

Moment-independence global sensitivity analysis for the system with fuzzy failure state and its Kriging method

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Abstract: For the system with the fuzzy failure state, the effects of the input random variables and the fuzzy failure state on the fuzzy probability of failure for the structural system are studied, and the moment-independence global sensitivity analysis (GSA) model is proposed to quantitatively measure these effects. According to the fuzzy random theory, the fuzzy failure state is transformed into an equivalent new random variable for the system, and the complementary function of the membership function of the fuzzy failure state is defined as the cumulative distribution function (CDF) of the new random variable. After introducing the new random variable, the equivalent performance function of the original problem is built. The difference between the unconditional fuzzy probability of failure and conditional fuzzy probability of failure is defined as the moment-independent GSA index. In order to solve the proposed GSA index efficiently, the Kriging-based algorithm is developed to estimate the defined moment-independence GSA index. Two engineering examples are employed to verify the feasibility and rationality of the presented GSA model, and the advantages of the developed Kriging method are also illustrated.

Keywords: fuzzy uncertainty, fuzzy failure state, fuzzy probability of failure, moment-independence global sensitivity analysis (GSA), Kriging model.

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1. Introduction

Global sensitivity analysis (GSA) as an important branch of the reliability assessment has a large potential in reliability design and probability safety analysis, which can determine which of the input random variables effects output the most in the whole uncertainty range of the inputs [1–3]. According to the rankings of the input variables, the designer can give more attention or priority to the input variables with high sensitivity, and even neglect the input

variables with low sensitivity during the process of design and optimization [4,5]. Right now, more and more GSA methods are developed for system safety analysis, such as non-parametric techniques [5–7], but this sensitivity analysis lacks the independence of the model; variance-based sensitivity analysis indices [5,8,9] are used extensively, but this technique implicitly assumes that this moment is sufficient to describe output variability. However, any moment of a random variable provides a summary of its distribution with the inevitable “loss of resolution” that occurs when the information contained in the distribution is mapped into a single number [5,10]. In order to study the influence of the input variable on the entire distribution of the response output, moment-independent GSA [11–14] are presented. Among these methods, the one presented by Borgonovo [13] is used more widely.

Nevertheless, these methods deal with random variables under the binary state assumption, i.e. the boundary between the safety domain and the failure one is crisp and deterministic. However, in engineering practice such as wear of mechanism, fatigue and corrosion of structures, the transition from safety state to failure state is not sharp but a gradual failure process, which means that the failure state is fuzziness [3,15]. Researchers have made a large amount of achievements for the fuzzy reliability analysis [16–21]. In 1960s, Zadeh proposed the fuzzy sets theory [16], which is a powerful tool and is used extensively in fuzzy reliability analysis.

The fuzzy failure state is the common case in engineering. However, a few studies have focused on the GSA for this case at present. In this work, based on the complementary function of the membership function of the fuzzy failure state, a new random variable is introduced into the system input. And then the equivalent performance function is established. According to the established equivalent performance equation, the moment-independent GSA indices on the fuzzy probabi-

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lity of failure are presented to measure the influences of the uncertainties of the input variables and the fuzzy failure state on the fuzzy probability of failure.

The most crucial and difficult of obtaining the proposed GSA indices is how to obtain the unconditional and conditional fuzzy probability of failure efficiently [3]. It is well known that calculating the failure probability, especially the smaller one, is a computation consuming process. Additional double-loop nested sampling is further needed when calculating the conditional failure probability, and the computational cost associated with this process increases extremely with the dimensionality of input variables [22]. The Kriging method [23–26] is widely used in reliability analysis, which can adopt a small number of sample points to fit the original performance function well. It provides a new possible method to estimate the presented moment-independent GSA rapidly and properly. Based on the Kriging model, the highly efficient algorithm is developed to estimate the proposed moment-independent GSA model.

The organization of this work is as follows. Section 2 establishes the equivalent performance function for the system with the fuzzy failure state. Section 3 introduces the moment-independent GSA model on the fuzzy probability of failure. Section 4 gives a brief review of the Kriging meta-model and introduces the efficient Kriging-based algorithm to calculate the presented GSA model. The presented GSA model is used in two practical examples in Section 5. Section 6 presents the discussions and the conclusion of this paper.

2. Equivalent model for the structure with fuzzy failure state

Set $\mathbf{X} = (X_1, X_2, \dots, X_n)$ as the vector of the basic input random variables for a structure, and X_i ($i = 1, 2, \dots, n$) is the i th input random variable. Thus the performance function of this structure system can be written as

$$Z = g(\mathbf{X}) = g(X_1, X_2, \dots, X_n). \quad (1)$$

The failure probability of the structural system is defined by

$$P_f = \int_F f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \quad (2)$$

where $f_{\mathbf{X}}(\mathbf{x})$ is the joint probability density function (PDF) of the input variables \mathbf{X} , F denotes the failure domain determined by the performance function $Z = g(\mathbf{X})$, i.e. $F = \{\mathbf{X} : g(\mathbf{X}) \leq 0\}$.

When the failure state of the structural system is fuzziness, the fuzzy failure event of the structural system is expressed as

$$\tilde{E} = \{(z, u_{\tilde{E}}(z)) | z \in \Omega\} \quad (3)$$

where $z \in \Omega$ is the state function of the fuzzy state space Ω , \tilde{E} is the fuzzy set of Ω , $u_{\tilde{E}}(z)$ is the membership function of the fuzzy set \tilde{E} [5].

According to the definition of the fuzzy event: i) $u_{\tilde{E}}(z) = 1$, which means that the fuzzy event will happen, and the structural system is a failure completely; ii) $u_{\tilde{E}}(z) = 0$, which means that the fuzzy event will not happen, and the structural system is safe completely; iii) $0 < u_{\tilde{E}}(z) < 1$, which indicates that the structural system is in the fuzzy failure domain.

To illustrate the fuzzy failure state more clearly, the case of the two input variables X_1 and X_2 is taken as an example and the schematic diagram for the fuzzy failure state is shown in Fig. 1.

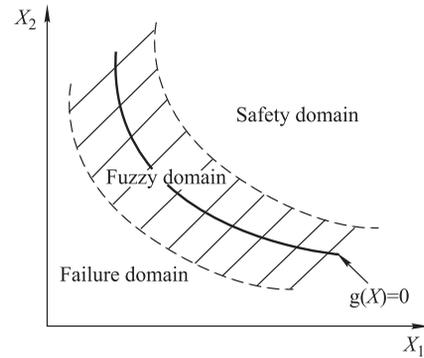


Fig. 1 Schematic diagram of the fuzzy failure state

The linear, Gaussian and quadratic membership functions of the fuzzy failure state are usually used in engineering. These three type membership functions are written as (4)–(6), respectively.

The definition of the linear membership function is

$$u_{\tilde{E}}(z) = \begin{cases} 1, & z \leq a_1 \\ \frac{a_2 - z}{a_2 - a_1}, & a_1 < z < a_2 \\ 0, & z \geq a_2 \end{cases} \quad (4)$$

where a_1 is the position parameter, and a_2 is the shape parameter. And we can obtain them by statistical analysis.

The definition of the Gaussian membership function is

$$u_{\tilde{E}}(z) = \begin{cases} 1, & z \leq b_1 \\ \exp \left[- \left(\frac{z - b_1}{b_2} \right)^2 \right], & z > b_1 \end{cases} \quad (5)$$

where b_1 is the position parameter, and b_2 is the shape parameter. And we can obtain them by statistical analysis.

The definition of the quadratic membership function is

$$u_{\tilde{E}}(z) = \begin{cases} 1, & z \leq c_1 \\ \left(\frac{c_2 - z}{c_2 - c_1} \right)^2, & c_1 < z < c_2 \\ 0, & z \geq c_2 \end{cases} \quad (6)$$

where c_1 is the position parameter, and c_2 is the shape parameter. And they can be obtained by statistical analysis.

Based on the probability formula of the fuzzy event [3,16], the fuzzy probability of the failure event for a structural system can be expressed as

$$P_f = \int_{-\infty}^{+\infty} u_{\tilde{E}}(z) f_Z(z) dz = \int_{-\infty}^{+\infty} u_{\tilde{E}}[g(\mathbf{x})] f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \quad (7)$$

where $u_{\tilde{E}}(z)$ is the membership function of fuzzy failure event \tilde{E} .

According to the definition of the membership function, it can be known that $u_{\tilde{E}}(z)$ is a monotony decrease function of z , and the range of $u_{\tilde{E}}(z)$ is $0 \leq u_{\tilde{E}}(z) \leq 1$. According to fuzzy random theory, the fuzzy failure state usually can be regarded as a variable. Thus the fuzzy event of the failure state is transformed into a new equivalent random variable X_{n+1} . Comparing the definitions of the membership function and the cumulative distribution function (CDF), the CDF of X_{n+1} can be expressed as

$$F_{X_{n+1}}(x_{n+1}) = 1 - u_{\tilde{E}}(x_{n+1}). \quad (8)$$

Furthermore, the PDF of the introduced random variable X_{n+1} can be expressed as

$$f_{X_{n+1}}(x_{n+1}) = -\partial u_{\tilde{E}}(x_{n+1}) / \partial x_{n+1}. \quad (9)$$

Then (7) can be rewritten as

$$P_f = \int_{-\infty}^{+\infty} [1 - F_{X_{n+1}}(x_{n+1})] f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f_{X_{n+1}}(x_{n+1}) f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} dx_{n+1}. \quad (10)$$

Thus the equivalent failure domain of the system with fuzzy failure state can be obtained as $g(\mathbf{X})\{\mathbf{X} | g(\mathbf{X}) \leq X_{n+1}\}$. The corresponding equivalent performance function is constructed as

$$Z_e = g(\mathbf{X}) - X_{n+1}. \quad (11)$$

3. Moment-independent GSA model for the structural system considering fuzzy failure state

The Borgonovo's moment-independent GSA [13] is defined by the average distance between the unconditional out density $f_Z(z)$ and the conditional out density $f_{Z|X_i}(z)$ when X_i is taken over its whole distribution region. Similar to this GSA, in this work, moment-independent GSA

on fuzzy probability of failure η_{X_i} is defined by the difference between the unconditional fuzzy probability of failure P_{f,Z_e} and conditional fuzzy probability of failure $P_{f,Z_e|X_i}$ which can be calculated by fixing the input random variable X_i at one given value x_i^* . The defined GSA index can measure the influence of the input variable X_i and fuzzy failure state on the fuzzy probability of failure. When random variable X_i is taken values according to the PDF $f_{X_i}(x_i)$, η_{X_i} can be expressed as

$$\eta_{X_i} = \frac{1}{2} E_{X_i} [|P_{f,Z_e} - P_{f,Z_e|X_i}|] = \frac{1}{2} \int_{-\infty}^{+\infty} \left| \int_F f_{Z_e}(z) dz - \int_F f_{Z_e|X_i}(z) dz \right| f_{X_i}(x_i) dx_i = \frac{1}{2} \int_{-\infty}^{+\infty} |P_{f,Z_e} - P_{f,Z_e|X_i}| f_{X_i}(x_i) dx_i \quad (12)$$

where $F = \{\mathbf{X} : g(\mathbf{X}) \leq X_{n+1}\}$.

The moment-independent GSA index of the fuzzy failure state on the fuzzy probability of failure $\eta_{X_{n+1}}$ is defined as

$$\eta_{X_{n+1}} = \frac{1}{2} E [|P_{f,Z_e} - P_{f,Z_e|X_{n+1}}|] = \frac{1}{2} \int_{-\infty}^{+\infty} |P_{f,Z_e} - P_{f,Z_e|X_{n+1}}| f_{X_{n+1}}(x_{n+1}) dx_{n+1}. \quad (13)$$

The moment-independent GSA of a group of the random variables $X_{i_1}, X_{i_2}, \dots, X_{i_r}$ is defined as

$$\eta_{i_1, i_2, \dots, i_r} = \frac{1}{2} E [|P_{f,Z_e} - P_{f,Z_e|X_{i_1}, X_{i_2}, \dots, X_{i_r}}|]. \quad (14)$$

The properties of the proposed moment-independent GSA on fuzzy probability of failure are derived as follows.

- (i) $\eta_{X_i} \geq 0; \eta_{X_{n+1}} \geq 0$.
- (ii) $\eta_{X_i} = 0; \eta_{X_{n+1}} = 0$ mean that the input random variable X_i or the fuzzy failure state has no influence on the fuzzy probability of failure.
- (iii) $\eta_{X_i} = \eta_{X_{ij}}$ means that adding a new random variable X_j on the influence of X_i has no additional influence on the fuzzy probability of failure.

4. Kriging method for the proposed GSA model

In engineering, the solution of the GSA model is usually a difficult problem due to the computational cost. The key of solving the proposed GSA indices is to estimate the unconditional fuzzy probability of failure and the conditional one of the fuzzy structural system. In order to overcome the shortcomings of large amounts of computation, the Kriging meta-model [23–26], which is a good alternative solution, is developed to calculate the presented GSA.

In order to better understand the Kriging-based method (KM) for the presented GSA indices, the principle of the Kriging meta-model method is briefly reviewed as follows.

The Kriging model treats the response model as a realization of a stochastic process function $g(\mathbf{x})$, which can be characterized by the linear regression part and the nonparametric part [26]. It can be expressed as

$$g(\mathbf{x}) = \mathbf{f}^T(\mathbf{x})\boldsymbol{\beta} + \mathbf{h}(\mathbf{x}) \quad (15)$$

where $\mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_m(\mathbf{x})]^T$ is the vector of the given basis function, $\boldsymbol{\beta} = [\beta_1, \beta_2, \dots, \beta_m]^T$ is the vector of the regression coefficients. $\mathbf{h}(\mathbf{x})$ is the realization of a stochastic process with zero mean, and covariance is defined as

$$\text{Cov}[\mathbf{h}(\mathbf{x}_i), \mathbf{h}(\mathbf{x}_j)] = \sigma_h^2 \mathbf{R}(\mathbf{x}_i, \mathbf{x}_j), \quad i, j = 1, 2, \dots, N \quad (16)$$

where N is the number of experimental points (sampled points), \mathbf{x}_i and \mathbf{x}_j are data points, σ_h^2 is the process variance of $\mathbf{h}(\mathbf{x})$, and $\mathbf{R}(\cdot, \cdot)$ is the correlation function. The squared-exponential function is used in this work, and it can be expressed as

$$\mathbf{R}(\mathbf{x}_i, \mathbf{x}_j) = \exp\left[-\sum_{t=1}^n \theta_t (\mathbf{x}_{i,t} - \mathbf{x}_{j,t})^2\right] \quad (17)$$

where n is the dimensionality of the input vector \mathbf{x} , $\mathbf{x}_{i,t}$ and $\mathbf{x}_{j,t}$ are the t th component of \mathbf{x}_i and \mathbf{x}_j , respectively, and θ_t denotes the multiplicative inverse of the correlation length in the t th direction and called the scale parameter.

The scalars $\boldsymbol{\beta}$ and σ_h^2 can be estimated as

$$\hat{\boldsymbol{\beta}} = (\mathbf{F}^T \mathbf{R}^{-1} \mathbf{F})^{-1} \mathbf{F}^T \mathbf{R}^{-1} \mathbf{g} \quad (18)$$

$$\hat{\sigma}_h^2 = \frac{1}{N} (\mathbf{g} - \mathbf{F} \hat{\boldsymbol{\beta}})^T \mathbf{R}^{-1} (\mathbf{g} - \mathbf{F} \hat{\boldsymbol{\beta}}) \quad (19)$$

where \mathbf{F} is a vector of $\mathbf{f}(\mathbf{x})$, \mathbf{g} is the vector of the output response at each experimental point, \mathbf{R} is the $N \times N$ matrix which represents the correlation between each pair of experimental points, i.e.,

$$\mathbf{R} = \begin{bmatrix} \mathbf{R}(\mathbf{x}_1, \mathbf{x}_1) & \cdots & \mathbf{R}(\mathbf{x}_1, \mathbf{x}_N) \\ \vdots & \ddots & \vdots \\ \mathbf{R}(\mathbf{x}_N, \mathbf{x}_1) & \cdots & \mathbf{R}(\mathbf{x}_N, \mathbf{x}_N) \end{bmatrix}. \quad (20)$$

The Kriging method provides not only a predicted value at a prediction point, but also an estimate of the prediction variance, which gives an uncertainty indication of the model [26].

The expected value $\mu_{\hat{g}}$ and variance $\sigma_{\hat{g}}^2$ of the Kriging meta-model at point \mathbf{x} are

$$\mu_{\hat{g}}(\mathbf{x}) = \mathbf{f}^T(\mathbf{x})\hat{\boldsymbol{\beta}} + \mathbf{r}^T(\mathbf{x})\mathbf{R}^{-1}(\mathbf{g} - \mathbf{F}\hat{\boldsymbol{\beta}}) \quad (21)$$

$$\sigma_{\hat{g}}^2(\mathbf{x}) = \sigma_h^2 - [\mathbf{f}^T(\mathbf{x}) \quad \mathbf{r}^T(\mathbf{x})] \begin{bmatrix} \mathbf{0} & \mathbf{F}^T \\ \mathbf{F} & \mathbf{R} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{f}(\mathbf{x}) \\ \mathbf{r}(\mathbf{x}) \end{bmatrix} \quad (22)$$

where $\mathbf{r}^T(\mathbf{x}) = [\mathbf{R}(\mathbf{x}, \mathbf{x}_1), \mathbf{R}(\mathbf{x}, \mathbf{x}_2), \dots, \mathbf{R}(\mathbf{x}, \mathbf{x}_N)]^T$ is the correlation vector between \mathbf{x} and N experimental points $(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N)$.

According to the concept of the Kriging meta-model method, we can construct the Kriging model for the equivalent performance function. Based on this meta-model, use the Monte Carlo simulation (MCS) to estimate the unconditional and conditional fuzzy probabilities of failure, in which the original performance function will not be called. Based on the definition of the presented GSA indices in (12) and (13), they can be obtained as

$$\eta_{X_i} = \frac{1}{2} \frac{1}{N} \sum_{j=1}^N |P_{f, Z_e} - P_{f, Z_e | X_i}(x_{i,j})| \quad (23)$$

where N is the number of sample points, $x_{i,j}$ is the j th sample points of the input random variable X_i .

The process of using the Kriging method to calculate the presented GSA indices can be divided into six steps, and the flowchart of the presented Kriging method is summarized in Fig. 2.

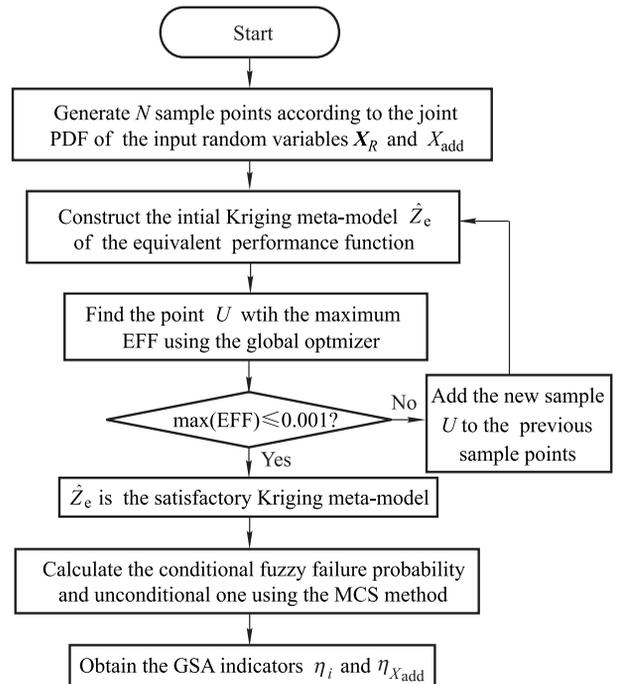


Fig. 2 Flowchart of the developed Kriging method

Step 1 Generate N_0 initial samples using the Latin hypercube sampling method [10] which has a low discrepancy property.

Step 2 Use these initial sample points to construct the initial Kriging meta-model. Then find the point with the maximum value of the expected feasibility function (EFF) [24], which is an indication of how well the true value of

the response is expected to satisfy the equality constraint [3].

Step 3 Judge whether the maximum value of EFF is smaller than the given threshold (0.001 in this work). If so, turn to Step 6 directly [3].

Step 4 Add this new sample (maximum value of EFF) to the training points and loop back to Step 2.

Step 5 Repeat Step 2 to Step 4 until the Kriging meta-model is satisfactory.

Step 6 Based on the constructed Kriging meta-model, use MCS to estimate unconditional and conditional fuzzy failure probabilities. And compute the GSA indices according to (23).

5. Engineering examples

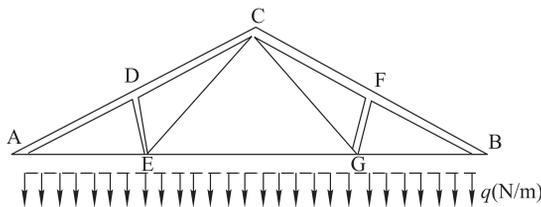
Two examples are used to verify the advantages of the presented GSA model and algorithm.

5.1 Example 1

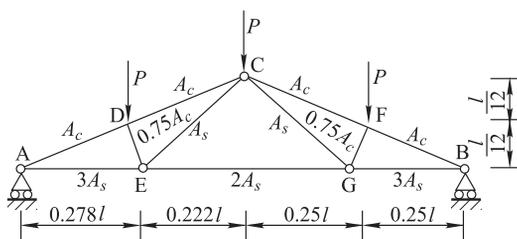
A roof truss [3,5,27] is shown in Fig. 3. The top boom and the compression bars are made by concrete, and the bottom boom and the tension bars are made by steel. We assume that the uniformly distributed load q applies on the roof truss, and the uniformly distributed load is transformed into the nodal load $P = ql/4$. According to the mechanical analysis, we can obtain the perpendicular deflection Δ_C of the node C , which is the function of input random variables,

$$\Delta_C = \frac{ql^2}{2} \left(\frac{3.81}{A_C E_C} + \frac{1.13}{A_S E_S} \right)$$

where A_C and A_S denote sectional areas, E_C and E_S denote elastic moduli and l denotes the length of the concrete and that of the steel bars.



(a) Structure diagram of the roof truss



(b) Simplified diagram of the roof truss

Fig. 3 Diagram of the roof truss

According to the safety and the applicability, we take Δ_C of the node C not exceeding 2 cm as the constraint condition. Thus the corresponding performance function can be expressed as

$$g(\mathbf{X}) = 0.02 - \Delta_C. \tag{24}$$

Table 1 shows the distribution parameters of the input random variables q, A_C, A_S, E_C, E_S and l .

Table 1 Distribution parameters of the input random variables

Random variable	Symbol	Mean	Standard deviation
q (N/m)	X_1	20 000	1 400
l /m	X_2	12	0.6
A_S /m ²	X_3	9.82×10^{-4}	9.82×10^{-5}
A_C /m ²	X_4	0.04	0.004
E_S /(N/m ²)	X_5	2×10^{11}	1.2×10^{10}
E_C /(N/m ²)	X_6	3×10^{10}	1.8×10^9

The failure event of displacement of node C is considered as the fuzzy event \tilde{E} . Assuming the membership function $u_{\tilde{E}}(z)$ of the fuzzy event \tilde{E} is a linear membership function, and the parameters a_1 and a_2 are taken -0.001 m and 0.001 m, respectively. According to the definition of the linear membership function, $u_{\tilde{E}}(z)$ can be written as

$$u_{\tilde{E}}(z) = \begin{cases} 1, & z \leq -0.001 \\ \frac{0.001 - z}{0.002}, & -0.001 < z < 0.001 \\ 0, & z \geq 0.001 \end{cases} \tag{25}$$

The sketch of the linear membership function is shown in Fig. 4.

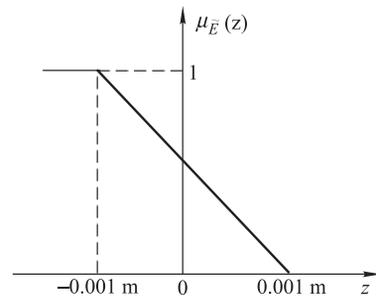


Fig. 4 Linear membership function

The fuzzy probability of failure for this example is $2.138 2 \times 10^{-3}$. The results of GSA indices calculated by the KM and MCS are shown in Table 2, and the histograms are given in Fig. 5 to illustrate the ranking conveniently.

Table 2 Results of the GSA indices for the roof truss

η_i	Method		η_i	Method	
	MCS	KM		MCS	KM
$\eta_q/10^{-3}$	1.167	1.074	$\eta_{E_S}/10^{-4}$	6.935	7.475
$\eta_l/10^{-3}$	1.659	1.413	$\eta_{E_C}/10^{-4}$	3.670	3.600
$\eta_{A_S}/10^{-3}$	1.256	1.212	$\eta_{X_{n+1}}/10^{-4}$	5.345	5.114
$\eta_{A_C}/10^{-4}$	5.546	6.015	-	-	-

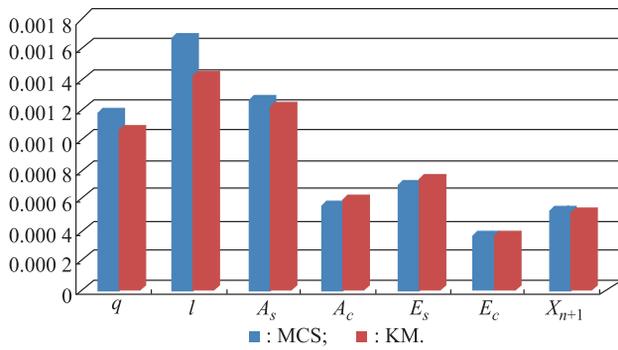


Fig. 5 Histogram of the moment-independent GSA indices for the roof truss

Table 2 and Fig. 5 show that the results obtained by the KM have a good consistency with the ones obtained by the MCS method. Using the MCS method is a double-loop sample process to calculate the GSA indices. For this example, the outer loop (the sample of X_i) requires 10^4 sample points, and the inner loop (computing the conditional failure probability $P_{f,Z_e|X_i}$) requires 10^5 sample points. Thus the total sample size for estimating the η_i by MCS is 10^9 . Nevertheless, the developed KM only requires fewer than 100 sample points to fit the equivalent performance function, then the GSA indices can be calculated using this mate-model, in which the original performance function does not need to be called. Obviously, the developed KM

is more efficient than the MCS.

The GSA indices ranking of input variables can be seen from Table 2 and Fig. 5, which is $l > A_s > q > E_s > A_c > E_c$. The GSA index of the fuzzy failure state is greater than E_c . According to the results, the designer can identify the influence of the fuzzy failure state and input variables on the fuzzy probability of failure. For example, the effect of the fuzzy failure state on the fuzzy failure state is great, so the fuzzy state of the roof truss should not be ignored. And the engineer should pay more attention to the fuzzy failure state and collect more information for the uncertainty of the failure state.

5.2 Example 2

A composite beam structure [28,29] is shown in Fig. 6. It contains 20 independent input variables. E_w denotes the Young’s moduli of the beam, and E_a denotes the Young’s moduli of the aluminous plate on its lower surface. L , A and B denote length, width and height of the beam, respectively, and C and D denote the width and height of the effective section, respectively. Six external loads P_i ($i = 1, \dots, 6$) are perpendicularly applied on the beam with the corresponding location L_i ($i = 1, \dots, 6$) shown in Fig. 6. Let S denote the permissible tensile stress of the beam. The distribution parameters of the input random variables are shown in Table 3.

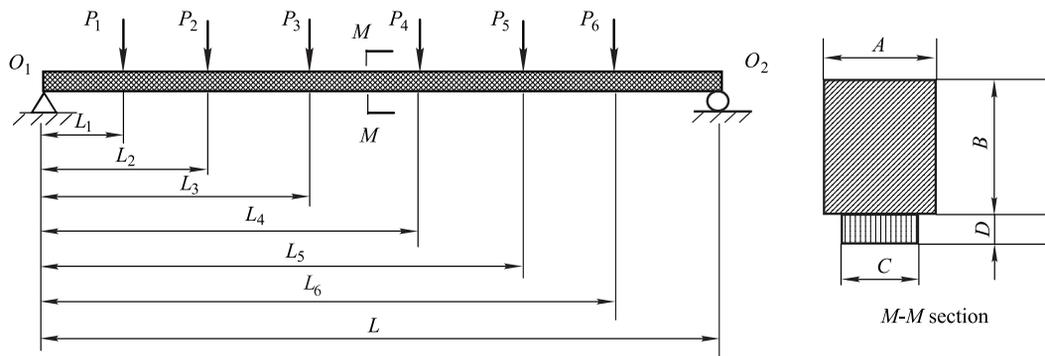


Fig. 6 Composite beam structure model

Table 3 Distribution parameters of the input variables of the composite beam [28,29]

Symbol	Input variable	Mean	Standard deviation	Distribution	Symbol	Input variable	Mean	Standard deviation	Distribution
X_1	A/mm	100	1	Normal	X_{11}	L/mm	1 400	2	Normal
X_2	B/mm	200	1	Normal	X_{12}	P_1/kN	15	1.5	Gumbel
X_3	C/mm	80	1	Normal	X_{13}	P_2/kN	15	1.5	Gumbel
X_4	D/mm	20	1	Normal	X_{14}	P_3/kN	15	1.5	Gumbel
X_5	L_1/mm	200	1	Normal	X_{15}	P_4/kN	15	1.5	Gumbel
X_6	L_2/mm	400	1	Normal	X_{16}	P_5/kN	15	1.5	Gumbel
X_7	L_3/mm	600	1	Normal	X_{17}	P_6/kN	15	1.5	Gumbel
X_8	L_4/mm	800	1	Normal	X_{18}	E_a/GPa	70	7	Normal
X_9	L_5/mm	1 000	1	Normal	X_{19}	E_w/GPa	8.75	0.875	Normal
X_{10}	L_6/mm	1 200	1	Normal	X_{20}	S/MPa	60	6	Gumbel

According to the mechanical analysis of composite material, we can obtain the maximum stress, which appears

in the middle cross section $M-M$, as

$$\sigma_{\max} = \frac{\left[L_3/L \sum_{i=1}^6 P_i(L-L_i) - P_1(L_2-L_1) - P_2(L_3-L_2) \right] \left[\frac{0.5AB^2 + DC(B+D)E_a/E_w}{AB + DCE_a/E_w} \right]}{\frac{1}{12}AB^3 + AB \left\{ \left[\frac{0.5AB^2 + DC(B+D)E_a/E_w}{AB + DCE_a/E_w} \right] - 0.5B \right\}^2 + \frac{1}{12} \frac{E_a}{E_w} CD^3 + \frac{E_a}{E_w} CD \left\{ 0.5D + B - \left[\frac{0.5AB^2 + DC(B+D)E_a/E_w}{AB + DCE_a/E_w} \right] \right\}^2}$$

Since the maximum stress can not exceed the permissible stress S , the performance function of the composite beam is constructed as follows:

$$g(\mathbf{X}) = S - \sigma_{\max}. \tag{26}$$

Set the failure event of stress as the fuzzy event \tilde{E} . Assuming the membership function $u_{\tilde{E}}(z)$ of the fuzzy event \tilde{E} is the Gaussian type, and the parameters b_1 and b_2 are taken as -5 MPa and 0.5 MPa, respectively. Thus the membership function $u_{\tilde{E}}(z)$ can be expressed as

$$u_{\tilde{E}}(z) = \begin{cases} 1, & z \leq -5 \\ \exp \left[- \left(\frac{z + 5}{0.5} \right)^2 \right], & z > -5 \end{cases} \tag{27}$$

The sketch of the Gaussian membership function for the fuzzy failure state is shown in Fig. 7.

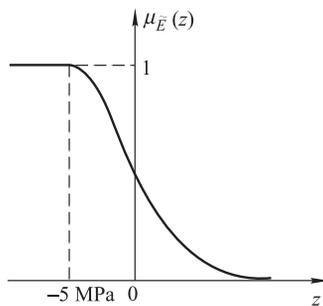


Fig. 7 Sketch of the Gaussian membership function

The fuzzy probability of failure for the composite beam is 3.2725×10^{-3} . The results of GSA indices on the fuzzy probability of failure calculated by the KM and MCS methods are shown in Table 4, and the histogram is shown in Fig. 8.

For this composite beam structure example, the MCS method requires 10^9 samples to calculate a η_i , while the proposed KM needs samples fewer than 200. The results in Table 4 and Fig. 8 test the applicability of the KM to

the proposed moment-independent GSA for the nonlinear response function once again.

Table 4 Results of the GSA indices for the composite beam

η_i	Method		η_i	Method	
	MCS	KM		MCS	KM
$\eta_A/10^{-4}$	3.860	3.653	$\eta_{P_1}/10^{-3}$	1.411	1.348
$\eta_B/10^{-4}$	4.143	4.041	$\eta_{P_2}/10^{-3}$	1.109	1.062
$\eta_C/10^{-4}$	1.120	1.001	$\eta_{P_3}/10^{-3}$	1.243	1.190
$\eta_D/10^{-4}$	5.221	5.553	$\eta_{P_4}/10^{-4}$	7.605	8.168
$\eta_{L_1}/10^{-5}$	2.196	1.898	$\eta_{P_5}/10^{-4}$	4.438	5.085
$\eta_{L_2}/10^{-5}$	1.786	1.860	$\eta_{P_6}/10^{-5}$	8.779	8.934
$\eta_{L_3}/10^{-5}$	3.255	3.177	$\eta_{E_a}/10^{-4}$	8.685	9.084
$\eta_{L_4}/10^{-5}$	1.380	1.486	$\eta_{E_w}/10^{-4}$	7.436	7.602
$\eta_{L_5}/10^{-5}$	2.845	2.716	$\eta_S/10^{-3}$	2.143	2.237
$\eta_{L_6}/10^{-5}$	1.950	2.123	$\eta_{X_{n+1}}/10^{-4}$	1.929	2.043
$\eta_L/10^{-4}$	1.478	1.432	-	-	-

In order to illustrate the results conveniently, all the input variables are classified into four sets according to their characteristics: the set of the cross section parameters (X_1, X_2, X_3, X_4), the set of the parameters describing the positions of the external loads and constraints ($X_5, X_6, X_7, X_8, X_9, X_{10}, X_{11}$), the set of the external loads ($X_{12}, X_{13}, X_{14}, X_{15}, X_{16}, X_{17}$) and the set of material parameters (X_{18}, X_{19}, X_{20}). It can be seen from Table 4 and Fig. 8 that the set of material parameters has the greatest influence on the fuzzy probability of failure, and the set of the parameters describing the positions of the external loads and constraints has the least influence on the fuzzy probability of failure. The influence of the introduced new random variable X_{n+1} is greater than the set of the parameters describing the positions. The strength of material S has the most prominent influence comparing with the other input variables and X_{n+1} . According to the results, the engineer can neglect the effect of L_i ($i = 1, \dots, 6$), and regard them as the determinate variables. That means that the dimension of the input uncertain variables can be reduced to 14-dimensions, which is extremely helpful to simplify the reliability design and safety analysis.

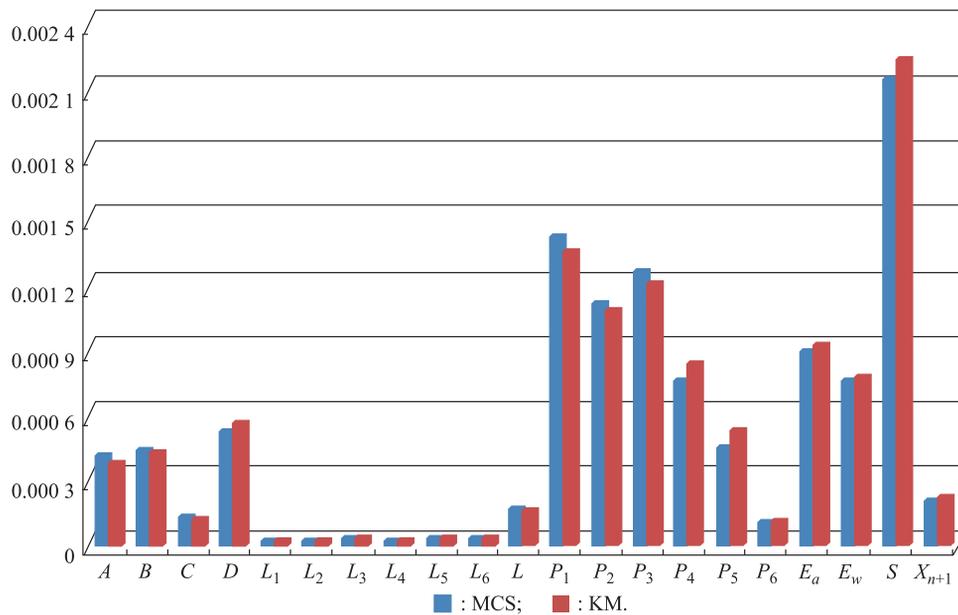


Fig. 8 Histogram of the moment-independent GSA results for the composite beam structure

6. Conclusions

For this work, we focus on establishing the moment-independent GSA model for the system considering the fuzzy failure state. According to the GSA indices, the engineer can not only identify the influence of input variables on the fuzzy probability failure, but also identify the influence of the fuzzy failure state on the fuzzy probability failure, which is the necessary information for the system reliability analysis. Firstly, the fuzzy failure state is transformed into an equivalent random variable as the new input random variable for the system. The cumulative distribution function of the new input variable can be obtained by the complementary function of the membership function of the fuzzy failure state. Thus the equivalent performance function can be constructed. Based on the definition of moment-independent GSA, the difference between the unconditional fuzzy probability of failure and the conditional fuzzy probability of failure is used to define the moment-independent GSA model.

Finally, the roof truss and the composite beam structure are applied to verify the feasibility and availability of the presented GSA model. Besides, the two examples also testify the accuracy and efficiency of the developed Kriging-based method.

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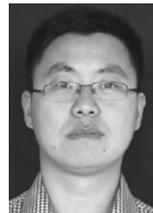
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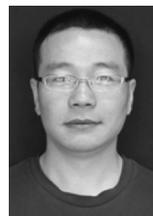
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