Hybrid heuristic algorithm for multi-objective scheduling problem

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Abstract: This research provides academic and practical contributions. From a theoretical standpoint, a hybrid harmony search (HS) algorithm, namely the oppositional global-based HS (OGHS), is proposed for solving the multi-objective flexible job-shop scheduling problems (MOFJSPs) to minimize makespan, total machine workload and critical machine workload. An initialization program embedded in opposition-based learning (OBL) is developed for enabling the individuals to scatter in a well-distributed manner in the initial harmony memory (HM). In addition, the recursive halving technique based on opposite number is employed for shrinking the neighbourhood space in the searching phase of the OGHS. From a practice-related standpoint, a type of dual vector code technique is introduced for allowing the OGHS algorithm to adapt the discrete nature of the MOFJSP. Two practical techniques, namely Pareto optimality and technique for order preference by similarity to an ideal solution (TOPSIS), are implemented for solving the MOFJSP. Furthermore, the algorithm performance is tested by using different strategies, including OBL and recursive halving, and the OGHS is compared with existing algorithms in the latest studies. Experimental results on representative examples validate the performance of the proposed algorithm for solving the MOFJSP.

Keywords: flexible job-shop scheduling, harmony search (HS) algorithm, Pareto optimality, opposition-based learning.

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1. Introduction

The traditional job-shop scheduling problem (JSP) is a classic discrete combinatorial optimization problem and has been attracting growing interest. To cater to the complexity of the real world, manufacturing systems need to be increasingly flexible [1–5]. The flexible JSP (FJSP) comes naturally, where operations are allowed to be processed on any machine among an available machine set rather than on a specified one. Compared with the traditional JSP,

which is a well-known non-deterministic polynomial (NP)-hard [6], the FJSP resembles real-world production scheduling more because of the assignment of the appropriate machine for each operation; the FJSP is therefore essentially NP-hard too.

A polynomial algorithm was initially proposed by Brucker et al. for solving the FJSP with two jobs [7]. Subsequently, meta-heuristic methods grew in popularity and usage for solving the FJSP and were widely utilized in finding reasonably good schedules with acceptable computational expense instead of determining exact solutions. Solving the FJSP should generally fulfill the operation arrangement and the machine assignment. The methodologies of solving the FJSP can be classified into two main categories, namely hierarchical and integrated methods, according to the principle of simultaneously considering the operation arrangement and the machine assignment sub-problems. Specifically, the former category treats the two sub-problems separately, and the latter, simultaneously. Hierarchical methods were originally introduced by Brandimarte [8]; they decomposed the FJSP into two sub-problems to mitigate the solving complexity. The operation arrangement sub-problem is solved by using some dispatching rules, and the machine assignment sub-problem is solved by using the tabu search (TS) algorithm. Many researchers have concentrated on solving the FJSP by employing integrated methods. Seyed et al. implemented the biogeography-based optimisation algorithm [9]. Ishikawa et al. devised a multi-space competitive distributed genetic algorithm [10]. Harmony search (HS), particle swarm optimization (PSO) and greedy algorithm were considered by Yuan et al. [11], Singh et al. [12] and Mati et al. [13], respectively. Research results demonstrate that integrated methods perform better than hierarchical methods do.

Real-world scheduling problems are multi-objective by nature [14–17]. Each particular decision-maker aims to maximize his or her own interest. However, decision-makers in scheduling problems substantially differ. Given

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that some conflicted objectives frequently need to be optimized simultaneously in practical situations, the multi-objective FJSP (MOFJSP) has become the latest highlighted issue. The MOFJSP, which contains the optimization and decision-making of schemes, is a complicated FJSP. The complete procedure of solving the MOFJSP can be generally divided into two areas, namely solving multi-objective optimization scheduling schemes and determining the most appropriate scheduling scheme among the optimal solutions.

Multi-objective optimisation problems (MOPs) are generally solved with the weighted aggregation or Pareto optimality. In recent years, Pareto optimality has been applied in the field of MOPs and generated many classic algorithms. Zitzler et al. [18], Corne et al. [19], Srinivas et al. [20] and Deb et al. [21] proposed Pareto-based evolutionary algorithms, such as the strength Pareto evolutionary algorithm (SPEA), the Pareto envelope-based selection algorithm (PESA)-II, the non-dominated sorting genetic algorithm (NSGA) and NSGA-II, respectively. NSGA-II is currently considered by many researchers worldwide due to its superior performance.

Many meta-heuristic approaches have been adopted for solving MOPs in view of their excellent performance in solving single-objective optimization problems. Among these, evolutionary algorithms seem particularly suitable for solving MOPs because they deal simultaneously with a set of trade-off solutions in a certain population. Li et al. [22], Rahmati et al. [23], Wang et al. [24] and Rohaninejad et al. [25] designed hybrid algorithms based on the genetic algorithm to successfully solve the MOFJSP. Furthermore, some meta-heuristics based on local search methods, such as TS and simulated annealing (SA), have been applied recently for the FJSP. Jia et al. [26], Zhang et al. [27], Vilcot et al. [28] and Li et al. [29] used TS in solving the MOFJSP. Kaplanoglu [30] presented an object-oriented approach for the MOFJSP along with SA. Xia et al. [31] implemented SA to enhance the local search performances of PSO, and Low et al. [32] designed a hybrid SA-TS algorithm to solve the MOFJSP. Gao et al. [33] investigated a Pareto-based HS algorithm to solve the MOFJSP.

In practice, the MOFJSP is a sophisticated optimization problem which involves not only MOPs but also multi-attribute decision-making (MADM). Firstly, the performance of the algorithm, keeping a balance between the global exploration and local exploitation in the universe of discourse, is significant to meta-heuristics [34]. Secondly, the convergence of the algorithm and the diversity of non-dominated solutions become essential problems because the Pareto optimality approach generates a large number of non-dominated solutions in MOPs. Finally, the selec-

tion of the most appropriate scheduling scheme according to decision-makers' intention is indispensable.

In this research, an oppositional global-based HS (OGHS) algorithm integrating HS and opposition-based learning (OBL) is proposed for solving the MOFJSP for three common objective functions. The HS algorithm, which is simple, easily implemented, and an excellent meta-heuristic based on population, has been successfully applied for optimizing real-world problems [11]. In the OGHS, significant initialization program-embedded OBL is developed for obtaining a satisfactory diversity of individuals in a population. Recursive halving based on OBL is implemented for shrinking the neighbourhood space rapidly. The proposed algorithm not only guarantees the diversity of non-dominated solutions but also ensures rapid convergence. The technique for order preference by similarity to an ideal solution (TOPSIS) is adopted for seeking the most appropriate scheduling scheme for the decision-makers of the MOFJSP.

The remainder of this paper is organized as follows. In Section 2, the mathematical model of the MOFJSP is described, and some basic concepts are defined. Section 3 introduces the groundwork related to the proposed algorithm. In Section 4, the particular scheme of the OGHS algorithm for the MOFJSP is elaborated. The performance examination of the OGHS and the comparison of its results with those of other approaches on certain benchmarks are presented in Section 5. Section 6 explains the decision-making process of the most appropriate scheduling scheme. Conclusions are summarized, and some future investigation perspectives are described in Section 7.

2. Problem description

The FJSP of n jobs on m machines is to organize execution as follows. Each job, denoted by J_i ($1 \leq i \leq n$), has a pre-defined sequence n_i of non-preemptible ordered operations. The machine is noted M_k , $k \in \{1, 2, \dots, m\}$. The execution of each operation of job J_i (noted O_{ij}) is permitted to be processed by one machine in a set of available machines (called M_{ij}), and $M_{ij} \subseteq M_k$ obviously. The FJSP naturally aims to allocate the operations for each job to available equipments, to determine their starting and completion time and to further optimize the pre-determined objectives.

The FJSP is classified into the total FJSP and the partial FJSP [35]. In the total FJSP, each operation is manipulated by any machine, whereas in the partial FJSP, at least an operation cannot be processed on all machines. At any rate, processes and machine constraints must be satisfied in a real environment.

In this work, our model of the MOFJSP, which involves

three sub-objectives, is provided as follows:

$$\begin{cases} f_1 = \min C_{\max} = \min(\max)\{c_i | i = 1, 2, \dots, n\} \\ f_2 = \min W_T = \min \left(\sum_{i=1}^n \sum_{j=1}^{n_i} \sum_{s=1}^m p_{ijs} x_{ijs} \right) \\ f_3 = \min W_M = \min \left(\max \left\{ \sum_{i=1}^n \sum_{j=1}^{n_i} p_{ijs} x_{ijs} \right\} \right) \end{cases} \quad (1)$$

where $C_{\max}$ is the maximum makespan, W_T is the total workload of machines, W_M is the workload of the critical machine, C_i is the makespan of J_i , n_i denotes the total number of job J_i , m represents the total number of machines, p_{ijs} denotes the processing time of O_{ij} on machine s and x_{ijs} is a decision variable.

A partial FJSP example with three jobs and three machines is listed in Table 1. The symbol “—” indicates that the assignment is impossible in this example.

Table 1 Execution time of an example of 3×3 partial FJSP

Job	Operation	M_1	M_2	M_3
J_1	O_{11}	3	—	2
	O_{12}	5	2	3
	O_{13}	—	2	4
J_2	O_{21}	4	5	3
	O_{22}	7	—	9
J_3	O_{31}	4	2	1
	O_{32}	—	5	3
	O_{33}	4	—	5

3. Related work

In the proposed algorithm, HS is considered the main algorithm. Pareto optimality is used to achieve the convergence of the algorithm through guiding the evolution of the population towards the Pareto front. OBL improves the quality of distribution by enhancing the diversity of the non-dominated solutions. Furthermore, recursive halving based on OBL accelerates the convergence speed by shrinking the neighbourhood space.

3.1 Pareto optimality

The idea of Pareto optimality is originally proposed by Edgeworth, generalized later by Vilfredo Pareto and eventually became a widely accepted concept, namely Pareto optimality [36–38].

A minimized MOP can be generally described as follows:

$$\text{Minimize } y = f(\mathbf{X}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_L(\mathbf{x})) \quad (2)$$

where $\mathbf{x} \in \mathfrak{R}^p$ is a possible solution in the universe of discourse, $\mathfrak{R}$ is the solution space and p represents the quantity of multi-dimensional $\mathbf{x}$. Moreover, $y \in \mathfrak{R}^L$, where

L denotes the quantity of sub-objectives. $f_q(\mathbf{x})$ is the q th sub-objective function, for $1 \leq q \leq L$. In practice, the feasible solutions are increased exponentially, whereas multiple sub-objectives are optimized simultaneously. Generally, a set of trade-off solutions can be obtained according to the conflicted sub-objectives rather than an exact solution. The trade-off solutions, also called Pareto optimal solutions [39], are at least better in one sub-objective than the others in the universe of discourse. Therefore, the Pareto optimal solutions in MOPs are those that cannot be improved along any dimension without simultaneously being deteriorated along other dimensions. The trade-off solutions are also defined as non-dominated solutions. Additionally, some Pareto optimality concepts used in this paper can be formulated as follows.

Definition 1 Pareto dominance. In a minimized MOP, a feasible solution x_A is defined to dominate another feasible solution x_B , denoted by

$$x_A \prec x_B \quad (3)$$

satisfying $\forall i = 1, 2, \dots, m, f_i(x_A) \leq f_i(x_B) \wedge \exists j = 1, 2, \dots, m, f_j(x_A) < f_j(x_B)$.

Definition 2 Non-dominated solution. A feasible solution x^* is deemed as a non-dominated solution if and only if no feasible solution x' which dominates x^* in a minimized MOP, i.e.,

$$\neg \exists x', x' \prec x^*. \quad (4)$$

Definition 3 Non-dominated solution set (NDSS). The NDSS P^* in a minimized MOP is defined as

$$P^* \stackrel{\text{def}}{=} \{\mathbf{x} \in \mathfrak{R}^p | \neg \exists x' \in \mathfrak{R}^p, f(x') \leq f(\mathbf{x})\}. \quad (5)$$

3.2 HS algorithm

3.2.1 Standard HS algorithm

The HS algorithm was first inspired by Geem et al. [40]. Considered as one of the nearest meta-heuristics based on population evolution methods due to its simplicity and easiness of implementation, the HS algorithm has drawn growing interest and extensive application successfully. Its effectiveness and advantage have been demonstrated in various practical applications, such as function optimization, FJSP [11] and engineering optimization [41]. To achieve a perfect state of music, musicians improvise their instruments' pitches. On the basis of the improvisation process done by skilled musicians, Geem [40] formalized three corresponding components in HS, specifically the generation of harmony memory (HM), the pitch adjustment and the replacement of HM. The first component guarantees that the best harmony will be preserved in a new HM. A new solution is produced via varying the

pitches slightly by a small random amount in the second component, where the randomization selection is allowed to increase the diversity of the solutions for the global optimality. The worst existing harmony is substituted by a new candidate harmony in the last component.

3.2.2 Variant of HS

To alleviate the problem of parameter adjustment which was difficult to specify a priori in HS, Mahdavi et al. [42] investigated a variant of HS, which updated dynamically the pitch adjusting rate (PAR) and the distance bandwidth. Mahamed et al. [43] designed a global-based HS (GHS) algorithm wherein the pitch adjustment was modified and the best harmony was straightforwardly reserved in the HM. Importantly, the GHS works efficiently not only in the continuous universe but also in the discrete space.

3.3 OBL

3.3.1 Definition of OBL

Tizhoosh [44] proposed OBL, which has been used to improve evolutionary algorithms such as PSO [45,46], differential evolution [47] and biogeography-based optimization [48,49]. The basic idea of OBL is to use opposite numbers instead of random ones to quickly evolve the population in the evolutionary process. For convenience of description, the multi-dimensional opposite number is defined as follows.

Definition 4 In an n -dimensional space, $P(x_1, x_2, \dots, x_n)$ is a certain point, $x_1, \dots, x_n \in \mathbb{R}^n$ and $x_i \in [a_i, b_i]$. Its opposite point $\hat{P}$ is defined by coordinates $\hat{x}_1, \dots, \hat{x}_n$ as

$$\hat{x}_i = a_i + b_i - x_i, \quad i = 1, 2, \dots, n.$$

A certain point $x \in [a, b]$ and $c = (a + b)/2$ in a one-dimensional space, and its opposite point, $\hat{x}$, is illustrated in Fig. 1.

Fig. 1 Opposite point

3.3.2 Recursive halving based on OBL

In the search space, a theoretical optimal solution must be considered, such as x_{opt} , which is obtained by finite iterations. However, searching for x_{opt} is a time-consuming process because of exponentially large non-dominated solutions in an MOP. In the MOFJSP, the trade-off solutions exist in NDSS as alternative scheduling schemes. Any non-dominated solution comes into being if and only if at least one of its objective is demoted while some others improved. Therefore, the non-dominated solutions are worthy

of solution rather than x_{opt} in view of the computational cost in engineering practice. Point x is an initial number generated by random selection, whereas its opposite number $\hat{x}$ is produced following Definition 4. x or $\hat{x}$ does not dominate each other in the iteration process. Meanwhile, the search range can be recursively halved by the non-dominated solutions [44], as illustrated in Fig. 2. See the specific steps of Algorithm 2. The optimal solution x_{opt} , which is theoretically approached by x or $\hat{x}$ in the searching process, only illustrates the existence of the optimal solution in Fig. 2.

Fig. 2 Schematic diagram of recursive halving based on OBL

4. OGHS approach

This section presents an exhaustive explanation of the OGHS algorithm. The MOFJSP is a complex combinatorial optimization problem. A discrete HS, namely GHS [43], is utilized in the OGHS. OBL is exploited to improve the quality of the initial population distribution and to reduce the search range by a recursive halving of the search space. The specific OGHS algorithm is elaborated as follows.

4.1 OGHS algorithm framework

The OGHS algorithm framework is illustrated in Fig. 3, which is integrated with both GHS and OBL.

4.2 Harmony vector representation for MOFJSP

A harmony vector

$$\mathbf{X} = \{x(1), x(2), \dots, x(s), x(s+1), \dots, x(2s)\} \quad (6)$$

where $-\zeta \leq x(j) \leq \zeta$, $j = 1, 2, \dots, 2s$. To solve the MOFJSP, $\mathbf{X}$ is firstly translated into a type of dual vector code [11] and subsequently decoded to a feasible scheme.

Fig. 3 Flow chart of OGHS

4.2.1 Encoding procedure

The harmony vector representation of the MOFJSP consists of a dual vector code [11], including the operation sequence and machine allocation corresponding to the operation arrangement sub-problem and the machine assignment sub-problem. The vector $\mathbf{X}$ is $2s$ -dimensional, where s is the total number of all operations in the MOFJSP. The first half part of $\mathbf{X}$

$$\mathbf{X}^{(1)} = \{x(1), x(2), \dots, x(s)\} \quad (7)$$

describes the operation sequence information, whereas the second part of $\mathbf{X}$

$$\mathbf{X}^{(2)} = \{x(s+1), x(s+2), \dots, x(2s)\} \quad (8)$$

represents the machine allocation information. For making encoding much more accessible to interpret, each operation is marked with a definite ID according to the job position, and evidently, the total number of the definite ID is

equal to the quantity of all the operations. Therefore, each operation can be demonstrated by one definite ID. For instance, the operation position 7, which corresponds to the definite ID=7, signifies the second operation of the third job, i.e., O_{32} , as shown in Fig. 4. The operation sequence vector

$$\mathbf{V}_o = \{v(1), v(2), \dots, v(s)\} \quad (9)$$

can be translated into an operation sequence [50], i.e., [1, 1, 1, 2, 2, 3, 3, 3], where all operations are marked with the same number satisfying the precedence constraints. Consequently, operation sequences ensure the feasibility of the schedule scheme without doubt.

Operation position	1	2	3	4	5	6	7	8
Definite ID	1	2	3	4	5	6	7	8
Job demonstration	J_1	J_1	J_1	J_2	J_2	J_3	J_3	J_3
Operation sequence	1	1	1	2	2	3	3	3
Operation demonstration	O_{11}	O_{12}	O_{13}	O_{21}	O_{22}	O_{31}	O_{32}	O_{33}
Machine allocation	2	3	1	3	1	2	2	1
Alternative machine set	$\{M_1, M_3\}$		$\{M_1, M_2\}$		$\{M_2, M_3\}$			

Fig. 4 Illustration of dual vector code

For the machine allocation vector,

$$\mathbf{V}_M = \{u(1), u(2), \dots, u(s)\} \quad (10)$$

is an arrangement of s integer values, i.e., a specific permutation [2, 3, 1, 3, 1, 2, 2, 1], as shown in Fig. 4. The machine allocation vector, $u(j)$ ($1 \leq j \leq s$), indicates that the $u(j)$ th machine is chosen by the operation j . For instance, $u(2) = 3$ reveals that machine M_3 is chosen for the operation O_{12} , as shown in Fig. 4.

4.2.2 Mapping the harmony vector into dual vector

The random keys [51] based on the largest position value rule are applied to implement the mapping procedure of the operation sequence. Firstly, a random number drawn uniformly from $[-1, 1]$ is assigned to each ID. Secondly, the position is visited in a non-increasing value. Thirdly, the operation ID is converted by using the job symbol. Finally, the feasible operation permutation is generated according to the constraints of the same job.

For instance, the conversion of the operation permutation for the problem in Table 1 is illustrated in Fig. 5. In detail, a random number is assigned to each ID as follows:

$$\mathbf{X}^{(1)} = \{0.5, 0.4, 0.1, -0.1, 0.6, 0.7, -0.3, -0.4\}. \quad (11)$$

Operation position	1	2	3	4	5	6	7	8
$X^{(1)}$	0.5	0.4	0.1	-0.1	0.6	0.7	-0.3	-0.4
Sorting	0.7	0.6	0.5	0.4	0.1	-0.1	-0.3	-0.4
Operation ID arrangement	6	5	1	2	3	4	7	8
Operation arrangement	O_{31}	O_{22}	O_{11}	O_{12}	O_{13}	O_{21}	O_{32}	O_{33}
Operation sequence	3	2	1	1	1	2	3	3
Feasible operation arrangement	O_{31}	O_{21}	O_{11}	O_{12}	O_{13}	O_{22}	O_{32}	O_{33}

Fig. 5 Generation procedure of feasible operation arrangement

All random numbers are sorted in the descending order as [0.7, 0.6, 0.5, 0.4, 0.1, -0.1, -0.3, -0.4].

Hence, the corresponding ID arrangement and the operation arrangement are generated respectively as [6, 5, 1, 2, 3, 4, 7, 8] and [O_{31} , O_{22} , O_{11} , O_{12} , O_{13} , O_{21} , O_{32} , O_{33}].

All the operations existing in a certain job are marked with the same numerical code, and subsequently, the operation sequence is acquired as [3, 2, 1, 1, 1, 2, 3, 3].

Consequently, the feasible operation arrangement is generated, i.e., [O_{31} , O_{21} , O_{11} , O_{12} , O_{13} , O_{22} , O_{32} , O_{33}].

The mapping procedure of machine allocation is implemented as follows: converting the real number $x(s+j) \in [-\zeta, \zeta]$ to the integer $u(j) \in [1, e(j)]$ by linear transformation, where $e(j)$ denotes the largest number of the machine in its feasible machine set for the operation j . The values of $u(j)$ are computed by Yuan et al. [11].

4.2.3 Decoding procedure

The plug-greedy decoding algorithm is introduced to ensure that the decoded solutions are active schedules. The basic procedure is described as follows: the decoding sequence abides by the arrangement order of operations. The first operation in the arrangement is arranged. Then, the second one in the arrangement is planned on the feasible machine in the earliest competent time, and so on. For every machine, the earliest available idle time interval is arranged preferentially. Without such an idle time at present, the operation is executed at the end of the machine.

4.3 Initialization of HM

Population initialization is of great importance in the HS algorithm because the quality of the initial solutions in HM considerably affects the convergence rate and solution distribution. Random initialization has been generally adopted in the absence of any priori information. Unfortunately, this method cannot effectively guarantee that the initial population obtains the information of the solution space, and the algorithm efficiency would be consequently limited to a certain extent. The initialization strategy integrating random and OBL technique is applied to allow the initial population to use the information of the search

space. Furthermore, the oppositional vector is produced according to the initial feasible solution. And the solutions in HM are sorted according to Definition 1.

To guarantee the feasibility of the solutions in the OGHS, the dual vector code of the initial solution must satisfy the rule that the number of operation of job J_i is equal to n_i and the total number of all the operations is equal to s .

4.3.1 Generating a random code

A random harmony vector

$$\mathbf{X} = \{x(1), x(2), \dots, x(n)\} \quad (12)$$

can be produced by using the following equation:

$$x(j) = x_{\min}(j) + (x_{\max}(j) - x_{\min}(j)) \times \text{rand}(0, 1),$$

$$j = 1, 2, \dots, n \quad (13)$$

where $x_{\min}$ and $x_{\max}$ represent the bound values for the positions and $\text{rand}(0, 1)$ denotes a random function valued from 0 to 1.

The feasibility of the random solution is verified through traversing all the operations in order in the dual vector code. In case the number of operations of job J_i is not equal to n_i or the total number of all the operations is not equal to s , the code is adjusted according to the initial number of n_i or of s correspondingly.

4.3.2 Oppositional operation sequence

Consciously, the non-dominated solution has an important effect on the global optimization. Then, OBL is employed to generate an oppositional harmony vector according to the non-dominated one. Because the oppositional harmony vector in our algorithm is only involved with the operation sequence, the operations are traversed in order and sorted with accelerating position values. For example, the oppositional operation arrangement in Fig. 5 is easily received as [3, 3, 2, 1, 1, 1, 2, 3]. The procedure generating and oppositional operation sequence is shown in Algorithm 1.

Algorithm 1 The procedure generating oppositional operation permutation

Step 1 Generate $\mathbf{X}^{(1)} = -2\text{rand}(1, N) + 1$;

Step 2 $[A, B] = \text{Sort}(\mathbf{X}^{(1)})$;

Step 3 Generate operation arrangement according to B ;

Step 4 Transform operation arrangement into the operation sequence;

Step 5 Generate oppositional operation arrangement according to the operation sequence.

4.3.3 Generating harmony vector

The procedure generating harmony vector representation includes two parts, namely the operation sequence and ma-

chine assignment. The transformation is derived from the approach of Yuan et al. [11].

4.4 Improvisation of a new harmony

Similar to the FJSP, the MOFJSP is a classic discrete combinatorial optimisation problem in the universe of discourse. As mentioned in 3.2.2, GHS can efficiently work in the discrete search space. Memory consideration, pitch adjustment and random selection constitute the main procedure of improvising a new harmony in GHS.

Considering the memory, a new vector $X' = (x'(1), x'(2), \dots, x'(2s))$ is generated from HM. The decision variable $x'(i)$ is determined through any value in the range $(x'(i) - x^{\text{HMS}}(i))$. The HMS parameter indicates the size of HM. The probabilities of selecting, either the historic HM or the entire search space, are HM considering rate (HMCR) or 1-HMCR, respectively. The feasibility of the new vectors selected randomly from the entire search space must necessarily be verified. Specifically, the new vector representation is firstly transformed into a dual vector code and subsequently checked and adjusted according to the rule in Section 4.3.1.

After choosing a value from the HM, the new vector representation, $X' = (x'(1), x'(2), \dots, x'(2s))$, should be examined to determine whether it should be pitch-adjusted. Specifically, the probability of selecting a non-dominated solution is $\text{PAR} \times \text{HMCR}$. Moreover, the probability of doing nothing is 1-PAR.

4.5 Recursive halving of search space

The effective neighbourhood of the local search is based on the recursive halving method, which is motivated by the narrowing of the search area. The procedure of generating an acceptable neighbourhood depending upon recursive halving based on OBL is shown in Algorithm 2.

Algorithm 2 The procedure of generating an acceptable neighbourhood in virtue of recursive halving based on OBL.

Step 1 Initialization:

$A_{\text{left}} = a_0, B_{\text{right}} = b_0, x_{\text{random}}$.

Step 2 Definition of the opposition point of x_{random} :

$\hat{x}_{\text{random}} = A_{\text{left}} + B_{\text{right}} - x_{\text{random}}$.

Step 3 Recursive halving based on OBL.

Update A_{left} and B_{right} by this way:

If x_{random} is non-dominated, $A_{\text{left}} = A_{\text{left}}, B_{\text{right}} = (A_{\text{left}} + B_{\text{right}})/2$;

Else if $\hat{x}_{\text{random}}$ is non-dominated, $A_{\text{left}} = (A_{\text{left}} + B_{\text{right}})/2, B_{\text{right}} = B_{\text{right}}$.

Step 4 Update the opposition of x_{random} .

Step 5 Recursive halving until reaching the termination criterion.

4.6 Complexities of OGHS algorithm

The complexities of the OGHS algorithm includes the space and the time complexities.

The difference between the OGHS and the standard HS algorithm is that the former combines the OBL and the recursive halving technique, and the two strategies do not increase the storage space. Therefore, the space complexity of the OGHS and HS is barely different [52]. Because the HM is the population space in the iterative process, the OGHS algorithm only needs the newly generated individual space to update the original space. Therefore, the space complexity is of constant value, i.e., the space complexity of the OGHS algorithm is $O(1)$.

The time complexity of the proposed algorithm is composed of two components. One is the time complexity of the standard HS algorithm, and the other is the time complexity of non-dominated sorting. As mentioned above, the most critical operations of the HS algorithm are the generation of HM, the pitch adjustment and the replacement of HM. The HS algorithm is of one-order linear progressive time complexity of the maximum iteration number [52]. Suppose the population size is N and the number of non-dominated solutions is r in an iterative process of the OGHS. The preservation rate and the replacement rate of population individuals are denoted by α and β , respectively. Evidently, $\alpha + \beta = 1$. During a non-dominated sorting phase, the second individual is compared only with the first. The third individual is compared at most with the first and the second, and so on. The N th individual needs $(N - 1)$ comparative operations at most. In the worst scenario, where all the individuals are non-dominated solutions, the total number of comparative times can be computed as $[1 + 2 + 3 + \dots + (N - 1)]$, i.e., $N \cdot (N - 1)/2$. Thus, the time complexity of the proposed algorithm is $T(N) = N^2 + \alpha \cdot N + \beta \cdot N$. Consequently, the time complexity of the OGHS algorithm is expressed as $O(rN^2)$.

4.7 Updating of HM

All harmony vectors are assessed after improvising a new harmony. In the current iteration, if a new harmony vector is non-dominated through the existing harmony in the HM, the new vector is included in the HM, and the dominated vector is excluded from the HM. Otherwise, no action should be implemented.

4.8 Termination

If the termination criterion is satisfied, then the algorithm ends and outputs the computational results. The last HM is updated after non-dominated sorting. The archive set is the final result of the OGHS algorithm.

5. Experiment results and discussion

First, this section tests the performance of the proposed algorithm by using different strategies, namely OBL and recursive halving, and subsequently analyzes the computational outcomes of the OGHS algorithm and the comparison results by using other approaches on benchmarks extracted from publications. The OGHS algorithm is manipulated in Matlab R2009a and performed on a personal computer with 3.19 GHz CPU and 3.47 GB RAM. HMS = 50, HMCR = 0.95, and PAR = 0.3.

5.1 Performance examination for strategies

To examine the performance of the OBL and the recursive halving technique embedded in the HS, the widely adopted multi-objective test functions of DTLZ1-4 and DTLZ7, which are suitable for evaluating population diversity and iterative convergence in a high-dimensional space [53,54], are optimized in this section. DTLZ1-4 problems are continuous functions, whereas DTLZ7 is a non-continuous function. The performance metrics of CPU time, generational distance (GD) and inverted generational distance (IGD) [55] are used to evaluate the OGHS. The five problems of DTLZ are tested by using HS, HS+OBL and HS+OBL+halving (i.e., OGHS), respectively. Each algorithm runs three times. The results, including performance metrics of CPU time and GD at the 5 000th iteration and their average values, are listed in Table 2 and Table 3. The convergence curves, portraying the different combinations between the HS and different strategies, are illustrated in Figs. 6–10. Furthermore, the mean and variance values of IGD, corresponding to the 100th, 500th and 3 000th generations, are listed in Table 4.

Table 2 Values of CPU time at the 5 000th iteration

Problem	Algorithm	Time/s			
		1	2	3	Average
DTLZ1	HS	11.370 1	11.504 6	10.928 2	11.267 6
	HS+OBL	11.733 0	11.669 0	11.797 5	11.733 2
	OGHS	11.080 0	11.153 3	11.064 3	11.099 2
DTLZ2	HS	11.142 2	11.116 9	10.970 5	11.076 5
	HS+OBL	11.031 8	10.966 2	11.100 8	11.032 9
	OGHS	11.186 1	11.055 9	11.323 1	11.188 4
DTLZ3	HS	11.059 4	10.894 9	11.180 0	11.044 8
	HS+OBL	11.127 5	11.278 7	10.986 8	11.131 0
	OGHS	11.275 6	10.867 7	11.225 6	11.123 0
DTLZ4	HS	11.226 3	10.967 8	10.802 0	10.998 7
	HS+OBL	11.054 1	11.339 2	10.878 1	11.090 5
	OGHS	11.187 7	11.312 8	11.200 1	11.233 5
DTLZ7	HS	11.864 5	10.802 2	11.920 9	11.529 2
	HS+OBL	11.348 2	11.101 3	12.288 2	11.579 2
	OGHS	10.903 1	11.395 0	11.062 1	11.120 1

From Table 2, the values of CPU time for different combinations between the HS and different strategies at the 5 000th iteration belong to the same order of magnitude.

Table 3 Values of GD at the 5 000th iteration

Problem	Algorithm	GD			
		1	2	3	Average
DTLZ1	HS	0.018 3	0.014 0	0.007 8	0.013 4
	HS+OBL	0.013 3	0.014 4	0.007 8	0.011 8
	OGHS	0.009 7	0.010 0	0.008 9	0.009 5
DTLZ2	HS	0.017 1	0.017 0	0.018 1	0.017 4
	HS+OBL	0.015 4	0.014 9	0.017 8	0.016 0
	OGHS	0.015 7	0.016 4	0.016 3	0.016 1
DTLZ3	HS	0.021 7	0.060 7	0.081 1	0.054 5
	HS+OBL	0.018 6	0.022 7	0.021 5	0.020 9
	OGHS	0.019 3	0.021 3	0.020 3	0.020 3
DTLZ4	HS	0.101 5	0.051 1	0.078 7	0.077 1
	HS+OBL	0.075 0	0.058 3	0.061 7	0.065 0
	OGHS	0.055 0	0.059 7	0.052 2	0.055 6
DTLZ7	HS	0.043 4	0.064 3	0.050 2	0.052 6
	HS+OBL	0.023 7	0.056 5	0.052 3	0.044 2
	OGHS	0.021 6	0.033 1	0.035 6	0.030 1

The time performance of OGHS for DTLZ1 and DTLZ7 is better than the HS's though the time complexity of the OGHS is inferior to the HS's. The main reason is that recursive halving speeds up the searching process. From the values of GD at the 5 000th iteration in Table 3, the average values of the HS+OBL algorithm are much better than those of the HS algorithm, and the average values of the OGHS algorithm outperform those of HS+OBL except DTLZ2. As shown in Figs. 6–10, the speeds of the convergence of the OGHS algorithm are considerably better than those of HS and HS+OBL. The unsatisfactory result of DTLZ4 must be interpreted because of its complicated Pareto front.

(a) HS algorithm

(b) HS+OBL algorithm

(c) HS+OBL+halving algorithm

(a) HS algorithm

Fig. 6 Convergence curves of DTLZ1

(a) HS algorithm

(b) HS+OBL algorithm

(b) HS+OBL algorithm

(c) HS+OBL+halving algorithm

(c) HS+OBL+halving algorithm

Fig. 7 Convergence curves of DTLZ2

(a) HS algorithm

Fig. 8 Convergence curves of DTLZ3

(b) HS+OBL algorithm

(c) HS+OBL+halving algorithm

Fig. 9 Convergence curves of DTLZ4

(a) HS algorithm

(b) HS+OBL algorithm

(c) HS+OBL+halving algorithm

Fig. 10 Convergence curves of DTLZ7

According to the IGD metrics, as depicted in Table 4, the best performances are optimized by the OGHS except DTLZ2. Therefore, the OBL and the recursive halving technique embedded in the OGHS not only improve the distribution of solutions but also accelerate the convergence speed during the evolution.

Table 4 IGD mean and variance values at generation 100, 500, and 3 000

Problem	Algorithm	Value	Generation		
			100	500	3 000
DTLZ1	HS	Mean	0.109 6	0.060 4	0.056 0
		Variance	0.001 9	0.000 5	0.001 5
	HS+OBL	Mean	0.084 0	0.060 8	0.047 7
		Variance	0.001 1	0.000 5	0.000 7
	OGHS	Mean	0.088 5	0.060 8	0.034 7
		Variance	0.001 0	0.000 7	0.000 5
DTLZ2	HS	Mean	0.123 9	0.099 5	0.091 3
		Variance	0.007 1	0.002 7	0.003 1
	HS+OBL	Mean	0.111 4	0.097 9	0.096 2
		Variance	0.006 1	0.003 0	0.002 8
	OGHS	Mean	0.104 3	0.091 3	0.091 6
		Variance	0.003 4	0.002 5	0.002 3
DTLZ3	HS	Mean	0.366 5	0.238 9	0.119 0
		Variance	0.026 8	0.008 4	0.004 6
	HS+OBL	Mean	0.184 2	0.148 4	0.105 9
		Variance	0.007 4	0.005 3	0.004 2
	OGHS	Mean	0.148 5	0.103 3	0.097 1
		Variance	0.004 0	0.002 9	0.002 9
DTLZ4	HS	Mean	0.083 4	0.053 5	0.049 5
		Variance	0.020 0	0.005 9	0.005 4
	HS+OBL	Mean	0.068 2	0.052 4	0.048 4
		Variance	0.009 3	0.004 6	0.003 2
	OGHS	Mean	0.063 1	0.049 9	0.040 1
		Variance	0.010 0	0.006 1	0.002 6
DTLZ7	HS	Mean	1.413 9	1.048 6	0.249 3
		Variance	0.091 0	0.112 5	0.010 9
	HS+OBL	Mean	0.900 9	0.636 0	0.253 7
		Variance	0.083 6	0.063 6	0.008 5
	OGHS	Mean	0.662 0	0.508 8	0.223 6
		Variance	0.038 1	0.033 2	0.005 8

5.2 Benchmark instance tests

Three representative MOFJSP instances are considered in our experiments, namely two types of benchmarks

(medium scale, i.e., problem 10×10 , and large scale, i.e., problem 15×10) taken from Kacem, et al. [35,56] and a benchmark (larger scale, i.e., problem 20×10) presented by Brandimarte [8]. To demonstrate the efficiency of our algorithms, the outcomes of the OGHS are compared with those of four well-known algorithms, i.e., PSO+SA [31], the artificial immune algorithm (AIA) [57], the multi-objective genetic algorithm (MOGA) [58] and the Pareto-

based discrete artificial bee colony (P-DABC) algorithm [59].

5.2.1 Computational results

Comparison results on two Kacem problems with three objectives is detailed in Table 5. Furthermore, a comparison of experiment results on Brandimarte problem 20×10 is listed in Table 6.

Table 5 Comparison results for Kacem instances

Kacem problem		Algorithm											
		PSO+SA	AIA	MOGA				P-DABC			OGHS		
f_1		7	7	8	7	8	7	8	7	8	7	8	7
10×10	f_2	44	43	42	42	41	45	41	43	42	42	41	43
	f_3	6	5	5	6	7	5	7	5	5	6	7	5
15×10	f_1	12	11	11	12	11	–	12	11	–	11	11	–
	f_2	91	93	91	95	98	–	91	93	–	94	91	–
	f_3	11	11	11	10	10	–	11	11	–	10	11	–

Table 6 Comparison results for Brandimarte instance

Brandimarte problem		AIA	P-DABC										OGHS				
			1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
20×10	f_1	312	311	313	319	324	411	417	424	431	474	484	346	453	463	466	479
	f_2	2 591	2 288	2 286	2 280	2 279	2 265	2 253	2 240	2 230	2 223	2 210	2 260	2 215	2 214	2 212	2 210
	f_3	306	299	311	307	307	349	360	386	402	444	454	342	412	422	438	454

5.2.2 Analysis of results

Table 5 shows that the OGHS algorithm obtains the non-dominated solutions for the two Kacem instances. For the problem 10×10 , the OGHS algorithm outperforms PSO+SA. In addition, the results are not worse than P-DABC and MOGA. The solution, i.e., $\{f_1 = 7, f_2 = 43, f_3 = 5\}$, dominates the solution $\{f_1 = 7, f_2 = 45, f_3 = 5\}$ generated by MOGA. For the larger-scale problem 15×10 , the OGHS algorithm achieves two non-dominated solutions. For the problem 20×10 , one non-dominated solution set, where the candidate solutions are denoted with 1–5 successively, is shown in Table 6. The five solutions of the OGHS are not dominated by the results of AIA, and the three solutions, i.e., $\{f_1 = 346, f_2 = 2\,260, f_3 = 342\}$, $\{f_1 = 453, f_2 = 2\,215, f_3 = 412\}$ and $\{f_1 = 479, f_2 = 2\,210, f_3 = 454\}$, dominate the results $\{f_1 = 411, f_2 = 2\,265, f_3 = 349\}$, $\{f_1 = 474, f_2 = 2\,223, f_3 = 444\}$ and $\{f_1 = 484, f_2 = 2\,210, f_3 = 454\}$ obtained by P-DABC, respectively. Furthermore, the non-dominated solution which provides the minimal total workload of machines is improved.

In summary, the OGHS algorithm is either outstanding or the same for the three test instances. In addition, it can obtain more than one scheme corresponding to the same non-dominated solution for all the problems in most cases. The proposed algorithm thus performs well in larger-scale problems.

6. Decision-making of the most appropriate solution

In engineering practice, choosing a satisfying solution to formulate a scheduling program from an NDSS is a crucial step. The decision-making of the most appropriate solution for the MOFJSP from a finite set of alternatives is an MADM problem. TOPSIS, which includes seven steps [60], is a useful technique in dealing with MADM problems in the real world [61–63]. The detailed procedures of TOPSIS for the 20×10 problem are explained as follows.

Step 1 The decision matrix $D = \{x_{ij}\}_{m \times n}$ of the problem 20×10 is expressed as

$$D = \begin{bmatrix} 346 & 2\,260 & 342 \\ 453 & 2\,215 & 412 \\ 463 & 2\,214 & 422 \\ 466 & 2\,212 & 438 \\ 479 & 2\,210 & 454 \end{bmatrix}$$

where x_{ij} indicates the performance of the i th alternative with respect to the j th attribute.

Step 2 The normalized value $R = \{r_{ij}\}_{m \times n}$ can be calculated using the linear normalization formula:

$$r_{ij} = \frac{x_{ij}}{\sum_{i=1}^m x_{ij}}. \quad (14)$$

Furthermore, the decision matrix R can be expressed in matrix format as

$$R = \begin{bmatrix} 0.156\ 8 & 0.203\ 4 & 0.165\ 4 \\ 0.205\ 3 & 0.199\ 4 & 0.199\ 2 \\ 0.209\ 8 & 0.199\ 3 & 0.204\ 1 \\ 0.211\ 1 & 0.199\ 1 & 0.211\ 8 \\ 0.217\ 0 & 0.198\ 9 & 0.219\ 5 \end{bmatrix}.$$

Step 3 The weighted normalized value $V = \{v_{ij}\}_{m \times n}$ is calculated as

$$V = R \cdot \omega^T$$

where $\omega = (\omega_1, \omega_2, \omega_3)$, and ω_j is the weight of the j th attribute, $0 \leq \omega_j \leq 1$ and $\sum_{j=1}^3 \omega_j = 1$.

Step 3a AHP is adopted to calculate a priori weight ω^a , and the a priori attribute weight matrix is listed in Table 7.

Table 7 A priori attribute weight matrix

	Makespan	Total workload	Critical machine workload
Makespan	1.000 0	0.250 0	0.166 7
Total workload	4.000 0	1.000 0	0.333 3
Critical machine workload	6.000 0	3.000 0	1.000 0

Table 7 shows the approximation value of the maximum eigenvalue $\lambda_{\max} \approx 3.053\ 7$, the consistency index $CI = 0.026\ 8$ and the random consistency index $RI = 0.58$. Accordingly, the consistency ratio is

$$CR = \frac{CI}{RI} = 0.046\ 2 < 0.1.$$

Hence, the normalized a priori weight value is

$$\omega^a = (0.085\ 2, 0.270\ 6, 0.644\ 2).$$

Step 3b The objective weight of attribute ω^c is calculated by using the information entropy of the system, E_j , where the mathematical expressions are provided as follows:

$$E_j = -\frac{1}{\ln n} \sum_{i=1}^m r_{ij} \ln r_{ij} \quad (16)$$

$$\omega_j^c = \frac{(1 - E_j)}{\sum_{j=1}^n (1 - E_j)}. \quad (17)$$

The result is $E = (1.459\ 2, 1.465\ 0, 1.460\ 9)$. The objective weight is $\omega^c = (0.331\ 5, 0.335\ 7, 0.332\ 8)$.

Step 3c With integration of the subjective and objective weights, the weight distribution values of each attribute are received from the following formula:

$$\omega_j = \frac{\omega_j^a \omega_j^c}{\sum_{j=1}^n \omega_j^a \omega_j^c}. \quad (18)$$

Thus, the weight is $\omega = (0.084\ 7, 0.272\ 4, 0.642\ 9)$.

Consequently, the decision matrix is

$$V = \begin{bmatrix} 0.013\ 3 & 0.055\ 4 & 0.106\ 3 \\ 0.017\ 4 & 0.054\ 3 & 0.128\ 1 \\ 0.017\ 8 & 0.054\ 3 & 0.131\ 2 \\ 0.017\ 9 & 0.054\ 2 & 0.136\ 2 \\ 0.018\ 4 & 0.054\ 2 & 0.141\ 1 \end{bmatrix}.$$

Step 4 The values for the ideal solution, V^+ , and the nonideal solution, V^- , defined respectively as $V^+ = \min\{v_{ij} | i = 1, 2, 3, 4, 5\}$, and $V^- = \max\{v_{ij} | i = 1, 2, 3, 4, 5\}$, are obtained:

$$V^+ = (0.013\ 3, 0.054\ 2, 0.106\ 3)$$

$$V^- = (0.018\ 4, 0.055\ 4, 0.141\ 1).$$

Step 5 The Euclidean distances $\bar{A}^+$ and $\bar{A}^-$ from V^+ and V^- , respectively, are calculated by using the following equations for alternative i ($i = 1, 2, 3, 4, 5$):

$$\bar{A}_i^+ = \sqrt{\sum_{j=1}^n (v_{ij} - V_j^+)^2} \quad (19)$$

$$\bar{A}_i^- = \sqrt{\sum_{j=1}^n (v_{ij} - V_j^-)^2}. \quad (20)$$

As a result,

$$\bar{A}^+ = (0.001\ 2 \quad 0.022\ 2 \quad 0.025\ 3 \quad 0.030\ 3 \quad 0.035\ 2)$$

and

$$\bar{A}^- = (0.035\ 2 \quad 0.013\ 1 \quad 0.010\ 0 \quad 0.005\ 1 \quad 0.001\ 2).$$

Step 6 The relative proximity C_i^* to the ideal solution is calculated as follows:

$$C_i^* = \frac{\bar{A}_i^-}{\bar{A}_i^+ + \bar{A}_i^-} \quad (21)$$

where $0 \leq C^* \leq 1$. Therefore, the relative proximity is $C^* = (0.967\ 0, 0.371\ 1, 0.283\ 3, 0.144\ 1, 0.033\ 0)$.

Step 7 The best alternative is ranked according to C_i^* in descending order. The priority ranking for candidate solutions calculated by using the TOPSIS is as follows:

$$1 \prec 2 \prec 3 \prec 4 \prec 5.$$

Therefore, the 1st candidate solution is considered as the most appropriate solution. In addition, the most appropriate scheduling scheme is described in Table 8. In the table,

as for the form $Mk(t_{i,j}, tf_{i,j})$, Mk denotes the processing machine, $t_{i,j}$ denotes the start time, and $tf_{i,j}$ denotes the completion time. The symbol “–” indicates that the process does not exist. This approach enables the construction of the most appropriate scheduling scheme for

the MOFJSP according to the preferences of the decision-maker. Compared with the above, with respect to the AIA algorithm, the most appropriate solution is a non-dominated one. Meanwhile, with respect to the P-DABC, the selected candidate solution dominates one result.

Table 8 Most appropriate scheduling scheme for problem 20×10 presented by Brandimarte ($f_1 = 346$, $f_2 = 2260$, $f_3 = 342$)

Operation		$Mk(t_{i,j}, tf_{i,j})$						
$i = 1$	$O_{i1} \sim O_{i7}$	M2(87,97)	M8(99,116)	M8(116,130)	M1(131,141)	M10(142,160)	M9(160,166)	M7(166,175)
	$O_{i8} \sim O_{i14}$	M1(196,203)	M9(203,214)	M4(290,306)	M2(316,321)	M4(322,328)	–	–
$i = 2$	$O_{i1} \sim O_{i7}$	M8(0,17)	M5(17,23)	M2(57,67)	M5(74,83)	M3(122,133)	M1(141,149)	M7(149,154)
	$O_{i8} \sim O_{i14}$	M1(168,175)	M7(201,210)	M1(271,281)	M3(281,292)	M4(306,322)	M5(322,330)	–
$i = 3$	$O_{i1} \sim O_{i7}$	M7(0,5)	M5(8,17)	M10(44,62)	M1(79,90)	M4(90,96)	M4(96,112)	M1(161,168)
	$O_{i8} \sim O_{i14}$	M1(203,213)	M5(213,219)	M3(223,234)	M3(234,248)	–	–	–
$i = 4$	$O_{i1} \sim O_{i7}$	M1(26,34)	M1(34,41)	M1(41,47)	M8(130,144)	M4(144,160)	M8(175,192)	M4(197,204)
	$O_{i8} \sim O_{i14}$	M5(204,209)	M3(209,223)	M7(223,232)	M1(281,291)	–	–	–
$i = 5$	$O_{i1} \sim O_{i7}$	M8(17,34)	M4(34,50)	M5(56,65)	M10(65,71)	M4(160,167)	M2(173,189)	M7(189,194)
	$O_{i8} \sim O_{i14}$	M8(209,223)	M5(223,229)	M10(229,239)	M4(239,245)	M5(245,254)	M1(291,299)	M1(307,314)
$i = 6$	$O_{i1} \sim O_{i7}$	M2(0,5)	M10(6,24)	M1(63,73)	M3(94,108)	M5(108,117)	M7(125,134)	M8(192,209)
	$O_{i8} \sim O_{i14}$	M8(223,237)	M2(237,242)	M7(242,247)	M1(247,253)	–	–	–
$i = 7$	$O_{i1} \sim O_{i7}$	M8(34,48)	M8(48,65)	M5(65,74)	M4(74,90)	M1(106,116)	M2(125,130)	M2(135,140)
	$O_{i8} \sim O_{i14}$	M5(149,155)	M4(211,217)	M1(223,229)	M2(229,237)	M7(237,242)	M4(276,283)	M10(283,301)
$i = 8$	$O_{i1} \sim O_{i7}$	M1(0,10)	M10(24,30)	M5(38,47)	M7(66,75)	M9(75,86)	M2(130,135)	M7(154,159)
	$O_{i8} \sim O_{i14}$	M2(159,164)	M2(164,172)	M4(217,223)	M2(267,277)	M3(292,303)	M5(306,312)	–
$i = 9$	$O_{i1} \sim O_{i7}$	M8(65,87)	M2(82,87)	M1(90,100)	M4(112,128)	M5(128,134)	M7(134,143)	M2(145,153)
	$O_{i8} \sim O_{i14}$	M9(166,177)	M9(177,183)	M2(209,219)	M5(254,263)	–	–	–
$i = 10$	$O_{i1} \sim O_{i7}$	M4(0,16)	M1(73,79)	M3(80,94)	M2(105,110)	M9(110,121)	M9(128,137)	M5(143,149)
	$O_{i8} \sim O_{i14}$	M4(204,211)	M2(219,229)	M3(270,281)	M1(314,322)	M1(322,332)	–	–
$i = 11$	$O_{i1} \sim O_{i7}$	M9(0,11)	M5(23,32)	M2(97,105)	M4(128,144)	M1(149,155)	M5(155,164)	M3(164,174)
	$O_{i8} \sim O_{i14}$	M3(185,199)	M1(237,247)	M4(247,254)	–	–	–	–
$i = 12$	$O_{i1} \sim O_{i7}$	M10(0,6)	M1(10,16)	M4(50,66)	M9(66,72)	M7(75,80)	M7(80,89)	M2(140,145)
	$O_{i8} \sim O_{i14}$	M8(254,268)	M4(270,276)	M5(286,292)	M2(292,308)	–	–	–
$i = 13$	$O_{i1} \sim O_{i7}$	M2(5,10)	M3(14,28)	M9(37,43)	M5(47,56)	M2(120,125)	M4(167,174)	M1(175,186)
	$O_{i8} \sim O_{i14}$	M8(237,254)	M1(254,261)	M1(261,271)	M7(271,280)	–	–	–
$i = 14$	$O_{i1} \sim O_{i7}$	M5(0,8)	M1(16,26)	M2(26,42)	M2(110,120)	M7(120,125)	M1(125,131)	M4(174,190)
	$O_{i8} \sim O_{i14}$	M4(190,197)	M7(210,219)	M3(248,259)	–	–	–	–
$i = 15$	$O_{i1} \sim O_{i7}$	M1(47,57)	M1(57,63)	M2(67,72)	M2(72,80)	M5(83,89)	M9(89,95)	M2(153,158)
	$O_{i8} \sim O_{i14}$	M5(164,169)	M4(223,239)	M3(259,270)	M2(277,287)	M7(287,292)	–	–
$i = 16$	$O_{i1} \sim O_{i7}$	M8(82,99)	M1(100,106)	M3(108,122)	M9(122,128)	M8(161,175)	M7(175,180)	M7(180,189)
	$O_{i8} \sim O_{i14}$	M2(189,194)	M1(229,237)	M2(242,252)	M4(254,270)	M5(270,278)	M1(332,339)	M5(339,344)
$i = 17$	$O_{i1} \sim O_{i7}$	M2(18,26)	M9(26,37)	M7(37,46)	M8(144,161)	M10(161,167)	M5(169,175)	M1(186,196)
	$O_{i8} \sim O_{i14}$	M7(196,201)	M2(252,262)	M10(262,280)	M4(283,290)	M3(303,317)	M1(339,346)	–
$i = 18$	$O_{i1} \sim O_{i7}$	M2(10,18)	M4(23,29)	M5(32,38)	M10(38,44)	M3(44,58)	M5(95,103)	M9(137,148)
	$O_{i8} \sim O_{i14}$	M2(194,204)	M2(262,267)	M8(285,299)	M1(299,307)	–	–	–
$i = 19$	$O_{i1} \sim O_{i7}$	M3(0,14)	M2(42,52)	M7(52,61)	M10(71,89)	M3(133,144)	M1(155,161)	M3(174,185)
	$O_{i8} \sim O_{i14}$	M2(204,209)	M5(229,234)	M5(234,243)	M2(287,292)	M5(292,297)	M5(297,306)	–
$i = 20$	$O_{i1} \sim O_{i7}$	M4(16,23)	M2(52,57)	M7(61,66)	M3(66,80)	M5(89,95)	M1(116,122)	M5(134,143)
	$O_{i8} \sim O_{i14}$	M1(213,223)	M8(268,285)	M9(285,291)	M2(308,316)	M3(317,328)	M2(328,344)	–

7. Conclusions and future work

(i) The OGHS algorithm, which integrates HS and OBL, is proposed for solving the MOFJSP. The theoretical contribution is reflected in the improved performance of the proposed algorithm. Meanwhile, the practical contribution is that the complicated MOFJSPs are successfully solved by using the proposed algorithm.

(ii) In the OGHS algorithm, Pareto optimality is employed to solve the non-dominated solution set for the MOFJSP. OBL is developed to enhance the performance of the algorithm through improving the distribution quality of the individuals in the initial population. Furthermore, recursive halving based on OBL is utilized to shrink the neighbourhood space and to accelerate convergence.

(iii) A practical implication for the MOFJSP is that

TOPSIS is adapted to decide the most appropriate scheduling scheme.

Further examinations need to be conducted in consideration of the following aspects.

(i) The robustness and the time efficiency of the proposed algorithm should be further improved.

(ii) Some excellent heuristics and hybrid algorithms are developed to solve the MOFJSP, especially for large-scale instances. Importantly, the performance of different coding schemes and algorithm strategies directly affects the efficiency and effectiveness of the algorithm. Further work should investigate thoroughly the relevant performance measures corresponding to different strategies.

(iii) A simultaneous investigation of additional objectives and uncertainties for the MOFJSP is vital.

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