

Modeling and solution based on stochastic games for development of COA under uncertainty

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Abstract: Developing a course of action (COA) is a key step in military planning. In most extant studies on the COA development, only the unilateral actions of friendly forces are considered. Based on stochastic games, we propose models that could deal with the complexities and uncertainties of wars. By analyzing the equilibrium state of both opponent sides, outcomes preferable to one side could be achieved by adopting the methods obtained from the proposed models. This research could help decision makers take the right COA in a state of uncertainty.

Keywords: course of action (COA), uncertainty of antagonism outcomes, stochastic games.

DOI: 10.21629/JSEE.2019.02.08

1. Introduction

The antagonism in an operation is fierce; both sides attempt to transform the initial situation to their own advantages, which is unquestionably contradictory [1]. The evolution of war greatly depends on the joint actions of both sides, which leads to the mutual influence on these actions [2–4]. This confrontation should be considered in planning a course of action (COA) [5]. The COA is utilized to optimize and organize actions that formulate plans under conditions of selectable operational actions of both sides, interrelationships between actions and situations, and resource constraints [6]. An information war is characterized by various operational elements, broad operations space, uncertainties, complicated relationships among operation elements, and the mutual influence actions arising from both sides [7,8]. Given the difficulty that a commander experiences in developing an operational plan from a series of feasible plans, he/she has to be

provided with scientific and efficient decision-making supporting methods [9,10].

Currently, studies on issues pertaining to the COA focus on operation research and artificial intelligence [11–15]. By incorporating timing information, Wagenhals [16] transformed the model into a discrete event system model in the form of a Petri net, revealing the effect of the order and time of action events on the desired results and the effects of the situation. Tian [17] proposed a method based on a greedy algorithm to solve the optimal or near optimal operation process in the case of uncertainty [18], thereby allowing the user to observe the effect of an action on a desired effect. Recognizing that the core problem of the process of action is to choose the best combination of actions based on alternative action plans, Wan [19] established the COA modeling solution framework. The first module establishes the mobile network model, the second chooses the optimal plan from all the feasible combinations described in the first module, and the third tests the action process. Considering the conflict and resource constraints, Du [20] established a behavioral process model based on the sequence game theory which is more suited to confrontation and transformed it into a matrix game model. Sumile [21] outlined a target set of command and control (C2) modeling and simulation (M&S) capabilities critical to providing commanders with adequate situational awareness (SA) to enable globally integrated operations. Sumile also provided a conceptual framework to formulate a collaborative information environment (CIE) that can support COA modeling and validation while planning during execution. In order to optimize the attack process against targets with complex relationships and highly dynamic features, Lei [22] described military targets and their relationships as a system of systems (SoS) which was modeled by using the dynamic bayesians network, and proposed a genetic algorithm based method to model the target-attacking

Manuscript received September 14, 2017.

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This work was supported by the Natural Science Foundation of China (71471174).

COA under a resource constraint situation. Based on the discrete system modeling method, Haider et al. [23] proposed a formalism based on a timing influence network (TIN) to simulate the uncertainties in a dynamic context. Zaidi et al. [24] extended the TIN, extending the two aspects of time-varying and looping to expand the description of TIN. Haider et al. [25] used the TIN to model an uncertain situation, and an intelligent algorithm, e.g., particle swarm optimization and evolution, to identify the effective action process in that uncertain situation. Rafi et al. [26] used the activation TIN to describe the continuous evolution of an action and its environment, and in the case of a set of prerequisites, developed two algorithms to optimize these actions. Zaidi et al. [27] proposed a comprehensive theory of influence network, including consistency, time problems and design constraints that affect the dynamic evolution of constants. Based on classical artificial intelligence planning techniques and the principles of anytime and anywhere, Tang et al. [28] proposed a set of heuristic methods to select search nodes that take into account the preferences of emergency managers in the planning process. Considering the planning of military operations that historically focused on actions and their direct consequences, Pousi [29] studied the application of multi-criteria decision analysis (MCDA) to the planning of effects-based operations (EBOs). Yaman [30] developed a fuzzy cognitive map (FCM) to build EBOs, taking into account the constraints of problem characteristics and management. Considering the impact of conflict games, resource constraints, and execution time, Chen et al. [31] established a COA development model based on TINs and the game theory to solve a given problem by converting it into a standard matrix game model. In these studies, the influence of the actions of each side toward a war situation is considered and the transformation of the war situation is achieved by implementing those actions. However, the fact that targeting actions the enemy may choose could also affect the war situation remains unconsidered [32].

The counterparts of the red and the blue sides (hostile sides) are unlikely to obtain complete information and stochastic factors in one operation [33–35]. Hence, the transformation paths of a war situation are uncertain even if the current situation and actions from both sides are known, which we define as uncertainty of confrontation outcomes [36]. This could be largely explained by two aspects: on the one hand, accidental factors and stochastic events may cause this uncertainty; on the other hand, the information received from a commander is incomplete for making the right decision. To address the deficiencies in developing COAs, this study analyzes and works out solutions for uncertainties in antagonism outcomes by stochastic games models [37].

2. Problem analysis

War situations evolve under the joint action of both counterparts. Owing to the uncertainties resulting from stochastic factors [38], it is difficult to ensure war situations. Instead, estimating possible situations and their probabilities is possible. As shown in Fig. 1, in situation S_1 , the red and blue sides take actions ϕ and ψ respectively, and the war situation is transformed into S_2, S_3 in a corresponding probability of p_1, p_2 .

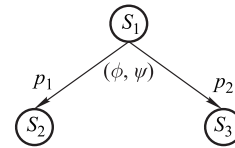


Fig. 1 Example of war situation transformation

The uncertainty of a war situation transformation makes it difficult for a commander to predict the transformation paths after he/she executes operational actions [38,39], and therefore the final outcome of the war is unknown and so is the decision-making effect of operational actions. According to the game theory, the uncertain situation of a confrontation outcome is actually the uncertainty of the system transformation for people inside the game. Thus, a stochastic-game model can be built to address the system transformation uncertainty that is described as a function of situation transformation.

3. Model formulation

3.1 Model description

3.1.1 War situation

A war situation includes information about the environment, disposition, and actions of both sides involved in an operational region. Actions from both sides have a direct influence on the situation, in which the foremost step in planning operational actions is to describe the situation, especially those significant elements which may influence the war situation. Drawing from cybernetics, a war situation is represented by the status variable S . Given that a war situation involves various elements, S is rather intricate. S could be a high-dimensional function $S = (s_1(t), s_2(t), \dots, s_m(t))$ related to time. Appropriate operational actions require reasonably simplified variables according to real-time conditions and refined key elements that may affect the operation actions, wherein the refined elements of the war situation should be discrete. Meanwhile, the war situation for each phase divided by an operational section can be represented by a multi-dimensional vector. Each element in the vector denotes a factor that may exert influence on a war situation. The transformation

of war situation is determined by the operational actions from both sides, which can be described as consumption of the operational resources and changes in military arrangement. The entire operational action is divided into K sections with the initial status S_1 , and possible action sets for the red and the blue sides being Φ_1, Ψ_1 . After implementing possible actions ϕ_i^1 and ψ_j^1 from the sets, the situation may be transformed into several situations $\{ijS_{mij}^2\}_{mij=1}^{M_{ij}=1}$ in the next section.

As shown in Fig. 2, in the k th ($1 \leq k \leq K$) section, after adopting actions (ϕ_i^k, ψ_j^k) from both the red and the blue sides, the situation for the $(k+1)$ th section could be transformed into several possible resultant situations. The transformation can be illustrated as a tree diagram $T = (S, E)$, where S is the nodes set representing different war situations, and E is the linking edges set denoting system transformation under the actions of both the red and the blue sides. Linking edge $\{(\phi_i, \psi_j), p_l\}$ represents a transformation into a target situation with a probability of p_l under the action (ϕ_i, ψ_j) .

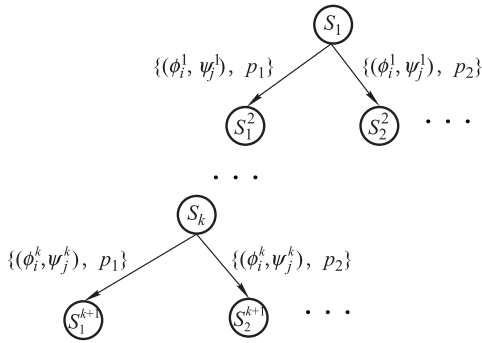


Fig. 2 War situation transformation in an operation

3.1.2 Action

Action is a decisive variable for a decision maker in a game, as well as a basic element for a game. In order to distinguish the term “action” from the theory of games, the basic operational actions for the red and the blue sides are defined as “tasks”. In each section of the operation, both the red and the blue sides have a set of tasks that can be adopted. In section k , the red side has h_k selectable tasks $\{rt_1^k, \dots, rt_{h_k}^k\}$, while the blue side has l_k selectable tasks $\{bt_1^k, \dots, bt_{l_k}^k\}$. In this study, the decisive variable for both the red and the blue sides in a section is the task vector, which meets the resource requirement and action constraints. Therefore, the action is defined as binary-realization of the task vector. In the situation S_k , the action set of the red side is denoted by $\Phi(S_k)$, and abbreviated as Φ_k , and $\Phi^{(k)} = \{\phi_1^{(k)}, \phi_2^{(k)}, \dots, \phi_{f_k}^{(k)}\}$. In addition, the action set of the blue side is denoted by $\Psi(S_k)$ and abbreviated as Ψ_k . Furthermore, the possible actions for both

sides need to be analyzed and decided by military experts in accordance with real-life situations.

3.1.3 War situation transformation probability

The war situation transformation is co-affected by both the red and the blue sides, with a certain uncertainty which is described by transformation probability. With the situation S_k and the actions ϕ_i^k, ψ_j^k defined above, the probability for system transformation situation S' is expressed as $P_{\psi_j^k, \phi_i^k}(S'|S_k)$, where $\sum_{S' \in S_{k, \phi_i^k, \psi_j^k}} P_{\psi_j^k, \phi_i^k}(S'|S_k) = 1$.

$S_{k, \phi_i^k, \psi_j^k}$ denotes the set of possible system situation transformations after the system S_k has experienced actions ϕ_i^k, ψ_j^k from both the red and the blue sides. The probability can be estimated by military experts, either from prior experience or from operational experiments. Similar to the transformation probability described above, we can further redefine the action sets of both the red and the blue sides. The action set for the red side is defined as $\Phi(S_k) = \{\phi_i^k \in \Phi_k | \exists S' \in S, P_{\psi_j^k, \phi_i^k}(S'|S_k) \neq 0\}$, while that of the blue side is defined as $\Psi(S_k) = \{\psi_j^k \in \Psi_k | \exists S' \in S, P_{\psi_j^k, \phi_i^k}(S'|S_k) \neq 0\}$.

3.1.4 Strategy

There exists an effect of mutual dependence between the actions of both sides and the war situation. The latter determines the actions of both sides, while the actions from both sides determine the transformation path of the war situation. Therefore, the strategy should reflect real conditions, namely a strategy offering action policies corresponding to all situations for the player. Strategy is defined as the reflection of the action set Φ, Ψ from the situation S , which is denoted by $\pi_r : S \rightarrow \Phi, \pi_b : S \rightarrow \Psi$.

$$\Phi = \{\Phi_1, \Phi_2, \dots, \Phi_M\}, \Psi = \{\Psi_1, \Psi_2, \dots, \Psi_M\}$$

represent the sets of possible actions for the red and the blue sides in situation m ($1 \leq m \leq M$).

Vectors $\pi_r^i = (\phi_{i_1}^1, \phi_{i_2}^2, \dots, \phi_{i_m}^m, \dots, \phi_{i_M}^M)$, $\pi_b^j = (\psi_{j_1}^1, \psi_{j_2}^2, \dots, \psi_{j_m}^m, \dots, \psi_{j_M}^M)$ stand for the i th and j th strategies of the red and the blue sides respectively, where the action $\phi_{i_m}^1$ of the red side and $\psi_{j_m}^m$ of the blue side correspond to the i th and j th strategies in the situation S_m .

3.1.5 Effect function

Effect function of the red and the blue sides is the function for the final war situation. Both sides expect the final situation to be in line with their initial expectations. Therefore, there may exist a preferred ranking for the possible confrontation outcomes. Besides, the counterparts in a war have antagonistic targets for operations, suggesting that the victory of one side represents failure for the other. There-

fore, we define the game model between the two sides as a zero-sum game with the red side as the decision-maker of Max type, and the blue side as the decision-maker of the Min type. Effect function stands for the various possible final situations, where the possible N_K situations at the final section constitute vector S_{N_K} and the effect function $f(S_{N_K}) \rightarrow \mathbb{R}$ should be determined by military experts.

3.2 Mathematical model

In summary, with the uncertainty of the confrontational outcome, the development of the COA can be modeled as a stochastic game, which can be described by a quintuple, $\Gamma = (N, S, \Omega, P, u)$.

- i. $N = \{R, B\}$ denotes the red (R) and blue (B) sides in the game;
- ii. $S = \{S_1, S_2, \dots, S_M\}$ denotes the set of the war situation;
- iii. $\Omega = \{\Phi, \Psi\}$ denotes the set for the actions of both sides;
- iv. $\Phi = \{\Phi_1, \Phi_2, \dots, \Phi_M\}$, and Φ_m denotes the set of feasible actions of the red side;
- v. $\Psi = \{\Psi_1, \Psi_2, \dots, \Psi_M\}$, and Ψ_m denotes the set of feasible actions of the blue side;
- vi. P denotes the probability of the war situation transformation;
- vii. $P_{\psi_j^k, \phi_i^k}(S_l | S_k)$ denotes the probability that the red and the blue sides are in the situation S_k and have executed actions ϕ_i^k, ψ_j^k , respectively;
- viii. U denotes the effect function;
- ix. $u = f(S_{N_K})$ denotes the effect function of the red side.

4. Definition and solution to a balanced strategy

4.1 Nature for equilibrium strategy

4.1.1 Definition of equilibrium

Owing to the mutual effect of actions from both the red and the blue sides, a rational decision maker would search for an equilibrium strategy for his/her own side and set it as his/her decision-making criterion [40]. The COA models with uncertainty of the confrontation outcome are π_r^i, π_b^j , and the mixed strategies are σ_r^i, σ_b^j .

Definition 1 The equilibrium strategy (σ_r^*, σ_b^*) has the following requirements.

- i. For the red side, the equilibrium strategy σ_r^* has the following requirements: $u(\sigma_r^*, \sigma_b^*) \geq u(\sigma_r^k, \sigma_b^*)$, $\sigma_r^k \in \Delta_r$, Δ_r is the mixed strategy for the red side.
- ii. For the blue side, the equilibrium strategy σ_b^* has the following requirements: $u(\sigma_r^*, \sigma_b^*) \leq u(\sigma_r^*, \sigma_b^n)$, $\sigma_b^n \in \Delta_b$, Δ_b is the mixed strategy for the blue side.

4.1.2 Existence for an equilibrium

Statement 1 Given the stochastic games $\Gamma = (N, S, \Omega, P, U)$, an equilibrium solution exists if the action set and the situation set of the war are finite sets [41].

The strategy set of the red side is π_r , as the situation set $S = \{S_1, S_2, \dots, S_M\}$ is finite, as well as an action set Φ_m ($1 \leq m \leq M$) in any war situation. Therefore, the pure strategy set $\pi_r^i = \{\pi_r^1, \pi_r^2, \dots, \pi_r^I\}$ for the red side is a finite set. Similarly, the pure strategy set $\pi_b^j = \{\pi_b^1, \pi_b^2, \dots, \pi_b^J\}$ is also finite. For the mixed strategy set σ_r^k of the red side, we define the probability of the pure strategy π_r^i ($1 \leq i \leq I$) as p_k^i ,

where $0 \leq p_k^i \leq 1$ and $\sum_{i=1}^I p_k^i = 1$, and the mixed strategy

σ_r^k is degraded into the pure strategy π_r^i if $p_k^i = 1$. When the mixed strategy for the blue side is σ_b^n , we decide the probability of the pure strategy π_b^j ($1 \leq j \leq J$) as q_n^j ,

$0 \leq q_n^j \leq 1$, and $\sum_{j=1}^J q_n^j = 1$, when $q_n^j = 1$ the mixed strategy

is degraded into the pure strategy π_b^j . When the red and the blue sides adopt the pure strategies π_r^i and π_b^j , respectively, the earned value of the red side can be calculated by effect function a_{ij} , and the earned value of the blue side is $-a_{ij}$ owing to the nature of the zero-sum game. When the red and the blue sides adopt the mixed strategies σ_r^k and σ_b^n , the expected return for the red side is $\sum_{j=1}^J \sum_{i=1}^I p_k^i q_n^j a_{ij}$,

and that of the blue side is $-\sum_{j=1}^J \sum_{i=1}^I p_k^i q_n^j a_{ij}$. We define

set-valued function $F : \Delta \rightarrow \Delta$, where Δ denotes the Descartes product by Δ_r and Δ_b . In addition, $\sigma = (\sigma_r, \sigma_b)$ denotes the strategy combination of the both side. For any mixed strategy σ_b^n , F will make the red side to select its own strategy σ_r^k to maximize the expected return

$$u_{kn} = \sum_{j=1}^J \sum_{i=1}^I p_k^i q_n^j a_{ij}, \text{ namely } \arg \max_{p_k^i} \sum_{i=1}^I \sum_{j=1}^J q_n^j a_{ij}.$$

Furthermore, for any mixed strategy σ_r^k , F will make the blue side to select its own strategy σ_b^n to maximize

$$\text{the expected return } u_{nk} = -\sum_{j=1}^J \sum_{i=1}^I p_k^i q_n^j a_{ij}, \text{ namely}$$

$$\arg \min_{q_n^j} \sum_{i=1}^I \sum_{j=1}^J p_k^i a_{ij}.$$

To prove the equilibrium in the model $\Gamma = (N, S, \Omega, P, U)$, we need to prove that there exists a fixed point in $F : \Delta \rightarrow \Delta$, namely, it has to meet the requirement of the Kakutani theorem. Besides, the earned value of the players is a linear function of its mixed strategy, and therefore

the mixed strategy is consecutive. To verify that the situation meets the requirements of the Kakutani theorem is to testify the following conditions:

(i) $F(\sigma)$ is convex.

In order to maximize the expected return $\sum_{j=1}^J \sum_{i=1}^I p_k^i q_n^j a_{ij}$ of the red side, we define $u_i = \sum_{j=1}^J q_n^j a_{ij}$

and $u = \sum_{i=1}^I p_k^i u_i$. In order to maximize u , p_k must belong to the following set:

$$P = \{p_k : p_k^i \geq 0, \sum p_k^i = 1, \text{ if } u_i < \max_t u_t, p_k^i = 0\}$$

namely $p_k \in P(q_n)$.

Similarly, for the blue side, in order to maximize its expected return, q_n must belong to the following set:

$$Q = \{q_n : q_n^j \geq 0, \sum q_n^j = 1, \text{ if } u_j < \max_t u_t, q_n^j = 0\},$$

namely $q_n \in P(p_k)$.

As F stands for the optimal response function for strategies of both the red and the blue sides, it has the following nature:

$$F(\sigma) = \begin{pmatrix} P(q) \\ Q(p) \end{pmatrix}.$$

Obviously, $F(\sigma)$ is convex.

(ii) $F(\sigma)$ has upper semi continuity, for

$$\widehat{\sigma}^n \in F(\sigma^n), (\sigma^n, \widehat{\sigma}^n) \rightarrow (\sigma, \widehat{\sigma}), \widehat{\sigma} \in F(\sigma).$$

$$\text{Set } \sigma^n = \begin{pmatrix} p^n \\ q^n \end{pmatrix}, \widehat{\sigma}^n = \begin{pmatrix} w^n \\ v^n \end{pmatrix}.$$

As $\widehat{\sigma}^n \in F(\sigma^n)$, namely, $\widehat{\sigma}^n$ is the optimal response for σ^n .

Therefore, for any strategy p' of the red side,

$$u(w^n, q^n) > u(p', q^n).$$

And for any strategy q' ,

$$u(p^n, v^n) > u(p^n, q').$$

When $\sigma^n \rightarrow \sigma, \widehat{\sigma}^n \rightarrow \widehat{\sigma}$, we can obtain $p^n \rightarrow p, q^n \rightarrow q, w^n \rightarrow w, v^n \rightarrow v$.

Then, for any strategy p' of the red side, $u(w, q) > u(p', q)$. Namely, w is the optimal response for q . For any strategy q' of the blue side, $u(p, v) > u(p, q')$. Namely, v is the optimal response for p .

$\widehat{\sigma} = \begin{pmatrix} w \\ v \end{pmatrix}$ is the optimal response for $\sigma = \begin{pmatrix} p \\ q \end{pmatrix}$, namely $\widehat{\sigma} \in F(\sigma)$.

Conditions (i) and (ii) hold true, and therefore, the stochastic game model $\Gamma = (N, S, \Omega, P, U)$ for the COA exists when the antagonism outcome is uncertain.

4.1.3 Refinement of balance

Statement 1 explains the existence of an equilibrium solution for the model. However, owing to the incredible balance caused by the transformation from stochastic games into strategic games, we need to analyze which balance is rational. In order to facilitate the analysis of the equilibrium of the game, we expand the general game tree diagram by adding chance nodes to describe the uncertainty of situation transformation, where “ \diamond ” denotes the chance nodes as shown in Fig. 3.

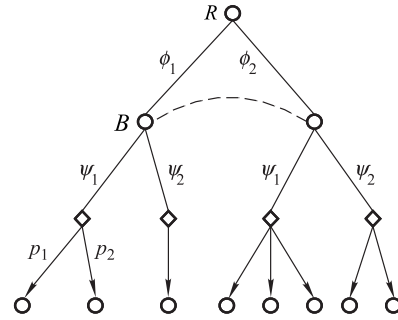


Fig. 3 Tree diagram of stochastic games

Nodes in the tree are divided into decisive nodes and chance nodes. The former stands for the position wherein a decision maker locates when he/she needs to make a decision, while the latter represents the uncertainty of war situation transformation or possible situation nodes. Linking edge starting with a decisive node stands for action of the players (such as ϕ_1, ϕ_2); linking edge starting with a chance node stands for the probability of situation transformation under actions from both sides (such as p_1, p_2). With the premise of uncertain confrontation outcomes, the stochastic game of the red and the blue sides takes on the nature of an expanded game. In an expanded game, the transformation from the expanded game to a strategic game is an “incredible threat,” and a refinement for the balance is subsequently required. Currently, the most acceptable concept of equilibrium is a subgame perfect equilibrium, which requires every proper subgame to represent the Nash equilibrium balance [42]. We define a proper subgame and a subgame perfect equilibrium according to the definitions of proper subgames and subgame perfect equilibrium from expanded games.

Definition 2 The proper subgame of a stochastic game is defined as a tree that contains structures starting with situation nodes, followed by the sequencing nodes and edges linking those nodes.

Definition 3 The perfect Nash equilibrium of the subgame requires that each anatomical side of the equilibrium strategy should constitute a Nash equilibrium in each proper subgame.

A subgame perfect equilibrium in stochastic games proposes a reasonable equilibrium after a stochastic game has been transformed into a matrix game. We need to analyze the condition of the perfect subgame based on its definition, which lays the foundation for its solution.

Statement 2 A subgame perfect equilibrium in a stochastic game $\Gamma = (N, S, \Omega, P, U)$ is a Nash equilibrium from the root nodes to the leaf nodes.

(\Rightarrow) suppose that there exists a subgame perfect equilibrium that does not contain a full path, then there must be a node that fails to lead the nodes following it to the path of the balanced strategy alongside the path directing it to its destination. This is contrary to the definition of the subgame perfect equilibrium, and thus the hypothesis should be rejected. Therefore, there does not exist a subgame perfect equilibrium that does not contain a full path.

Statement 2 proposes a solution to a subgame perfect equilibrium with the uncertainty of confrontation outcome, which is to work out the equilibrium by a transformation into a matrix game and analyze if the solution constitutes a path that links the root to the final sub-nodes of a game tree structure.

4.2 Solution to a balanced strategy

According to the definitions of strategy and effect function, we can obtain a strategy π_r^i of both sides and a strategy π_b^j of the blue side. The payment value of both sides is $u(\pi_r^i, \pi_b^j)$. The payment matrix is then calculated and so is the balanced strategy for both sides. The specific solutions are as follows:

(i) The strategy sets of the red and blue sides π_r, π_b , respectively, are determined according to the situation set S and actions set Ω .

In addition, the i th strategy of the red side is $\pi_r^i = (\phi_{i1}^1, \phi_{i2}^2, \dots, \phi_{im}^m, \dots, \phi_{iM}^M)$, which denotes the action ϕ_{im}^1 that the red side adopts in the system situation S_m . The j th strategy of the blue side is $\pi_b^j = (\psi_{j1}^1, \psi_{j2}^2, \dots, \psi_{jm}^m, \dots, \psi_{jM}^M)$, which denotes the action ψ_{jm}^m in the system situation S_m .

(ii) The payment matrix is determined according to the set of strategies and effect functions.

For the i th strategy of the red side and the j th strategy of the blue side, the probability of the final situation can be determined according to the probability of situation transformation, and subsequently, its payment value can be determined according to the effect function, and the entire payment matrix can be determined in a similar way.

(iii) The balanced strategy and the earned value of the red and the blue sides can be determined by the payment matrix.

After the payment has been determined, it is easy to work out balanced strategies for both sides and their earned values.

(iv) Analyze the balanced strategy to work out the subgame perfect equilibrium.

Analyze if the equilibrium strategy contains the full path starting from the root nodes to the leaf nodes. The strategy containing the full path is known as a subgame perfect equilibrium.

5. Computational experiments

5.1 Example design

Suppose that an operation is divided into two sections, and the initial section is S_0 . In S_0 , the red and the blue sides can adopt actions ϕ_1^0, ϕ_2^0 and ψ_1^0, ψ_2^0 respectively. The detailed numerical example is shown in Fig. 4. After the adoption of ϕ_1^0 and ψ_1^0 , the system situation will be transformed into S_2 . After the adoption of ϕ_2^0 and ψ_1^0 , the system situation will be transformed into S_3 ; after the adoption of ϕ_2^0 and ψ_2^0 , the system situation will be transformed into S_4 with a probability of 0.3, and into S_5 with a probability of 0.7. In S_1 , the red and the blue sides can adopt actions ϕ_1^1 and ψ_1^1 , and subsequently, the system situation will be transformed into S_6 and S_7 with a probability of 0.5 respectively. In S_2 , the red and the blue sides can adopt actions ϕ_2^2 and ψ_2^2 , and the system situation will be transformed into S_8 and S_9 with probabilities of 0.4 and 0.6, respectively. In S_3 , the red side can adopt actions ϕ_1^3 and ϕ_2^3 , and the blue side can adopt ψ_1^3 . Following the adoption of ϕ_1^3 and ψ_1^3 , the system will be transformed into S_{10} ; after the adoption of ϕ_2^3 and ψ_1^3 , the system situation will be transformed into S_{11} and S_{12} with probabilities of 0.8 and 0.2 respectively.

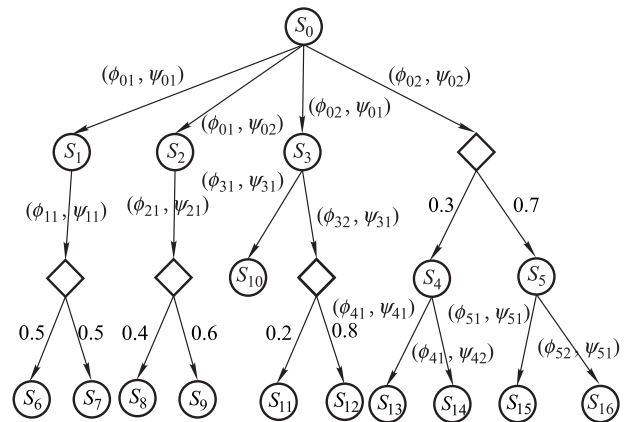


Fig. 4 Example of a stochastic game

In S_4 , the red side can adopt action ϕ_1^4 while the blue

side adopts ψ_1^4 and ψ_2^4 . Following the adoption of ϕ_1^4 and ψ_1^4 , the system will be transformed into S_{13} ; after the adoption of ϕ_1^4 and ψ_2^4 , the system will be transformed into S_{14} . In S_5 , the red side can adopt actions ϕ_1^5 and ϕ_2^5 , and the blue side ψ_1^5 . After the adoption of ϕ_1^5 and ψ_1^5 , the system will be transformed into S_{15} ; after the adoption of ϕ_2^5 and ψ_1^5 , the system situation will be transformed into S_{16} .

5.2 Example solution

The strategy sets for both sides are determined according to the action set of the situation set.

In S_0 , S_3 , and S_5 , two actions can be adopted by the red side, whereas only one action can be adopted in S_1 , S_2 and S_4 , resulting in eight strategies for the red side. In S_0 and S_4 , the blue side can adopt two actions, and the red side can only adopt one action in S_1 , S_2 , S_3 , and S_5 , resulting in four strategies for the blue side. Strategy is the reflection from an action space into a situation space. For instance, if the red side chooses actions $\phi_1^0, \phi_1^1, \dots, \phi_1^5$ in situation S_0, S_1, \dots, S_5 respectively, then the strategy is described as $(1, 1, 1, 1, 1, 1)$, which is the first action in every situation. The strategy of the blue side is described in the same way.

Hence, the strategy sets of the red side are as follows:

$$\{(1, 1, 1, 1, 1, 1), (1, 1, 1, 1, 1, 2), (1, 1, 1, 2, 1, 1), \\ (2, 1, 1, 1, 1, 1), (1, 1, 1, 2, 1, 2), (2, 1, 1, 1, 1, 2), \\ (2, 1, 1, 2, 1, 1), (2, 1, 1, 2, 1, 2)\}.$$

The strategy sets of the blue side are as follows:

$$\{(1, 1, 1, 1, 1, 1), (1, 1, 1, 1, 2, 1), \\ (2, 1, 1, 1, 1, 1), (2, 1, 1, 1, 2, 1)\}.$$

The payment matrix is determined according to the strategy set and the effect function. The details are shown in Table 1.

Table 1 Payment matrix of the red side

	(1,1,1,1,1,1)	(1,1,1,1,2,1)	(2,1,1,1,1,1)	(2,1,1,1,2,1)
(1,1,1,1,1,1)	5	5	4.8	4.8
(1,1,1,1,1,2)	5	5	4.8	4.8
(1,1,1,2,1,1)	5	5	4.8	4.8
(2,1,1,1,1,1)	7	7	1.6	2.1
(1,1,1,2,1,2)	5	5	4.8	4.8
(2,1,1,1,1,2)	7	7	7.9	8.5
(2,1,1,2,1,1)	10.4	10.4	1.6	2.1
(2,1,1,2,1,2)	10.4	10.4	7.9	8.5

Calculating the balanced strategy, we obtain the equilibrium strategy for the red side $(2, 1, 1, 2, 1, 2)$, and that of the blue $(2, 1, 1, 1, 1, 1)$. The earned value for the red side is 7.9, suggesting that in S_0 , the red side adopts ϕ_2^0 and the blue side adopts ψ_2^0 ; if the situation is transformed

into S_4 , then the red side adopts ϕ_1^4 and the blue side adopts ψ_1^4 ; if the situation is transformed into S_5 , then the red side adopts ϕ_2^5 and the blue side adopts ψ_1^5 .

$$\phi_2'(n, m) \phi_3'(n, k) \psi(n, l)$$

6. Conclusions

War is accompanied by fierce confrontation, and the evolution of the battlefield situation depends on the joint actions of both sides. Hence the strategies and correctness of their actions are closely interrelated. Under the condition of uncertainty of the confrontation outcome, we build a stochastic-game-based model to develop a COA and describe the operation action sequence from the aspects of the war situation, actions, situation transformation, and effect function. We propose the definition of equilibrium and analyze the existence of a balanced solution, as well as the conditions for a subgame perfect equilibrium. Moreover, we also propose a solution to it. Based on this work, we put forward an example and explain its solution for application.

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