Robustness of iterative learning control for a class of fractional-order linear continuous-time switched systems in the sense of L^p norm

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Abstract: For a class of fractional-order linear continuous-time switched systems specified by an arbitrary switching sequence, the performance of PD $^\alpha$ -type fractional-order iterative learning control (FOILC) is discussed in the sense of L^p norm. When the systems are disturbed by bounded external noises, robustness of the PD $^\alpha$ -type algorithm is firstly analyzed in the iteration domain by taking advantage of the generalized Young inequality of convolution integral. Then, convergence of the algorithm is discussed for the systems without any external noise. The results demonstrate that, under some given conditions, both convergence and robustness can be guaranteed during the entire time interval. Simulations support the correctness of the theory.

Keywords: fractional-order, switched systems, iterative learning control, L^p norm.

DOI: 10.21629/JSEE.2019.04.15

1. Introduction

Switched systems [1] are hybrid systems composed of a finite number of discrete-time or continuous-time subsystems, and a switching sequence. The switching sequence is usually arbitrary and defines a specific subsystem being activated during a subinterval of time. In engineering, switched systems have numerous applications in the control of chemical processes, power systems, process control, traffic systems, and many other fields [2,3]. Current studies have typically focused on the analysis of stability, controllability, reachability and observability of switched systems [4–7], etc. Designing a controller to achieve trajectory tracking of switched systems is one aspect of the stability issue, but research on this problem is still rare.

Therefore, it is necessary to further study tracking control for switched systems.

In actual systems, some switched systems are usually repetitive. For example, in addition to the occasional accidents, daily traffic flows are very similar and display intrinsic repeatability. Consequently, traffic control systems can be regarded as switched systems and have strong repeatability in operation [8]. For repetitive dynamical systems, iterative learning control (ILC) [9] can provide a systematic scheme that enhances tracking performance by enough learning times in a fixed time interval. As a result, ILC can be applied to track a given trajectory for switched systems. Research on ILC for switched systems has just started, and there are a few publications addressing the issue. In [10,11], a D-type ILC algorithm had been applied to a class of linear and nonlinear continuous-time switched systems, respectively, and the convergence conditions of the algorithms were obtained. Further, Xu et al. [12] studied the problem of a PD-type ILC for linear continuous-time switched systems in the presence of a fixed initial shift. Yang et al. [13] addressed the convergence performance of first-order and higher-order PD-type ILC for a class of linear continuous-time switched systems in the sense of L^p norm. For a class of nonlinear switched systems with state delays, the stability of high-order PID-type iterative learning control law was given in [14]. Research on ILC for discrete-time switched systems had also been conducted. For instance, Bu et al. [15–18] designed respectively P-type, D-type, and PD-type ILC algorithms for a class of linear discrete-time switched systems. In [19], for nonlinear discrete-time switched systems with input constraints, an open-closed-loop ILC algorithm with convergence analysis was presented and was applied to a switched freeway traffic system. In addition, in order to reduce the influence of measurement noise, Cao et al. [20] proposed an ILC with an attenuation factor, and Wang et al. [21] designed a

Manuscript received October 25, 2018.

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This work was supported by the National Natural Science Foundation of China (61201323), the Special Fund Project for Promoting Scientific and Technological Innovation in Xuzhou City (KC18013), and the Cultivation Project of Xuzhou Institute of Technology (XKY2017112).

hybrid ILC with state information.

The above research has mainly focused on integer-order switched systems. Now, some scholars begin to study ILC for fractional-order switched systems. For example, for a class of fractional-order distributed parameter switched systems, Lan et al. [22] proposed a P-type ILC scheme. In the real world, many natural phenomena exhibit fractional-order dynamical behaviors, which cannot be described by integer-order equations, and can only be described by fractional-order differential equations. Systems described by fractional differential equations are called fractional-order systems, which are a generalization of integer-order systems. Compared with the conventional integer-order model, the fractional-order model can more accurately describe the systems with memory and genetic effects, and better reveal the essential characteristics and behavior of the systems with fractional-order characteristics [23]. In recent years, many scholars have paid great attention to the research on ILC for fractional-order systems, and have achieved fruitful results [24–31].

For ILC algorithms of continuous-time switched systems, most of above papers use the λ -norm to measure the tracking error. The λ -norm is a supremum norm with a negative exponential weighting. In a way, the essential features of the tracking error cannot be objectively and quantitatively described by the λ -norm [32]. Some results have indicated that the monotonic convergence of the control algorithm can be guaranteed theoretically in the sense of the λ -norm when the parameter is sufficiently large. However, in the repetitive operation of the systems, the maximum of the tracking error lies beyond the engineering permissible range, which will cause the output of the systems to fail to track the desired output [33,34]. Therefore, to make the convergence of the systems independent on the parameter value λ , we use the L^p norm to analyze the performance of fractional-order iterative learning control (FOILC) in this paper.

Robustness is an important performance criterion for ILC algorithms. Since external noises are inevitable in engineering practice, studying the robustness of ILC algorithms is necessary. In view of this, for a class of fractional-order linear continuous-time switched systems with a fixed arbitrary switching sequence, in the sense of the L^p norm, we discuss the robustness and convergence of the PD^α -type FOILC algorithm using the generalized Young inequality of convolution integral in this paper.

The rest of the paper is organized as follows. Section 2 presents some preliminary definitions and lemmas. Section 3 presents the fundamental problem with respect to the switched systems and exhibits the main analysis of the proposed PD^α -type FOILC algorithm including the robustness and convergence. In order to confirm the feasibility

and the effectiveness of the algorithm, the numerical simulations are given in Section 4 and the conclusions are drawn in Section 5.

2. Preliminaries

In order to prove the proposed algorithm, some related mathematical knowledge is given in this section.

Definition 1 For a continuous time-varying vector function

$$\mathbf{f} : [0, T] \rightarrow \mathbf{R}^n,$$

$$\mathbf{f}(t) = [f^1(t), f^2(t), \dots, f^n(t)]^T,$$

its λ -norm and L^p norm are given respectively by

$$\begin{cases} \|\mathbf{f}(t)\|_\lambda = \sup_{0 \leq t \leq T} e^{-\lambda t} (\max_{1 \leq i \leq n} |f^i(t)|), & \lambda > 0 \\ \|\mathbf{f}(t)\|_p = \left[\int_0^T (\max_{1 \leq i \leq n} |f^i(t)|)^p dt \right]^{\frac{1}{p}}, & 1 \leq p \leq \infty \end{cases}.$$

Lemma 1 [35] Suppose the functions $\mathbf{g}(t) \in L^q$ and $\mathbf{h}(t) \in L^p$, $t \in [0, T]$ are Lebesgue integral, and the convolution integral of functions $\mathbf{g}$ and $\mathbf{h}$

$$(\mathbf{g} * \mathbf{h})(t) = \int_0^T \mathbf{g}(t - \tau) \mathbf{h}(\tau) d\tau$$

exists, then the generalized Young inequality of convolution integral is

$$\|(\mathbf{g} * \mathbf{h})(t)\|_r \leq \|\mathbf{g}(t)\|_q \|\mathbf{h}(t)\|_p$$

where $1 \leq p, q, r \leq \infty$, $\frac{1}{r} = \frac{1}{p} + \frac{1}{q} - 1$. Particularly, when $r = p$, the inequality is reformulated as

$$\|(\mathbf{g} * \mathbf{h})(t)\|_p \leq \|\mathbf{g}(t)\|_1 \|\mathbf{h}(t)\|_p.$$

Definition 2 [36] Riemann-Liouville's fractional integral of order $\alpha > 0$ for a function $\mathbf{f}$ is described as

$${}_t D_t^{-\alpha} \mathbf{f}(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^t (t - \tau)^{\alpha-1} \mathbf{f}(\tau) d\tau$$

where $\Gamma(\cdot)$ is the Gamma function.

The Caputo derivative is given by

$${}_t^C D_t^\alpha \mathbf{f}(t) = {}_t D_t^{-(\lceil \alpha \rceil - \alpha + 1)} \left[\frac{d^{\lceil \alpha \rceil + 1}}{dt^{\lceil \alpha \rceil + 1}} \mathbf{f}(t) \right]$$

where $\alpha \in \mathbf{R}^+$, $\lceil \alpha \rceil$ is the integral part of α .

Lemma 2 [37] Assume the function $\mathbf{f}(\mathbf{x}(t), t)$ is continuous, then the initial value problem

$$\begin{cases} {}_t^C D_t^\alpha \mathbf{x}(t) = \mathbf{f}(\mathbf{x}(t), t) \\ \mathbf{x}(t_0) = \mathbf{x}_0 \end{cases}$$

where $0 < \alpha < 1$, is equivalent to the following nonlinear Volterra integral equation

$$\mathbf{x}(t) = \mathbf{x}_0 + \frac{1}{\Gamma(\alpha)} \int_{t_0}^t (t - \tau)^{\alpha-1} \mathbf{f}(\mathbf{x}(\tau), \tau) d\tau.$$

3. Problem statements and analysis

Considering a class of fractional-order linear continuous-time switched systems with bounded external noises

$$\begin{cases} {}^C_0 D_t^\alpha \mathbf{x}_k(t) = \mathbf{A}_{\sigma(t)} \mathbf{x}_k(t) + \mathbf{B}_{\sigma(t)} \mathbf{u}_k(t) + \boldsymbol{\xi}_k(t) \\ \mathbf{y}_k(t) = \mathbf{C}_{\sigma(t)} \mathbf{x}_k(t) + \boldsymbol{\eta}_k(t) \end{cases} \quad (1)$$

where $t \in [0, T]$, $\alpha \in (0, 1)$, $\cdot^{(\alpha)}$ denotes ${}^C_0 D_t^\alpha$; $\mathbf{x}_k(t) \in \mathbf{R}^n$, $\mathbf{u}_k(t) \in \mathbf{R}^p$, and $\mathbf{y}_k(t) \in \mathbf{R}^q$ are the state vector, the input vector, and the output vector at the k th repetitive operation of the systems, respectively. $\mathbf{A}_{\sigma(t)}$, $\mathbf{B}_{\sigma(t)}$, and $\mathbf{C}_{\sigma(t)}$ with appropriate dimensions are state matrices, input matrices, and output matrices of subsystems, respectively. The symbol $\sigma(t)$ is a random switching sequence defined as $\sigma(t) : [0, T] \rightarrow M = \{1, 2, \dots, m\}$, where m is the number of subsystems of the switched systems. That is, the matrices group $[\mathbf{A}_{\sigma(t)} \ \mathbf{B}_{\sigma(t)} \ \mathbf{C}_{\sigma(t)}]$ is an arbitrary element of the following set $\{[\mathbf{A}_1 \ \mathbf{B}_1 \ \mathbf{C}_1], \dots, [\mathbf{A}_m \ \mathbf{B}_m \ \mathbf{C}_m]\}$. The vectors $\boldsymbol{\xi}_k(t)$ and $\boldsymbol{\eta}_k(t)$ are the bounded state disturbance and the bounded measurement disturbance, respectively, and satisfy $\|\boldsymbol{\xi}_k(t)\|_p \leq b_\xi$, $\|\boldsymbol{\eta}_k(t)\|_p \leq b_\eta$, $\|\boldsymbol{\eta}_k^{(\alpha)}(t)\|_p \leq b_{\eta\alpha}$, $k = 1, 2, \dots$, where b_ξ , b_η , and $b_{\eta\alpha}$ are positive constants.

Some basic assumptions of the switched systems are given in the following.

Assumption 1 Systems (1) can operate repetitively on the finite time interval $[0, T]$, and the initial state can be reset for all iterations, that is,

$$\mathbf{x}_k(0) = \mathbf{x}_d(0), \quad k = 1, 2, \dots$$

where $\mathbf{x}_d(0)$ is the initial value of the desired state.

Assumption 2 The given desired output $\mathbf{y}_d(t)$ is invariant in the process of iteration.

Assumption 3 The switching sequence $\sigma(t)$ remains iteration-invariant once it is randomly specified at the initial iteration.

Assumption 4 For the given desired output $\mathbf{y}_d(t)$, there exists only a desired control input $\mathbf{u}_d(t)$, such that

$$\begin{cases} {}^C_0 D_t^\alpha \mathbf{x}_d(t) = \mathbf{A}_{\sigma(t)} \mathbf{x}_d(t) + \mathbf{B}_{\sigma(t)} \mathbf{u}_d(t) \\ \mathbf{y}_d(t) = \mathbf{C}_{\sigma(t)} \mathbf{x}_d(t) \end{cases}.$$

Assumption 5 $\mathbf{C}_i \mathbf{B}_i$ ($i = 1, 2, \dots, m$) are full rank matrices.

In this paper, we consider the following PD $^\alpha$ -type fractional-order iterative learning control law

$$\mathbf{u}_{k+1}(t) = \mathbf{u}_k(t) + \mathbf{F}_p \mathbf{e}_k(t) + \mathbf{F}_d \mathbf{e}_k^{(\alpha)}(t) \quad (2)$$

where $\mathbf{F}_p$ and $\mathbf{F}_d$ represent the gain matrices of proportional learning and differential learning respectively; $\mathbf{e}_k(t) = \mathbf{y}_d(t) - \mathbf{y}_k(t)$ is the tracking error of systems at the k th iteration.

Without loss of generality, we suppose that the arbitrary switching sequence $\sigma(t)$ is given by

$$\sigma(t) = i = \begin{cases} 1, & t \in [0, t_1] \\ 2, & t \in [t_1, t_2] \\ \vdots \\ m, & t \in [t_{m-1}, T] \end{cases}. \quad (3)$$

The sequence indicates that each subsystem only activates once during the interval $[0, T]$.

Thus, the systems (1) with m subsystems can be described as

$$\begin{cases} {}^C_0 D_t^\alpha \mathbf{x}_k(t) = \mathbf{A}_i \mathbf{x}_k(t) + \mathbf{B}_i \mathbf{u}_k(t) + \boldsymbol{\xi}_k(t) \\ \mathbf{y}_k(t) = \mathbf{C}_i \mathbf{x}_k(t) + \boldsymbol{\eta}_k(t) \end{cases} \quad (4)$$

where $i \in \{1, 2, \dots, m\}$.

Let $t_0 = 0$ and $t_m = T$. From Lemma 2, the state of systems (4) can be reformulated as

$$\begin{aligned} \mathbf{x}_k(t) = & \mathbf{x}_k(t_{i-1}) + \frac{1}{\Gamma(\alpha)} \int_{t_{i-1}}^t (t - \tau)^{\alpha-1} \mathbf{A}_i \mathbf{x}_k(\tau) d\tau + \\ & \frac{1}{\Gamma(\alpha)} \int_{t_{i-1}}^t (t - \tau)^{\alpha-1} \mathbf{B}_i \mathbf{u}_k(\tau) d\tau + \\ & \frac{1}{\Gamma(\alpha)} \int_{t_{i-1}}^t (t - \tau)^{\alpha-1} \boldsymbol{\xi}_k(\tau) d\tau, \\ & t \in [t_{i-1}, t_i]; \quad 1 \leq i \leq m. \end{aligned} \quad (5)$$

Remark 1 From (5), the state of switched systems depends on the initial conditions, the system matrices, the switching order and the switching time.

Remark 2 For the initial condition of the subsystems, there are $\mathbf{x}_k(0) = \mathbf{x}_d(0)$ and $\mathbf{x}_k(t_i) \neq \mathbf{x}_d(t_i)$, $i = 1, 2, \dots, m-1$.

Theorem 1 Assume that the systems (4) satisfy Assumptions 1–5, when the control law (2) is applied to the systems (4), if

- i) $M_i = \Gamma(\alpha) - \|(t)^{\alpha-1} \mathbf{A}_i\|_1 > 0$, $i = 1, 2, \dots, m$
- ii) $\rho_i = \|\mathbf{I} - \mathbf{F}_d \mathbf{C}_i \mathbf{B}_i\| + \beta_i < 1$

where

$$\begin{aligned} \beta_i = & \frac{\|\mathbf{F}_p \mathbf{C}_i + \mathbf{F}_d \mathbf{C}_i \mathbf{A}_i\| \|(t)^{\alpha-1} \mathbf{B}_i\|_1}{M_i}, \\ & i = 1, 2, \dots, m, \end{aligned}$$

then the tracking error is uniform boundedness and the output of the systems asymptotically falls into a small neighborhood of the desired output during the interval $[0, T]$ as $k \rightarrow \infty$.

Proof Let

$$\Delta \mathbf{x}_k(t) = \mathbf{x}_d(t) - \mathbf{x}_k(t) \quad (6)$$

and

$$\Delta \mathbf{u}_k(t) = \mathbf{u}_d(t) - \mathbf{u}_k(t). \quad (7)$$

From (4), we obtain

$$\begin{aligned} \mathbf{e}_k^{(\alpha)}(t) &= \mathbf{y}_d^{(\alpha)}(t) - \mathbf{y}_k^{(\alpha)}(t) = \\ C_i(\mathbf{x}_d^{(\alpha)}(t) - \mathbf{x}_k^{(\alpha)}(t)) - \boldsymbol{\eta}_k^{(\alpha)}(t) &= \\ C_i\Delta \mathbf{x}_k^{(\alpha)}(t) - \boldsymbol{\eta}_k^{(\alpha)}(t) &= \\ C_i\mathbf{A}_i\Delta \mathbf{x}_k(t) + C_i\mathbf{B}_i\Delta \mathbf{u}_k(t) - C_i\boldsymbol{\xi}_k(t) - \boldsymbol{\eta}_k^{(\alpha)}(t). \end{aligned} \quad (8)$$

According to (2), we have

$$\Delta \mathbf{u}_{k+1}(t) = \Delta \mathbf{u}_k(t) - \boldsymbol{\Gamma}_p \mathbf{e}_k(t) - \boldsymbol{\Gamma}_d \mathbf{e}_k^{(\alpha)}(t). \quad (9)$$

Taking (8) into (9), we further get

$$\begin{aligned} \Delta \mathbf{u}_{k+1}(t) &= (\mathbf{I} - \boldsymbol{\Gamma}_d C_i \mathbf{B}_i) \Delta \mathbf{u}_k(t) - \\ &(\boldsymbol{\Gamma}_p C_i + \boldsymbol{\Gamma}_d C_i \mathbf{A}_i) \Delta \mathbf{x}_k(t) + \\ &\boldsymbol{\Gamma}_d C_i \boldsymbol{\xi}_k(t) + \boldsymbol{\Gamma}_p \boldsymbol{\eta}_k(t) + \boldsymbol{\Gamma}_d \boldsymbol{\eta}_k^{(\alpha)}(t). \end{aligned} \quad (10)$$

When $t \in [t_i, t_{i+1}]$, the L^p norm of the function $\mathbf{f}$ is denoted by $\|\mathbf{f}[t_i, t_{i+1}]\|_p$; When $t \in [0, T]$, the L^p norm of the function $\mathbf{f}$ is denoted by $\|\mathbf{f}\|_p$. Obviously, when $[t_i, t_{i+1}] \subset [0, T]$, there is $\|\mathbf{f}[t_i, t_{i+1}]\|_p \leq \|\mathbf{f}\|_p$.

Step 1 $t \in [0, t_1]$.

In this case, the subsystem 1 is active. The state of the systems (4) has the following expression:

$$\begin{aligned} \mathbf{x}_{k+1}(t) &= \mathbf{x}_{k+1}(0) + \\ &\frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} \mathbf{A}_1 \mathbf{x}_{k+1}(\tau) d\tau + \\ &\frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} \mathbf{B}_1 \mathbf{u}_{k+1}(\tau) d\tau + \\ &\frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} \boldsymbol{\xi}_{k+1}(\tau) d\tau. \end{aligned} \quad (11)$$

Considering Assumption 1, we get

$$\begin{aligned} \Delta \mathbf{x}_{k+1}(t) &= \mathbf{x}_d(t) - \mathbf{x}_{k+1}(t) = \\ &\frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} \mathbf{A}_1 \Delta \mathbf{x}_{k+1}(\tau) d\tau + \\ &\frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} \mathbf{B}_1 \Delta \mathbf{u}_{k+1}(\tau) d\tau - \\ &\frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} \boldsymbol{\xi}_{k+1}(\tau) d\tau. \end{aligned} \quad (12)$$

When $M_1 = \Gamma(\alpha) - \|(t)^{\alpha-1} \mathbf{A}_1\|_1 > 0$, taking the L^p norm ($t \in [0, t_1]$) on both sides of (12) and adopting the generalized Young inequality of convolution integral, we can obtain

$$\frac{\| \Delta \mathbf{x}_{k+1}[0, t_1] \|_p \leq \frac{\|(t)^{\alpha-1} \mathbf{B}_1\|_1 \|\Delta \mathbf{u}_{k+1}[0, t_1]\|_p + \|(t)^{\alpha-1}\|_1 b_\xi}{M_1}}{M_1}. \quad (13)$$

From (10), we have

$$\begin{aligned} \Delta \mathbf{u}_{k+1}(t) &= (\mathbf{I} - \boldsymbol{\Gamma}_d C_1 \mathbf{B}_1) \Delta \mathbf{u}_k(t) - \\ &(\boldsymbol{\Gamma}_p C_1 + \boldsymbol{\Gamma}_d C_1 \mathbf{A}_1) \Delta \mathbf{x}_k(t) + \\ &\boldsymbol{\Gamma}_d C_1 \boldsymbol{\xi}_k(t) + \boldsymbol{\Gamma}_p \boldsymbol{\eta}_k(t) + \boldsymbol{\Gamma}_d \boldsymbol{\eta}_k^{(\alpha)}(t). \end{aligned} \quad (14)$$

Taking the L^p norm ($t \in [0, t_1]$) on both sides of (14), we can derive

$$\begin{aligned} \|\Delta \mathbf{u}_{k+1}[0, t_1]\|_p &\leq \\ \|\mathbf{I} - \boldsymbol{\Gamma}_d C_1 \mathbf{B}_1\| \|\Delta \mathbf{u}_k[0, t_1]\|_p + \\ \|\boldsymbol{\Gamma}_p C_1 + \boldsymbol{\Gamma}_d C_1 \mathbf{A}_1\| \|\Delta \mathbf{x}_k[0, t_1]\|_p + \\ \|\boldsymbol{\Gamma}_d C_1\| b_\xi + \|\boldsymbol{\Gamma}_p\| b_\eta + \|\boldsymbol{\Gamma}_d\| b_{\eta\alpha}. \end{aligned} \quad (15)$$

Taking (13) into (15) yields

$$\begin{aligned} \|\Delta \mathbf{u}_{k+1}[0, t_1]\|_p &\leq \\ (\|\mathbf{I} - \boldsymbol{\Gamma}_d C_1 \mathbf{B}_1\| + \beta_1) \|\Delta \mathbf{u}_k[0, t_1]\|_p + \\ \gamma_1 b_\xi + \|\boldsymbol{\Gamma}_p\| b_\eta + \|\boldsymbol{\Gamma}_d\| b_{\eta\alpha} = \\ \rho_1 \|\Delta \mathbf{u}_k[0, t_1]\|_p + \gamma_1 b_\xi + \|\boldsymbol{\Gamma}_p\| b_\eta + \|\boldsymbol{\Gamma}_d\| b_{\eta\alpha} \end{aligned} \quad (16)$$

where

$$\begin{aligned} \beta_1 &= \frac{\|\boldsymbol{\Gamma}_p C_1 + \boldsymbol{\Gamma}_d C_1 \mathbf{A}_1\| \|(t)^{\alpha-1} \mathbf{B}_1\|_1}{M_1}, \\ \gamma_1 &= \frac{\|\boldsymbol{\Gamma}_p C_1 + \boldsymbol{\Gamma}_d C_1 \mathbf{A}_1\| \|(t)^{\alpha-1}\|_1}{M_1} + \|\boldsymbol{\Gamma}_d C_1\|. \end{aligned}$$

From (16), we get

$$\begin{aligned} \|\Delta \mathbf{u}_{k+1}[0, t_1]\|_p &\leq \rho_1^k \|\Delta \mathbf{u}_1[0, t_1]\|_p - \\ &\frac{\rho_1^k \gamma_1 b_\xi + \|\boldsymbol{\Gamma}_p\| b_\eta + \|\boldsymbol{\Gamma}_d\| b_{\eta\alpha}}{1 - \rho_1} + \\ &\frac{\gamma_1 b_\xi + \|\boldsymbol{\Gamma}_p\| b_\eta + \|\boldsymbol{\Gamma}_d\| b_{\eta\alpha}}{1 - \rho_1}. \end{aligned} \quad (17)$$

Using the condition ii) of Theorem 1, computing the supremum on both sides of (17), and taking $k \rightarrow \infty$, it yields

$$\begin{aligned} \lim_{k \rightarrow \infty} \sup_{0 \leq t \leq t_1} \|\Delta \mathbf{u}_{k+1}[0, t_1]\|_p &\leq \\ \frac{\gamma_1 b_\xi + \|\boldsymbol{\Gamma}_p\| b_\eta + \|\boldsymbol{\Gamma}_d\| b_{\eta\alpha}}{1 - \rho_1}. \end{aligned} \quad (18)$$

Combining (13) with (18), we have

$$\lim_{k \rightarrow \infty} \sup_{0 \leq t \leq t_1} \|\Delta \mathbf{x}_{k+1}[0, t_1]\|_p \leq \frac{\|(t)^{\alpha-1} \mathbf{B}_1\|_1 (\gamma_1 b_\xi + \|\mathbf{F}_p\| b_\eta + \|\mathbf{F}_d\| b_{\eta\alpha})}{M_1(1-\rho_1)} + \frac{\|(t)^{\alpha-1}\|_1 b_\xi}{M_1}. \quad (19)$$

According to (4), we obtain

$$e_{k+1}(t) = C_1(\mathbf{x}_d(t) - \mathbf{x}_{k+1}(t)) - \boldsymbol{\eta}_{k+1}(t) = C_1 \Delta \mathbf{x}_{k+1}(t) - \boldsymbol{\eta}_{k+1}(t). \quad (20)$$

Taking the L^p norm ($t \in [0, t_1]$) on both sides of (20) and computing the supremum, then substituting (19) into it, it can be concluded that

$$\lim_{k \rightarrow \infty} \sup_{0 \leq t \leq t_1} \|\Delta e_{k+1}[0, t_1]\|_p \leq \left(\frac{\gamma_1 \|\mathbf{C}_1\| \|(t)^{\alpha-1} \mathbf{B}_1\|_1}{M_1(1-\rho_1)} + \frac{\|\mathbf{C}_1\| \|(t)^{\alpha-1}\|_1}{M_1} \right) b_\xi + \left(\frac{\|\mathbf{C}_1\| \|\mathbf{F}_p\| \|(t)^{\alpha-1} \mathbf{B}_1\|_1}{M_1(1-\rho_1)} + 1 \right) b_\eta + \frac{\|\mathbf{C}_1\| \|\mathbf{F}_d\| \|(t)^{\alpha-1} \mathbf{B}_1\|_1}{M_1(1-\rho_1)} b_{\eta\alpha}. \quad (21)$$

The proof indicates that the tracking error of the subsystem 1 is uniform boundedness during the interval $[0, t_1]$.

Step 2 $t \in [t_1, t_2]$.

In this case, the subsystem 2 is active. Similar to (12), we have

$$\begin{aligned} \Delta \mathbf{x}_{k+1}(t) &= \mathbf{x}_d(t) - \mathbf{x}_{k+1}(t) = \Delta \mathbf{x}_{k+1}(t_1) + \\ &\frac{1}{\Gamma(\alpha)} \int_{t_1}^t (t-\tau)^{\alpha-1} \mathbf{A}_2 \Delta \mathbf{x}_{k+1}(\tau) d\tau + \\ &\frac{1}{\Gamma(\alpha)} \int_{t_1}^t (t-\tau)^{\alpha-1} \mathbf{B}_2 \Delta \mathbf{u}_{k+1}(\tau) d\tau - \\ &\frac{1}{\Gamma(\alpha)} \int_{t_1}^t (t-\tau)^{\alpha-1} \xi_{k+1}(\tau) d\tau. \end{aligned} \quad (22)$$

When $M_2 = \Gamma(\alpha) - \|(t)^{\alpha-1} \mathbf{A}_2\|_1 > 0$, using similar manipulations as Step 1, we get

$$\begin{aligned} \|\Delta \mathbf{x}_{k+1}[t_1, t_2]\|_p &\leq \\ &\frac{\|(t)^{\alpha-1} \mathbf{B}_2\|_1 \|\Delta \mathbf{u}_{k+1}[t_1, t_2]\|_p + \|(t)^{\alpha-1}\|_1 b_\xi}{M_2} + \\ &\frac{\Gamma(\alpha) \|\Delta \mathbf{x}_{k+1}(t_1)\|_p}{M_2} \end{aligned} \quad (23)$$

and

$$\begin{aligned} \|\Delta \mathbf{u}_{k+1}[t_1, t_2]\|_p &\leq \\ &(\|\mathbf{I} - \mathbf{F}_d \mathbf{C}_2 \mathbf{B}_2\| + \beta_2) \|\Delta \mathbf{u}_k[t_1, t_2]\|_p + \gamma_2 b_\xi + \\ &\|\mathbf{F}_p\| b_\eta + \|\mathbf{F}_d\| b_{\eta\alpha} + c_2 \|\Delta \mathbf{x}_k(t_1)\|_p = \\ &\rho_2 \|\Delta \mathbf{u}_k[t_1, t_2]\|_p + \gamma_2 b_\xi + \|\mathbf{F}_p\| b_\eta + \\ &\|\mathbf{F}_d\| b_{\eta\alpha} + c_2 \|\Delta \mathbf{x}_k(t_1)\|_p \end{aligned} \quad (24)$$

where

$$\begin{aligned} \beta_2 &= \frac{\|\mathbf{F}_p \mathbf{C}_2 + \mathbf{F}_d \mathbf{C}_2 \mathbf{A}_2\| \|(t)^{\alpha-1} \mathbf{B}_2\|_1}{M_2}, \\ \gamma_2 &= \frac{\|\mathbf{F}_p \mathbf{C}_2 + \mathbf{F}_d \mathbf{C}_2 \mathbf{A}_2\| \|(t)^{\alpha-1}\|_1}{M_2} + \|\mathbf{F}_d \mathbf{C}_2\|, \\ c_2 &= \frac{\Gamma(\alpha) \|\mathbf{F}_p \mathbf{C}_2 + \mathbf{F}_d \mathbf{C}_2 \mathbf{A}_2\|}{M_2}. \end{aligned}$$

From (24), we have

$$\begin{aligned} \|\Delta \mathbf{u}_{k+1}[t_1, t_2]\|_p &\leq \rho_2^k \|\Delta \mathbf{u}_1[t_1, t_2]\|_p - \\ &\rho_2^k \frac{\gamma_2 b_\xi + \|\mathbf{F}_p\| b_\eta + \|\mathbf{F}_d\| b_{\eta\alpha} + c_2 \|\Delta \mathbf{x}_k(t_1)\|_p}{1 - \rho_2} + \\ &\frac{\gamma_2 b_\xi + \|\mathbf{F}_p\| b_\eta + \|\mathbf{F}_d\| b_{\eta\alpha} + c_2 \|\Delta \mathbf{x}_k(t_1)\|_p}{1 - \rho_2}. \end{aligned} \quad (25)$$

From (19), we know

$$\begin{aligned} \lim_{k \rightarrow \infty} \|\Delta \mathbf{x}_{k+1}(t_1)\|_p &\leq \\ &\frac{\|(t)^{\alpha-1} \mathbf{B}_1\|_1 (\gamma_1 b_\xi + \|\mathbf{F}_p\| b_\eta + \|\mathbf{F}_d\| b_{\eta\alpha})}{M_1(1-\rho_1)} + \\ &\frac{\|(t)^{\alpha-1}\|_1 b_\xi}{M_1}. \end{aligned} \quad (26)$$

Computing the supremum on both sides of (25), and taking $k \rightarrow \infty$, we obtain

$$\begin{aligned} \lim_{k \rightarrow \infty} \sup_{t_1 \leq t \leq t_2} \|\Delta \mathbf{u}_{k+1}[t_1, t_2]\|_p &\leq \\ &\frac{\gamma_2 b_\xi + \|\mathbf{F}_p\| b_\eta + \|\mathbf{F}_d\| b_{\eta\alpha}}{1 - \rho_2} + \\ &\frac{c_2 \|(t)^{\alpha-1} \mathbf{B}_1\|_1 (\gamma_1 b_\xi + \|\mathbf{F}_p\| b_\eta + \|\mathbf{F}_d\| b_{\eta\alpha})}{M_1(1-\rho_1)(1-\rho_2)} + \\ &\frac{c_2 \|(t)^{\alpha-1}\|_1 b_\xi}{M_1(1-\rho_2)}. \end{aligned} \quad (27)$$

Combining (23), (26) and (27), we get

$$\begin{aligned} \lim_{k \rightarrow \infty} \sup_{t_1 \leq t \leq t_2} \|\Delta \mathbf{x}_{k+1}[t_1, t_2]\|_p &\leq \\ &\omega_2 b_\xi + \varepsilon_2 b_\eta + \lambda_2 b_{\eta\alpha} \end{aligned} \quad (28)$$

where

$$\omega_2 = \frac{\gamma_2 \|(t)^{\alpha-1} \mathbf{B}_2\|_1}{M_2(1-\rho_2)} + \frac{\|(t)^{\alpha-1}\|_1}{M_2} +$$

$$\begin{aligned}
& \frac{c_2 \|(t)^{\alpha-1} \mathbf{B}_2\|_1}{M_1 M_2 (1 - \rho_2)} \left(\frac{\gamma_1 \|(t)^{\alpha-1} \mathbf{B}_1\|_1}{1 - \rho_1} + \|(t)^{\alpha-1}\|_1 \right) + \\
& \frac{\Gamma(\alpha)}{M_1 M_2} \left(\frac{\gamma_1 \|(t)^{\alpha-1} \mathbf{B}_1\|_1}{1 - \rho_1} + \|(t)^{\alpha-1}\|_1 \right), \\
& \varepsilon_2 = \frac{\|(t)^{\alpha-1} \mathbf{B}_2\|_1 \|\mathbf{F}_p\|}{M_2 (1 - \rho_2)} + \\
& \frac{\|(t)^{\alpha-1} \mathbf{B}_1\|_1 \|\mathbf{F}_p\|}{M_1 M_2 (1 - \rho_1)} \left(\frac{c_2 \|(t)^{\alpha-1} \mathbf{B}_2\|}{1 - \rho_2} + \Gamma(\alpha) \right), \\
& \lambda_2 = \frac{\|(t)^{\alpha-1} \mathbf{B}_2\|_1 \|\mathbf{F}_d\|}{M_2 (1 - \rho_2)} + \\
& \frac{\|(t)^{\alpha-1} \mathbf{B}_1\|_1 \|\mathbf{F}_d\|}{M_1 M_2 (1 - \rho_1)} \left(\frac{c_2 \|(t)^{\alpha-1} \mathbf{B}_2\|}{1 - \rho_2} + \Gamma(\alpha) \right).
\end{aligned}$$

Similar to (21), one can conclude that

$$\begin{aligned}
& \lim_{k \rightarrow \infty} \sup_{t_1 \leq t \leq t_2} \|\mathbf{e}_{k+1}[t_1, t_2]\|_p \leq \\
& \omega_2 \|\mathbf{C}_2\| b_\xi + (\varepsilon_2 \|\mathbf{C}_2\| + 1) b_\eta + \lambda_2 \|\mathbf{C}_2\| b_{\eta\alpha}. \quad (29)
\end{aligned}$$

The proof indicates that the tracking error of the subsystem 2 is also uniform boundedness during the interval $[t_1, t_2]$.

Step 3 In this proof, we can also get that the tracking errors are also uniform boundedness during the intervals $[t_2, t_3], \dots, [t_{m-1}, T]$.

Therefore, if the conditions of Theorem 1 are given, the tracking error is uniform boundedness during the whole interval $[0, T]$. That is, with the increase of iteration numbers, the output of the systems asymptotically falls into a small neighborhood of the desired output during the whole interval $[0, T]$. $\square$

When the control law (2) is applied to the systems (4) without any external noise, the following conclusion can be obtained.

Theorem 2 Assume that the systems (4) satisfy Assumptions 1–5, when the control law (2) is applied to the systems (4) without any external noise, if

- i) $M_i = \Gamma(\alpha) - \|(t)^{\alpha-1} \mathbf{A}_i\|_1 > 0, i = 1, 2, \dots, m$
- ii) $\rho_i = \|\mathbf{I} - \mathbf{F}_d \mathbf{C}_i \mathbf{B}_i\| + \beta_i < 1$.

where

$$\begin{aligned}
\beta_i &= \frac{\|\mathbf{F}_p \mathbf{C}_i + \mathbf{F}_d \mathbf{C}_i \mathbf{A}_i\| \|(t)^{\alpha-1} \mathbf{B}_i\|_1}{M_i}, \\
i &= 1, 2, \dots, m,
\end{aligned}$$

then the tracking error monotonically converges to zero and the output of the systems converges to the desired output during the interval $[0, T]$ as $k \rightarrow \infty$.

Remark 3 According to the proof process of Theorem 1, the results can be generalized to the arbitrary switching sequence with many times active for each subsystem.

Remark 4 The sufficient condition for the convergence of the PD^α -type algorithm is more conservative than the sufficient condition $\rho_i^* = \|\mathbf{I} - \mathbf{F}_d \mathbf{C}_i \mathbf{B}_i\| < 1$ ($i = 1, 2, \dots, m$) which is got in the sense of the λ -norm. However, the convergence analysis of the algorithm and the measure of errors are not dependent on the value of λ .

Remark 5 If the convergence conditions $\|\mathbf{I} - \mathbf{F}_d \mathbf{C}_i \mathbf{B}_i\| + \beta_i < 1$ ($i = 1, 2, \dots, m$) cannot be satisfied for all subsystems, a PD^α -type algorithm can be chosen as $\mathbf{u}_{k+1}(t) = \mathbf{u}_k(t) + \mathbf{F}_{pi} \mathbf{e}_k(t) + \mathbf{F}_{di} \mathbf{e}_k^{(\alpha)}(t)$, and the conditions can be rewritten as $\|\mathbf{I} - \mathbf{F}_{di} \mathbf{C}_i \mathbf{B}_i\| + \beta_i < 1$ ($i = 1, 2, \dots, m$).

4. Numerical simulations

Consider the following fractional-order linear continuous-time switched systems with three subsystems

$$\begin{cases} {}^C D_t^{0.8} \mathbf{x}_k(t) = \mathbf{A}_{\sigma(t)} \mathbf{x}_k(t) + \mathbf{B}_{\sigma(t)} \mathbf{u}_k(t) + \boldsymbol{\xi}_k(t) \\ \mathbf{y}_k(t) = \mathbf{C}_{\sigma(t)} \mathbf{x}_k(t) + \boldsymbol{\eta}_k(t) \end{cases}. \quad (30)$$

We use a random variable with the values 1, 2, and 3 to produce the switching sequence $\sigma(t)$, as shown in Fig. 1. When $\sigma(t) = i$ ($i = 1, 2, 3$), the i th subsystem is operated.

Fig. 1 Random switching sequence of $\sigma(t)$

The parameter matrices of the subsystems are given as

$$\begin{aligned}
\mathbf{A}_1 &= \begin{bmatrix} 0.5 & 0.3 \\ 0.2 & -0.1 \end{bmatrix}, \quad \mathbf{B}_1 = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}, \\
\mathbf{C}_1 &= [0.1 \quad 1.1]; \quad \mathbf{A}_2 = \begin{bmatrix} -0.1 & 0.2 \\ 0 & -0.4 \end{bmatrix}, \\
\mathbf{B}_2 &= \begin{bmatrix} 0.1 \\ 0.12 \end{bmatrix}, \quad \mathbf{C}_2 = [-0.1 \quad 1]; \\
\mathbf{A}_3 &= \begin{bmatrix} 0.1 & 0.4 \\ -0.3 & 0.2 \end{bmatrix}, \quad \mathbf{B}_3 = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}, \\
\mathbf{C}_3 &= [-0.2 \quad 0.9].
\end{aligned}$$

Suppose that the systems (30) operate repetitively during the finite time interval $[0, 1]$, and the desired output is preset as $\mathbf{y}_d(t) = 0.8t \sin(2\pi t), t \in [0, 1]$. The initial state and the initial control input are chosen as $\mathbf{x}_k(0) =$

$\mathbf{x}_d(0) = [0, 0]^T$ ($k = 1, 2, \dots$) and $\mathbf{u}_1(t) = 1$ ($t \in [0, 1]$), respectively.

We choose the PD^α -type control law as follows:

$$\mathbf{u}_{k+1}(t) = \mathbf{u}_k(t) + 0.9\mathbf{e}_k(t) + 1.9\mathbf{e}_k^{(0.8)}(t). \quad (31)$$

Here we obtain

$$\|\mathbf{I} - \Gamma_d \mathbf{C}_1 \mathbf{B}_1\| + \beta_1 = 0.8028 < 1,$$

$$\|\mathbf{I} - \Gamma_d \mathbf{C}_2 \mathbf{B}_2\| + \beta_2 = 0.7927 < 1,$$

and

$$\|\mathbf{I} - \Gamma_d \mathbf{C}_3 \mathbf{B}_3\| + \beta_3 = 0.8429 < 1.$$

Obviously, the convergence conditions in Theorems 1 and 2 are satisfied.

Convergence When $\xi_k(t) = [0, 0]^T$ and $\eta_k(t) = 0$, that is, the control law (31) is applied to the switched systems (30) without any external noise, the outputs of the systems (30) at the 8th and 22th iterations are shown in Fig. 2. It can be seen from the tracking curve that, further iterations are performed, the output of the systems can exactly track the desired output. Fig. 3 illustrates the curve of the tracking error from the first iteration to the thirtieth iteration. It is evident that the tracking error monotonically converges to zero as the iteration number increases. These results show that the control law (31) is feasible and effective for the switched systems (30) without any external noise. Fig. 4 makes a comparison of the tracking errors made by $\alpha = \{0.2, 0.5, 0.8, 1\}$.

Fig. 2 Tracking effect when external noises are absent

Fig. 3 Tracking error when external noises are absent

Fig. 4 Tracking error for $\alpha = \{0.2, 0.5, 0.8, 1\}$

Compared with $\alpha = 1$, FOILC has the faster convergence speed.

Robustness The state disturbance vector $\xi_k(t)$ and the measurement disturbance vector $\eta_k(t)$ are chosen, respectively, as $\xi_k(t) = \begin{bmatrix} 0.08 \cos(1000k\pi ft) \\ 0.08 \sin(1000k\pi ft) \end{bmatrix}$ and $\eta_k(t) = 0.05 \sin(100k\pi ft)$, with $f = 100$ Hz. When the control law (31) is applied to the switched systems (30), the outputs of the systems (30) at the eighth and twentieth iterations are exhibited in Fig. 5 and the tracking error is shown in Fig. 6. Obviously, the output of the systems asymptotically falls into a small neighborhood of the desired output and the tracking error is uniform boundedness during the whole interval $[0, 1]$ as the iteration number increases.

Fig. 5 Tracking effect with disturbance of bounded noises

Fig. 6 Tracking error with disturbance of bounded noises

5. Conclusions

In the sense of the L^p norm, the performance of the PD^α -type FOILC algorithm is discussed for a class of fractional-order linear continuous-time switched systems with an arbitrary switching sequence. The results indicate that, under appropriate conditions, the PD^α -type algorithm can guarantee that the tracking error is uniform boundedness during the whole interval when bounded external noises are present, and the tracking error monotonically converges to zero when bounded external noises are absent. The method proposed in this paper can be used to further analyze the performance of FOILC algorithms for fractional-order nonlinear switched systems.

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