for the three cases, and CR value keeps decreasing step by step during each iteration until CR<0.1.

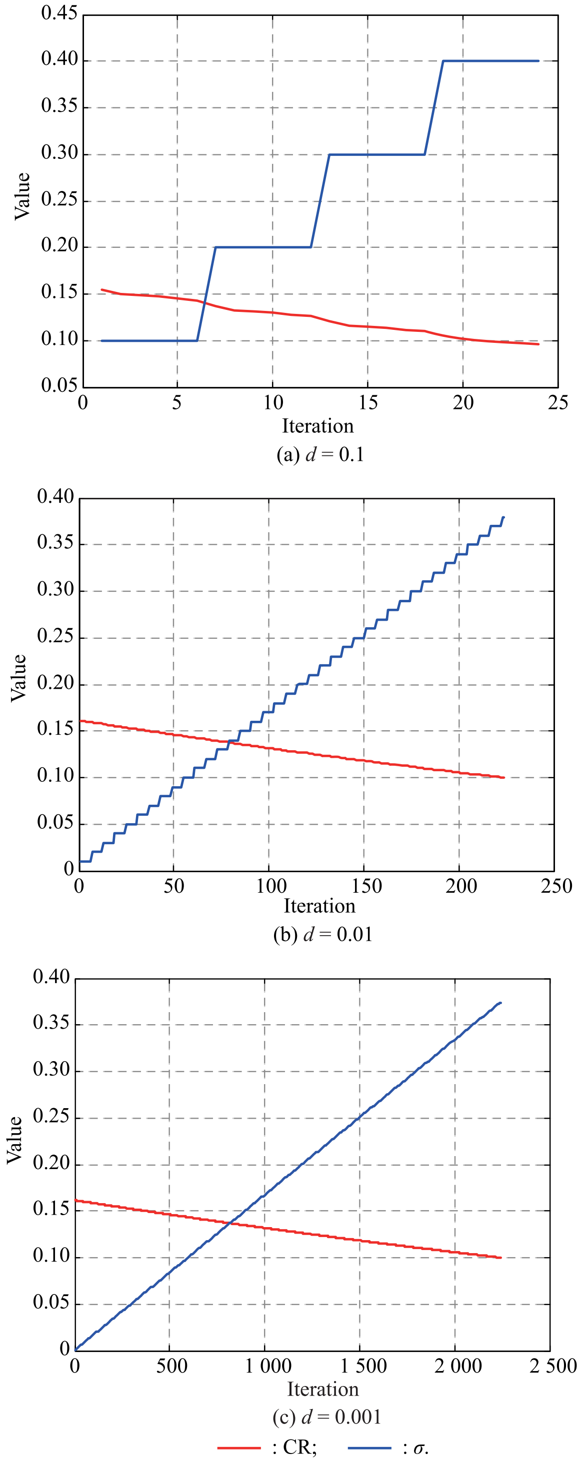

Fig. 4 CR and σ values for each iteration in Example 2

From the above two examples and Table 4, we can obtain the following results:

(i) The smaller d is, the smaller σ we obtain, which means more information of the original PCM can be preserved when d is smaller. However, as d gets smaller, more iterations are required, which means more time will be spent on obtaining the final revised matrix. We can also observe that for the three cases ($d = 0.1$, 0.01, 0.001), especially when d is smaller, e.g., when $d = 0.01$ or 0.001, the priority vector $\boldsymbol{\omega}$ is almost the same for both cases but the time spent on the latter case increases nearly 10 times as shown in Table 4. Therefore, in order to make the revised matrix acceptable, as well as time efficient, in general, it is suggested that d is set to be 0.01 in practical applications, and it is unnecessary to set d below 0.001 since the priority vector will not change much as $d = 0.001$.

Table 4 Comparing our approach with Xu and Wei [8], Cao et al. [6], Wu et al. [13]

Original matrix	Method	d	Total iterations (time)	Decision vector of the revised matrix	CR	σ
Example 1	Xu and Wei			{3.259 1,4.602 5,0.543 2}	0.098 4	0.397 5
	Our approach	0.1	7(0.2s)	{3.2,4.8,0.588 2}	0.094 4	0.3
Example 2	Xu and Wei			{1.822 9,2.942 8,4.661 8,7.733 6,2.142 3,0.527 2}	0.099 8	1.266 4
	Our approach	0.1	24(0.625s)	{1.6,2.9,4.6,8.6,2.4,0.625}	0.096 39	0.4
		0.01	224(4.6s)	{1.62,2.88,4.63,8.63,2.37,0.613 5}	0.099 66	0.38
		0.001	2 244(46.8s)	{1.626,2.874,4.626,8.626,2.374,0.615}	0.099 995	0.374
		0.5	4(0.2s)	{1.5,3,4.5,9,2.5,0.5}	0.093 218	0.5
Example 3	Xu and Wei			see Table 5	0.097 2	1.845
	Cao et al.			see Table 5	0.099 7	1.708 7
	Wu et al.			see Table 5	0.1	0.996 4
	Our approach	0.1	190(6.95s)	see Table 5	0.099 961	0.8
		0.01	1 952 (43.7s)	see Table 5	0.099 815	0.72
		0.001	19 453(419.8s)	see Table 5	0.099 985	0.718 6

Here, we have summarized some experiences on how to choose the parameter d. If the revision needs to be determined in a short time, we can set $d = 0.1$; if the revision asks for better optimization result, we can set $d = 0.001$. The selection of d also needs to consider the order of PCM and the CR value of the original PCM: when CR is much higher than 0.1 or the order of PCM is high (e.g., 8×8 PCM), more iterations are needed. We suggest to choose the parameter d by trial and error, that is, firstly set $d = 0.1$, and perform the proposed procedure to see if the acceptable result is obtained in time t; then estimate the time needed when d is smaller: about $10t$ for $d = 0.01$, $100t$ for $d = 0.001$ by experiences; finally, choose appropriate d according to the estimated time to perform the proposed procedure again so that the optimal result can be obtained in acceptable time.

(ii) Specifically, if this algorithm were to adapt Saaty's one to nine scales, we could set $d = 0.5$ or 1 so that the element of the revised matrix which is greater than 1 is integer or integer multiplied by 0.5. Here, for $d = 0.5$, as shown in Table 4, the revised matrix is obtained after four iterations in 0.2 s, and its $\lambda_{\max}$, CR, $\boldsymbol{\omega}$, and σ are

	1	2	3	4	$\boldsymbol{\omega}$
1	1	1.5	3	4.5	0.416 6
2	0.666 7	1	9	2.5	0.386 5
3	0.333 3	0.111 1	1	0.5	0.074 8
4	0.222 2	0.4	2	1	0.122 1

$$\lambda_{\max} = 4.248\,9, \quad \text{CR} = 0.093\,218 < 0.1, \quad \sigma = 0.5 < 1.$$

Besides, we have compared our approach with Xu and Wei [8], in which we use Algorithm I in [8] and set $\lambda = 0.99$. The comparison is presented in Table 4. As we can see, the criteria of modification effectiveness σ in our method is much smaller than that of Xu and Wei, even when $d = 0.5$, which means our method can preserve more original comparison information.

Example 3 The 8×8 PCM $\boldsymbol{A}$ and its $\lambda_{\max}$, and CR [6 – 8] are

	1	2	3	4	5	6	7	8
1	1	5	3	7	6	6	1/3	1/4
2	1/5	1	1/3	5	3	3	1/5	1/7
3	1/3	3	1	6	3	4	6	1/5
4	1/7	1/5	1/6	1	1/3	1/4	1/7	1/8
5	1/6	1/3	1/3	3	1	1/2	1/5	1/6
6	1/6	1/3	1/4	4	2	1	1/5	1/6
7	3	5	1/6	7	5	5	1	1/2
8	4	7	5	8	6	6	2	1

$$\lambda_{\max} = 9.668\,9, \quad \text{CR} = 0.169\,1 > 0.1.$$

For $d = 0.1$, 0.01, 0.001, the final results are presented in Table 4, and the decision vectors of the revised matrices are shown in Table 5.

(i) $d = 0.1$. The revised matrix is obtained after 190 iterations in 6.95 s, and $\lambda_{\max} = 8.986\,6$, CR=0.099 961<0.1, $\sigma = 0.8 < 1$.

(ii) $d = 0.01$. The revised matrix is obtained after 1 952 iterations in 43.7 s, and $\lambda_{\max} = 8.985\,3$, CR= 0.099 815<0.1, $\sigma = 0.72 < 1$.

(iii) d=0.001. The revised matrix is obtained after 19 453 iterations in 419.8 s, and $\lambda_{\max} = 8.987$, CR= 0.099 985<0.1, $\sigma = 0.718\,6 < 1$.

We have compared several existing methods with our approach in Table 4 and Table 5. From Table 4, we can see that our approach is the most efficient with the smallest σ, which means more information of the original matrix can be preserved than other existing methods.

Table 5 Final decision vectors comparing our approach with existing methods for Example 3

Element	Original value	Xu and Wei	Cao et al.	Wu et al.	Our approach		
					$d = 0.1$	$d = 0.01$	$d = 0.001$
a_{12}	5	4.524	4.441 2	4.081 6	4.3	4.28	4.282
a_{13}	3	2.339	2.368 2	2.003 7	2.3	2.28	2.282
a_{14}	7	7.523	7.674 3	7.971	7.7	7.72	7.718
a_{15}	6	5.888	5.855 9	5.054 2	5.3	5.28	5.282
a_{16}	6	5.686	5.607 9	5.333 2	5.3	5.28	5.282
a_{17}	0.333 3	0.425	0.420 1	0.499 1	0.454 5	0.438 6	0.438 2
a_{18}	0.25	0.292	0.296 8	0.321 7	0.303	0.304 9	0.304 7
a_{23}	0.333 3	0.326	0.321	0.403 8	0.333 3	0.336 7	0.336 4
a_{24}	5	4.516	4.422 4	4.044 2	4.3	4.28	4.282
a_{25}	3	2.671	2.617 5	2.069 2	2.3	2.28	2.282
a_{26}	3	2.58	2.539 2	2.339 9	2.3	2.28	2.282
a_{27}	0.2	0.222	0.226 8	0.249 6	0.232 6	0.233 6	0.233 5
a_{28}	0.142 9	0.147	0.148 6	0.149	0.158 7	0.159 2	0.159 2
a_{34}	6	6.749	6.914 9	6.969 8	6.7	6.72	6.718
a_{35}	3	3.46	3.535 1	3.899	3.7	3.72	3.718
a_{36}	4	4.188	4.277 4	4.784 9	4.3	4.32	4.328
a_{37}	6	4.155	4.510 5	5.045 3	5.3	5.28	5.282
a_{38}	0.2	0.249	0.248 7	0.179 2	0.232 6	0.233 6	0.233 5
a_{45}	0.333 3	0.373	0.380 5	0.333 3	0.434 8	0.438 6	0.438 2
a_{46}	0.25	0.287	0.292 7	0.332 8	0.303	0.304 9	0.304 7
a_{47}	0.142 9	0.134	0.131 3	0.148 5	0.147 1	0.148 4	0.148 3
a_{48}	0.125	0.104	0.103	0.119 8	0.114 9	0.114 7	0.114 7
a_{56}	0.5	0.561	0.572 2	0.837 9	0.769 2	0.781 2	0.78
a_{57}	0.2	0.197	0.196	0.239 3	0.232 6	0.233 6	0.233 5
a_{58}	0.166 7	0.147	0.144 4	0.154	0.149 3	0.148 8	0.148 9
a_{67}	0.2	0.204	0.205 5	0.234	0.232 6	0.233 6	0.233 5
a_{68}	0.166 7	0.153	0.149 4	0.146	0.149 3	0.148 8	0.148 9
a_{78}	0.5	0.501	0.500 4	0.684 4	0.384 6	0.380 2	0.380 2

The bias matrix [20] can be calculated by subtracting the modified PCM in Table 5 from the original PCM. In this paper, when the parameter $d = 0.001$, the bias matrix becomes

$$\begin{bmatrix} 0 & 0.718 & 0.718 & -0.718 & 0.718 & 0.718 & -0.104\,9 & -0.054\,7 \\ -0.033\,5 & 0 & -0.003\,1 & 0.718 & 0.718 & 0.718 & -0.033\,5 & -0.016\,3 \\ -0.104\,9 & 0.027\,3 & 0 & -0.718 & -0.718 & -0.328 & 0.718 & -0.033\,5 \\ 0.013\,3 & -0.033\,5 & 0.017\,8 & 0 & -0.104\,9 & -0.054\,7 & -0.005\,4 & 0.010\,3 \\ -0.022\,7 & -0.104\,9 & 0.064\,4 & 0.717\,9 & 0 & -0.28 & -0.033\,5 & 0.017\,8 \\ -0.022\,7 & -0.104\,9 & 0.018\,9 & 0.718\,1 & 0.717\,9 & 0 & -0.033\,5 & 0.017\,8 \\ 0.717\,9 & 0.717\,3 & -0.022\,7 & 0.256\,9 & 0.717\,3 & 0.717\,3 & 0 & 0.119\,8 \\ 0.718\,1 & 0.718\,6 & 0.717\,3 & -0.718\,4 & -0.715\,9 & -0.715\,9 & -0.630\,2 & 0 \end{bmatrix}$$

In the bias matrix, the largest perturbation a_{82} is smaller than 1, which means all elements are in the tolerance deviation interval in Table 2.

In Xu and Wei [8], when the parameter $\lambda = 0.98$, the bias matrix becomes

$$\begin{bmatrix} 0 & 0.476 & 0.661 & -0.523 & 0.112 & 0.314 & -0.091\,7 & -0.042 \\ -0.021 & 0 & 0.007\,3 & 0.484 & 0.329 & 0.42 & -0.022 & -0.004\,1 \\ -0.094\,2 & -0.067\,5 & 0 & -0.749 & -0.46 & -0.188 & 1.845^{*} & -0.049 \\ 0.009\,9 & -0.021\,4 & 0.018\,5 & 0 & -0.039\,7 & -0.037 & 0.008\,9 & 0.021 \\ -0.003\,2 & -0.041\,1 & 0.044\,3 & 0.319 & 0 & -0.061 & 0.003 & 0.019\,7 \\ -0.009\,2 & -0.054\,3 & 0.011\,2 & 0.515\,7 & 0.217\,5 & 0 & -0.004 & 0.013\,7 \\ 0.647\,1 & 0.495\,5 & -0.074 & -0.462\,7 & -0.076\,1 & 0.098 & 0 & -0.001 \\ 0.575\,3 & 0.197\,3 & 0.983\,9 & -1.6154^{*} & -0.802\,7 & -0.535\,9 & 0.004 & 0 \end{bmatrix}.$$

In the bias matrix, there are some relatively larger perturbations such as a_{37} and a_{84} (marked by asterisk symbol), which are greater than 1, thus these elements are out of the tolerance deviations interval described in Table 2.

In Cao et al. [6], when the parameter $\gamma = 0.98$, the bias matrix becomes

$$\begin{bmatrix} 0 & 0.558\,8 & 0.631\,8 & -0.674\,3 & 0.144\,1 & 0.392\,1 & -0.086\,8 & -0.046\,8 \\ -0.025\,2 & 0 & 0.012\,3 & 0.577\,6 & 0.382\,5 & 0.460\,8 & -0.026\,8 & -0.005\,7 \\ -0.088\,9 & -0.115\,3 & 0 & -0.914\,9 & -0.535\,1 & -0.277\,4 & 1.489\,5^{*} & -0.048\,7 \\ 0.012\,6 & -0.026\,1 & 0.022\,1 & 0 & -0.047\,2 & -0.042\,7 & 0.011\,6 & 0.022 \\ -0.004\,1 & -0.048\,7 & 0.050\,5 & 0.371\,9 & 0 & -0.072\,2 & 0.004 & 0.022\,3 \\ -0.011\,7 & -0.060\,5 & 0.016\,2 & 0.583\,5 & 0.252\,4 & 0 & -0.005\,5 & 0.017\,3 \\ 0.619\,6 & 0.590\,8 & -0.055 & -0.616\,1 & -0.102 & 0.133\,8 & 0 & -0.000\,4 \\ 0.630\,7 & 0.270\,5 & 0.979\,1 & -1.708\,7^{*} & -0.925\,2 & -0.693\,4 & 0.001\,6 & 0 \end{bmatrix}.$$

In the bias matrix, there are also some relatively larger perturbations such as a_{37} and a_{84} (marked by asterisk symbol), which are greater than 1, thus these elements are also out of the tolerance deviations interval described in Table 2.

In Wu et al. [13], the bias matrix becomes

$$\begin{bmatrix} 0 & 0.918\,4 & 0.996\,3 & -0.971 & 0.945\,8 & 0.666\,8 & -0.165\,8 & -0.071\,7 \\ -0.045\,0 & 0 & -0.070\,5 & 0.955\,8 & 0.930\,8 & 0.660\,1 & -0.049\,6 & -0.006\,1 \\ -0.165\,7 & 0.523\,5 & 0 & -0.969\,8 & -0.899 & -0.784\,9 & 0.954\,7 & 0.020\,8 \\ 0.017\,4 & -0.047\,3 & 0.023\,2 & 0 & 0 & -0.082\,8 & -0.005\,6 & 0.005\,2 \\ -0.031\,2 & -0.149\,9 & 0.076\,9 & -0.000\,3 & 0 & -0.337\,9 & -0.039\,3 & 0.012\,7 \\ -0.020\,8 & -0.094 & 0.041 & 0.995\,2 & 0.806\,5 & 0 & -0.034 & 0.020\,7 \\ 0.996\,4 & 0.993\,6 & -0.031\,5 & 0.266 & 0.821\,1 & 0.726\,5 & 0 & -0.184\,4 \\ 0.891\,5 & 0.288\,6 & -0.580\,4 & -0.347\,2 & -0.493\,5 & -0.849\,3 & 0.538\,9 & 0 \end{bmatrix}.$$

In the bias matrix, there are no such perturbations which are greater than 1, thus all elements are in the tolerance deviations interval described in Table 2. However, the maximum revision $\sigma(= 0.996\,4)$ is larger than our approach ($\sigma = 0.718\,6$), which means our approach is more efficient and can preserve more information of the original PCM.

To summarize, in the proposed method, all values are revised in the tolerance deviation interval described in Table 2, and the maximum revision is much lower than that of Xu and Wei, Cao et al., Wu et al., which means more original comparison information can be preserved than these three methods. Furthermore, the formula used by Xu and Wei [8], Cao et al. [6] are based on the priority vector ratios, which are calculated by the inconsistent original PCM. However, there are many different methods to derive priority vectors from the PCM [21], and different methods may yield different priority vectors, which may cause different revised PCM when different methods are used to calculate priority vectors. Compared with Xu and Wei, Cao et al., the proposed method is based on the original PCM instead of the priority vector, and if the only parameter d is certain, the same revised PCM will always be obtained. Also, the proposed method does not need to identify the inconsistent elements manually as Ergu et al., instead, it is very efficient to obtain satisfactory result by using Matlab programming, even if the PCM is a high-order matrix and the only parameter d is set very low, e.g., the 8×8 PCM in Example 3 can be solved within several minutes when $d = 0.001$.

5. Conclusions

In this paper, we propose a marginal optimization method to improve the inconsistent PCM in AHP for individual decision making problems, where the entries of the PCM are plain numbers. The method is feasible because the theorem offered in Section 3.2 guarantees that CR is monotonically decreasing in the iterative process. Also, the method has overcome the shortcomings in most existing methods which pay attention to reducing CR as fast as possible while the preservation of original comparison information is neglected. The effectiveness for both low order and high order PCM has also been demonstrated through several examples. By comparing with similar studies, the analytical results show that the proposed method is able to obtain a revised PCM with acceptable consistency, while preserve more original comparison information than existing methods (Xu and Wei [8]; Cao et al. [6]; Wu et al. [13], etc). Meanwhile, our approach is simple, intuitive and easy to be realized by computer programming, thus it may provide a new tool for the adjustment of inconsistent PCM in AHP.

A future research issue is on whether our methods could be mixed or integrated with other existing methods to further improve the efficiency especially for higher order comparison matrix such as PCM with an order larger than 10.

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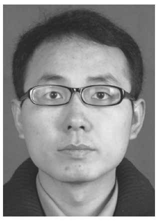

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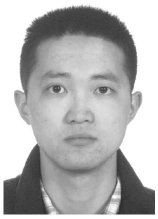

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