Journal of Systems Engineering and Electronics
Vol. 30, No. 4, August 2019, pp.792 – 798

EXCLUSIVELY AVAILABLE VIA
IEEE Xplore®
DIGITAL LIBRARY

Stabilization of switched systems with all unstable modes: application to the aircraft team problem

TAVAKOLIFAR Dariush[1], KHALOOZADEH Hamid[2,*], and AMJADIFARD Roya[3]

1. Department of Electrical Engineering, Science and Research Branch, Islamic Azad University, Tehran 1477893855, Iran;
2. Department of Systems and Control, K. N. Toosi University of Technology, Tehran 1969764499, Iran;
3. Department of Electrical & Computer Engineering, Kharazmi University, Tehran 1571914911, Iran

Abstract: The method of stabilizing switched systems based on the optimal control is applied, with all modes unstable, for a typical example of the multi-agent system. First, the optimal control method for stabilizing switched systems is introduced. For this purpose, a switching table rule procedure is constructed. This procedure is inspired by the optimal control that identifies the optimal trajectory for the switched systems. In the next step, the stability of a multi-agent system is studied, considering different unstable connection topologies. Finally, the optimal control method is successfully applied to an aircraft team, as an example of the multi-agent systems. Simulation results evaluate and confirm the successful application of this method in the aircraft team example.

Keywords: switched system, multi-agent system, stabilization, optimal control, unstable modes, aircraft team.

DOI: 10.21629/JSEE.2019.04.16

1. Introduction

A switched system is a hybrid system including discrete/continuous-time subsystems and a rule that supervises the switching between subsystems. Due to their wide applications in many areas, such as robot control, aircraft control, and networked control systems [1,2], switched systems have drawn an increasing attention in the past few decades. The design of switching laws plays a key role in the study of switched systems. It is noted that the switching laws can be classified into two categories: constrained switchings and arbitrary switchings [2]. Useful results on stabilization of switched systems were presented in [3 – 6]. These results can be divided into two main stability methods: the multiple Lyapunov functions (MLF) method [3,6] and the (average) dwell-time method [4,5]. In [7], the finite-time stabilization for a class of switched systems with the constrained state was studied via the MLF method.

In reality, due to the growing popularity of the digital technology, the sampled-data control technology has been considered in switching systems [8 – 11]. The global stabilization problem via sampled-data control for a class of switched systems was used in [8].

Many researchers have devoted to switched systems with unstable modes [12 – 15].

One of the most important applications of switched systems with unstable modes is multi-agent systems. Multi-agent systems are of great importance for many applications. In [13], multi-agent systems were investigated and a method for the stabilization of such systems was presented. An important motivation is to research multi-agent systems where each topology is unstable in itself. Therefore, the switching operation between these topologies can be viewed as a switched system with unstable subsystems [16 – 18]. The aircraft team is an example of multi-agent systems with a leader-following structure and a switching topology, where each following agent may run far away from the target without cooperation [15].

On the other hand, it is noted that the optimal control for switched systems has been widely studied in the recent years [19 – 22]. In [22], the only decision variables were the switching times. In [19], this optimization problem, by taking both the switching times and the switching sequence as decision variables, was generalized. The approach in [19] was still based on the construction of switching tables.

In [21], a feedback control rule that minimized a given quadratic cost was calculated in the case that the number of permissible switches approached to infinity and only one dynamic mode remained stable. Subsequently, this method is efficiently employed under the condition that all LTI dynamics are unstable, by readily solving a proper optimal control rule called the augmented optimal control problem that comprises a "dummy" stable dynamics.

Manuscript received May 21, 2018.
*Corresponding author.

This paper is based on the results presented in [21] and the method is applied to the aircraft team problem. The main contributions are summarized as follows.

(i) We provide a stabilizing switching law for the aircraft team problem, which is an example of a class of multi-agent systems in the leader-following structure through switching between two topologies and each topology is unstable in itself.

(ii) We compute a proper switching law, such that the problem is asymptotically stable with the optimal time cost for the aircraft team problem.

Through computing the suitable switching law, the time to achieve stability is reduced and therefore, the transmitted information between the aircraft is reduced. From a practical point of view, the reduction of transmitted information between the aircraft is very significant.

(iii) The method is successfully applied to the aircraft team problem with an optimal switching law and we show that this method leads to a better result compared to that in [15].

The rest of the paper is organized as follows. In Section 2, we provide some preliminaries and the problem formulation. Section 3 is devoted to the main results, which are illustrated by an example of multi-agent system. This section is followed by a conclusion in Section 4.

2. Stabilization of switched system

2.1 Basic definitions

Definition 1 [23] Autonomous switching hybrid systems (ASHS) are referred to systems that switch the signal from the preset space. These systems are represented by

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_{i(t)}\boldsymbol{x}(t),$$

$$i(t) \in \mathcal{S} = \{1, 2, \ldots, s\} \tag{1}$$

where $\boldsymbol{A}_i$ is a matrix $i(t)$ is a switching signal, $\boldsymbol{x} \in \mathbf{R}^n$ is the state.

Definition 2 [23] Controlled switching hybrid systems (CSHS) are systems in which the switching signal is determined by an external agent. These systems are built into the system by adding a continuous external input to the system (1) as follows:

$$\dot{\boldsymbol{x}}(t) = f_{i(t)}(\boldsymbol{x}(t), \boldsymbol{u}(t)). \tag{2}$$

Definition 3 (Equilibrium point) [24] For a switching system, the origin is an equilibrium point whenever $\forall t \geqslant 0, f(0, t) = 0$.

Definition 4 (Stability of equilibrium point) [24] $\boldsymbol{x} = 0$ in $\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_{i(t)}\boldsymbol{x}(t)$ is

(i) a stable equilibrium point if for $\varepsilon > 0$ there exists $\delta = \delta(\varepsilon, t_0) > 0$ such that

$$\|\boldsymbol{x}(t_0)\| < \delta \Rightarrow \|\boldsymbol{x}(t)\| < \varepsilon$$

$$\forall t \geqslant t_0 \geqslant 0;$$

(ii) an unstable equilibrium point if it is not stable;

(iii) an asymptotically stable equilibrium point if it is stable and there exists positive $\delta = \delta(t_0)$ such that for all $\boldsymbol{x}(t_0) < \delta$, we have $\lim\limits_{t\to\infty} \boldsymbol{x}(t) \to 0$;

(iv) an exponentially stable equilibrium point if there exist positive values δ, K and λ such that for all $\boldsymbol{x}(t_0) < \delta$ the following is satisfied:

$$\|\boldsymbol{x}(t)\| \leqslant K\|\boldsymbol{x}(t_0)\|\mathrm{e}^{-\lambda(t-t_0)}.$$

If the system, for any initial value $\boldsymbol{x}(t_0)$, is asymptotically (or exponentially) stable, then the equilibrium point will be asymptotically (or exponentially) stable. Therefore, by establishing a switching rule $i(\boldsymbol{x}, t)$, the switching system can be universally stabilized. It should be noted that if there is at least one stable subsystem, then the switching system has the capability of being exponentially stable.

Definition 5 (Dummy subsystem) [21] Consider the switching system $\{\boldsymbol{A}_i\}_{i\in\mathcal{S}}$. We define the dummy system as a switching system $\{\boldsymbol{A}_i\}_{i\in\widetilde{\mathcal{S}}}$ so that

(i) $|\widetilde{\mathcal{S}}| = |\mathcal{S}| + 1$;

(ii) The system $\{\boldsymbol{A}_i\}_{i\in\widetilde{\mathcal{S}}}$ contains all subsystems $\{\boldsymbol{A}_i\}_{i\in\mathcal{S}}$;

(iii) The system $\{\boldsymbol{A}_i\}_{i\in\widetilde{\mathcal{S}}}$ contains a stable subsystem $\boldsymbol{A}_{s+1} \in \{\boldsymbol{A}_i\}_{i\in\widetilde{\mathcal{S}}}$.

2.2 Switched systems

We show how to design a stabilizing law for switched systems (1), as mentioned in Definition 1, where $\boldsymbol{A}_i$ is unstable.

The suggested stabilizing method is based on the solution of the following optimal control problem. Therefore, the system performance index is considered as follows [21]:

$$J_N^* \triangleq \min_{I,\tau} F(I, \mathcal{T}) \triangleq \int_0^\infty \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Q}_{i(t)}\boldsymbol{x}(t)\mathrm{d}t$$
$$\text{s.t. } \dot{\boldsymbol{x}}(t) = \boldsymbol{A}_{i(t)}\boldsymbol{x}(t)$$
$$\boldsymbol{x}(0) = \boldsymbol{x}_0$$
$$i(t) = i_k \ \text{ for } \ \tau_k \leqslant t < \tau_{k+1}$$
$$k = 0, \ldots, N$$
$$i_{k+1} \in \mathrm{succ}(i_k) \quad k = 0, \ldots, N \tag{3}$$

where N represents the maximum number of allowed switches, which is finite; $\boldsymbol{Q}_i$ denotes positive definite weighting matrices; $\boldsymbol{x}_0$ is the initial known continuous state. Moreover, $\mathcal{T} \triangleq \{\mathcal{T}_1, \ldots, \mathcal{T}_N\}$ and $I \triangleq \{i_0, \ldots, i_N\}$

represent the set of switching times and the sequence of indices associated with discrete modes, respectively.

Corona et al. [21] showed that a feedback control law that minimized a given quadratic cost could also be calculated in the case where the only dynamic mode is stable. Moreover, they showed that the optimal control law for the optimization problem would be in the form of a state-feedback. In other words, it would be sufficient to consider the current system state so as to determine if a switch from linear dynamics $\boldsymbol{A}_{i_{k-1}}$ to $\boldsymbol{A}_i$, should take place.

In more detail, for a given mode $i \in \mathcal{S}$, when k switches are still available, it is conceivable to make a partition $\mathcal{C}_k^i$ of the state space $\mathbf{R}^n$ into s regions. The partition $\mathcal{C}_k^i$ is called a table. At any time $i_{N-k} = i$, they utilize table $\mathcal{C}_k^i$ to determine if a switch should take place. Once the continuous state $\boldsymbol{x}$ reaches a point in the region $\mathcal{R}_j$ for a certain $i \in \mathcal{S}\backslash\{i\}$, they will switch to the mode $i_{N-k-1} = j$. No switch will take place if the continuous system's state $\boldsymbol{x}$ falls in $\mathcal{R}_i$.

By presenting the virtual stable subsystem, it is assumed that at least one stable subsystem for applying the switching table method is satisfied. Then it is shown that by making use of this stable subsystem and solving the optimal control problem, we can calculate a switching law for a switching system with unstable subsystems.

In general, this method is based on the addition of a virtual subsystem, as mentioned in Definition 5.

In order to prove the stability of the switching systems with unstable subsystems, we need to provide some theorems in this direction, which is described in the following section.

Theorem 1 [21] Consider the optimal control problem (3) with the subsystems $\{\boldsymbol{A}_i\}_{i\in\widetilde{\mathcal{S}}}$. If the switching table $\widetilde{\mathcal{C}}_N$ only contains the subsystem areas $\{\boldsymbol{A}_i\}_{i\in\mathcal{S}}$, for each initial condition x_0, the optimal control rule obtained from solving the optimal virtual control $\widetilde{\mathcal{S}}$ will also be optimal for the optimal control problem with $\mathcal{S} \subset \widetilde{\mathcal{S}}$ subsystems. In other words, we consider the unstable switching system with a stable virtual subsystem and solve the problem of the optimal control. Finally, if a stable subsystem area does not appear in the switching table, the switching rule obtained for the original system will also be optimized.

Proof To prove this theorem, we need to use the basic definitions of the switching table $\mathcal{C}_N$. When the area of $\boldsymbol{A}_i$ does not appear in the $\mathcal{C}_N$ switching table, it means that switching to this subsystem does not have a lower cost, and even if the subsystem $\boldsymbol{A}_i$ is considered as the primary subsystem, it will be still impossible to activate this subsystem, and the system will switch to another subsystem. □

Theorem 2 [21] Consider $J_N^*(\boldsymbol{x}_0, i_0)$ as an optimal cost and response to the optimal control problem $OP(\mathcal{S})$ of form (3), which is described in (3) (with initial conditions $(\boldsymbol{x}_0, i_0)$). Now consider $\widetilde{J}_N^*(\boldsymbol{x}_0, i_0)$ as the optimal cost and response to the virtual optimal control problem $OP(\widetilde{\mathcal{S}})$ with the same initial conditions. The optimal cost $\widetilde{J}_N^*(\boldsymbol{x}_0, i_0, q)$ is a strictly increasing function in terms of q.

Proof The proof of this theorem is by contradiction. We consider two virtual optimal control problems $OP(\widetilde{\mathcal{S}})'$ and $OP(\widetilde{\mathcal{S}})''$ with two different values of q. In this regard, we consider q' and q'' as two coefficients for q ($q'' < q'$). Now assume that $\widetilde{J}_N^*(\boldsymbol{x}_0, q') = \widetilde{J}_N^*(\boldsymbol{x}_0, q'')$. Consider the optimal response for $OP(\widetilde{\mathcal{S}})'$ and obtain the cost by using $OP(\widetilde{\mathcal{S}})''$ weights. The cost is less than the value of $\widetilde{J}_N^*(\boldsymbol{x}_0, q'')$, which is contradictory, because based on the definition of the cost of the solution, $OP(\widetilde{\mathcal{S}})''$ will be the lowest. □

Theorem 3 (The stability of the switching system with unstable subsystems) [21] Consider the switching system $\{\boldsymbol{A}_i\}_{i\in\mathcal{S}}$ and the optimal control problem $OP(\mathcal{S})$ with $\boldsymbol{Q}_i > 0$. Moreover, consider the virtual system $\{\boldsymbol{A}_i\}_{i\in\widetilde{\mathcal{S}}}$ and the virtual optimal control $OP(\widetilde{\mathcal{S}})$ with $\widetilde{\boldsymbol{Q}}_i > 0$.

(i) The switching system $\{\boldsymbol{A}_i\}_{i\in\mathcal{S}}$ has the capability of asymptotic stability if there exists $\widetilde{q} \in \mathbf{R}^+$ such that the switching table $\mathcal{C}_k^i$ does not include the region associated with the stable subsystem $\widetilde{A}$.

(ii) If the switching system $\{\boldsymbol{A}_i\}_{i\in\mathcal{S}}$ has the capabi lity of the asymptotic stability function, then there exists $\widetilde{q} \in \mathbf{R}^+$ such that the switching table $\mathcal{C}_k^i$ does not include the region associated with the stable subsystem $\widetilde{A}$.

Proof [18] By proving Theorem 1, it is possible to use a switching table to stabilize the switching systems with unstable subsystems.

By proving Theorem 2, if the switching system $\{\boldsymbol{A}_i\}_{i\in\mathcal{S}}$, with unstable subsystems, has the capacity to stabilize, it is always possible to calculate a stabilizing switching rule through the switching table. This switching rule not only stabilizes the system (which in itself is the main goal), it also measures the square performance indicator.

Therefore, the process of computing the stabilizing switching law for the switching system, in which all the subsystems are unstable, are summarized in four stages listed as follows:

(i) Consider an optimal $OP(\mathcal{S})$ control problem, with an N-valued number for the switching system.

(ii) Create a virtual system $\{\boldsymbol{A}_i\}_{i\in\widetilde{\mathcal{S}}}$, and the new optimal control problem $OP(\widetilde{\mathcal{S}})$, where q is a large positive value. There is no limit to the choice of a stable subsystem, and we can consider each stable matrix as a stable subsystem.

It should be noted that in this virtual system, new values of $\boldsymbol{Q}$ will be considered to solve the optimal control problem, so that for all subsystems $i = 1, \dots, s$ we use the same $\boldsymbol{Q}_i$. However, for the added subsystem we use the value of $\boldsymbol{Q}_{s+1}$, where

$$\boldsymbol{Q}_{s+1} = q\widetilde{\boldsymbol{Q}}, \quad q \in \mathbf{R}^+, \text{ and } \widetilde{\boldsymbol{Q}} > 0.$$

In general, there is no analytical method for obtaining the appropriate q. In the examples, $q = 10^5$ is considered.

(iii) Obtain the switching table $\mathcal{C}_k^i$.

(iv) If the resulting table does not include the area associated with the stable subsystem $\boldsymbol{A}_{s+1}$, then using Theorem 3, it can be concluded that the original system $\{\boldsymbol{A}_i\}_{i\in\mathcal{S}}$ has the global asymptotic stability. In this case, by using the table $\mathcal{C}_k^i$, we will compute the stabilizing feedback control rule that will minimize the chosen quadratic performance index. □

2.3 Multi-agent systems

The cooperative system is usually recognized as a multi-agent system. Agents can communicate with one another through a sparse communication network. In this paper, we model the communication topology among agents by using a weighted directed graph $\mathcal{G} = (\mathbb{N}, \mathcal{E})$, in which $\mathbb{N} = \{0, 1, 2, \dots, n\}$ is the set of agents, $\mathcal{E}$ is the set of arcs, and $(j, i) \in \mathcal{E}$ is an arc from agent j to agent i. Moreover, for a directed graph, if a node i is a neighbor of node j, then node j can get information from the node i and not necessarily vice versa. The agents' dynamics are defined by

$$\dot{\boldsymbol{x}}_i = g_i(\boldsymbol{x}_i) + \underbrace{\sum_{j\neq i} a_{ij}(t)(\boldsymbol{x}_j - \boldsymbol{x}_i)}_{u_i^c}, \quad i \in \mathbb{N} \tag{4}$$

where u_i^c is the cooperative agent law i, $a_{ij}(t)$ represents the arc weight between agents i and j, and g_i represents a smooth function [15]. Moreover, $a_{ij}(t) = 0$ if (j, i) does not exist. Given a fixed target point, ξ, for every $i \in \mathbb{N}$ we define

$$\boldsymbol{V}_i = (\boldsymbol{x}_i - \boldsymbol{\xi})^{\mathrm{T}} \boldsymbol{P} (\boldsymbol{x}_i - \boldsymbol{\xi}) \tag{5}$$

where $\boldsymbol{P}$ is a symmetric positive definite matrix. Moreover, V_i measures the distance between the agent i and the target point, and ξ is a constant vector.

Assumption 1 There exist constant parameters $\lambda_0, \lambda_1 > 0$ such that

$$u_0^c = 0 \Rightarrow \dot{V}_0 \leqslant -\lambda_0 V_0,$$
$$u_i^c = 0 \Rightarrow \dot{V}_i \leqslant \lambda_1 V_i, \quad i \in \mathbb{N} - \{0\}. \tag{6}$$

It indicates that agent 0 is the leader, which has the target point's information, and can reach the target by itself, other agents are named the followers.

Theorem 4 [13]

Let a multi-agent system (4) satisfy Assumption 1. Then $\forall i \in \mathbb{N} = \{0, 1, 2, \dots, n\}$, such that $\lim\limits_{t\to\infty} \boldsymbol{x}_i(t) \to \boldsymbol{\xi}$, there exist cooperative laws u_i^c where at most k arcs can be activated simultaneously, where $k < n$ is a given positive integer.

Proof The design of u_i^c contains two factors: the topology and the arc weights. Since $km = n$, m connection topologies can be built as the following rule and switch among them.

Rule of building the nth topology.

(i) Pick agents $0, (i-1)k + 1, \dots, ik$

(ii) Set arcs $(0, (i-1)k + 1)$ and $(j, j+1)$, $\forall j = (i-1)k + 1, \dots, ik - 1$. □

Each topology can be seen a chain structure, i.e., each following agent has a unique neighbor. Agent 0 is involved in each topology and has no neighbor.

Finally, the multi-agent system can be considered as a switched system where each unstable mode corresponds to one of the topologies.

3. Practical example: an aircraft team

To verify the effectiveness of the proposed stabilizing method, we simulate a team of aircraft including three agents as a multiple system, as shown in Fig. 1.

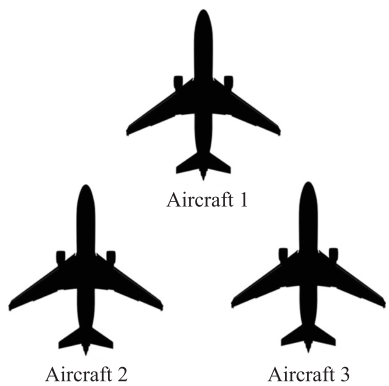


Fig. 1 Aircraft team

Therefore, it is desirable that the pitch of these three aircrafts reaches a certain amount and remains the same. The state space equations of pitch rates under suitable controllers are shown as follows [15]:

$$\dot{w}_1 = -a_1(w_1 - w^*)$$
$$\dot{w}_i = \underbrace{a_i(w_i - w^*) + b_i(w_1 - w_i)}_{u_i^c}, \quad i = 2, 3 \tag{7}$$

where w^* indicates the target pitch rate, a_1, a_2, a_3, b_2 and b_3 are positive coefficients.

Equation (7) indicates that the first aircraft is the leader and has the information necessary to reach w^*, while the two other aircraft will follow the leader and in the event of a disconnection with the leader, the aircraft 2 and aircraft 3 will not be able to reach w^* and will go towards instability. It should be noted that it is assumed that the aircraft cannot simultaneously transfer information to both aircraft. Moreover, aircraft 2 and aircraft 3 are not able to transmit information to one another, and they can only obtain information from the leader.

In this case, two topologies are used, so that a topology establishes the connection between agent 1 and agent 2, and the other topology establishes the connection between agent 1 and agent 3. The topologies will be shown as follows.

$$\begin{aligned}&\text{Topology 1}: \begin{cases}\dot{w}_1 = -a_1(w_1 - w^*)\\ \dot{w}_2 = a_2(w_2 - w^*) + b_2(w_1 - w_2)\\ \dot{w}_3 = a_3(w_3 - w^*)\end{cases}\\ &\text{Topology 2}: \begin{cases}\dot{w}_1 = -a_1(w_1 - w^*)\\ \dot{w}_2 = a_2(w_2 - w^*)\\ \dot{w}_i = a_3(w_3 - w^*) + b_3(w_1 - w_3)\end{cases}\end{aligned} \tag{8}$$

In the first topology, u_3^c is kept zero, while in the second topology, u_2^c is kept zero. Consequently, by replacing constant values and considering the topologies as the subsystems of switching, we will have the following equations.

$$\begin{aligned}&\text{Topology 1}: \dot{\boldsymbol{w}}(t) = \begin{bmatrix}-5 & 0 & 0\\ 10 & -9 & 0\\ 0 & 0 & 1\end{bmatrix}\boldsymbol{w}(t) + \begin{bmatrix}0.5\\ -0.1\\ -0.1\end{bmatrix}\\ &\text{Topology 2}: \dot{\boldsymbol{w}}(t) = \begin{bmatrix}-5 & 0 & 0\\ 0 & 1 & 0\\ 10 & 0 & -9\end{bmatrix}\boldsymbol{w}(t) + \begin{bmatrix}0.5\\ -0.1\\ -0.1\end{bmatrix}\end{aligned} \tag{9}$$

Define $\Omega_i = (w_i - w^*)^2$. The time derivatives under topology 1 and topology 2 are obtained as follows.

$$\begin{aligned}&\dot{\Omega}_1 = 2(w_1 - w^*)w_1 = \\ &2(w_1 - w^*)[-a_1(w_1 - w^*)] = \\ &-2a_1(w_1 - w^*)^2 = -2a_1\Omega_1\end{aligned} \tag{10}$$

$$\begin{aligned}&\dot{\Omega}_2 = 2(w_2 - w^*)\dot{w}_2 = \\ &2(w_2 - w^*)[a_2(w_2 - w^*) + b_2(w_1 - w_2)] = \\ &2a_2(w_2 - w^*)^2 + \underbrace{2b_2(w_2 - w^*)(w_1 - w_2)}_{2b_2(w_2 - w^*)(w_1 - w^* + w^* - w_2)}\end{aligned} \tag{11}$$

According to the Young inequality $2ab \leqslant a^2 + b^2$, we have

$$\begin{aligned}&2b_2(w_2 - w^*)(w_1 - w^*) \leqslant \\ &b_2(w_2 - w^*)^2 + b_2(w_1 - w^*)^2.\end{aligned}$$

Substituting the above equation into (11) results in

$$\begin{aligned}&\dot{\Omega}_2 \leqslant 2a_2(w_2 - w^*)^2 - 2b_2(w_2 - w^*)^2 + \\ &b_2(w_2 - w^*)^2 + b_2(w_1 - w^*)^2 \leqslant \\ &(2a_2 - b_2)(w_2 - w^*)^2 + b_2(w_1 - w^*)^2 \leqslant \\ &(2a_2 - b_2)\Omega_2 + b_2\Omega_1.\end{aligned} \tag{12}$$

Finally, the simpler form of the two topologies will be in the form of (10) and (11) shown as follows.

$$\begin{aligned}&\text{Topology 1}: \begin{cases}\dot{\Omega}_1 = -2a_1\Omega_1\\ \dot{\Omega}_2 \leqslant (2a_2 - b_2)\Omega_2 + b_2\Omega_1\\ \dot{\Omega}_3 = 2a_3\Omega_3\end{cases}\\ &\text{Topology 2}: \begin{cases}\dot{\Omega}_1 = -2a_1\Omega_1\\ \dot{\Omega}_2 = 2a_2\Omega_2\\ \dot{\Omega}_3 \leqslant (2a_3 - b_3)\Omega_3 + b_3\Omega_1\end{cases}\end{aligned} \tag{13}$$

In the first topology, we ensures that w_1 and w_2 tend to w^*, but w_3 may run far away. Moreover, the second topology guarantees that w_1 and w_3 reach w^*, while w_2 may escape. It should be noted that fixed values must be chosen so that the two conditions $2a_3 < b_3$ and $2a_2 < b_2$ are established. In [15], by obtaining a periodic switching rule between these two topologies, the pitch rate of each of the three aircraft has stabilized to a value of w^*. The switching rule is shown as follows.

$$\text{Topology 1}: \ \forall t \in [0.9k, 0.9k + 0.4]$$

$$\text{Topology 2}: \ \forall t \in [0.9k + 0.4, 0.9k + 0.9]$$

The simulation results of the system are presented in Fig. 2. In this simulation, we select $b_2 = b_3 = 10, a_1 = 5$ and $a_2 = a_3 = 1$ as constant values and $w^* = 0.1$ rad/s as the target pitch rate and $w_1(0) = 0.3$ rad/s, $w_2(0) = 0$ and $w_3(0) = -0.2$ rad/s is considered as the initial condition of states.

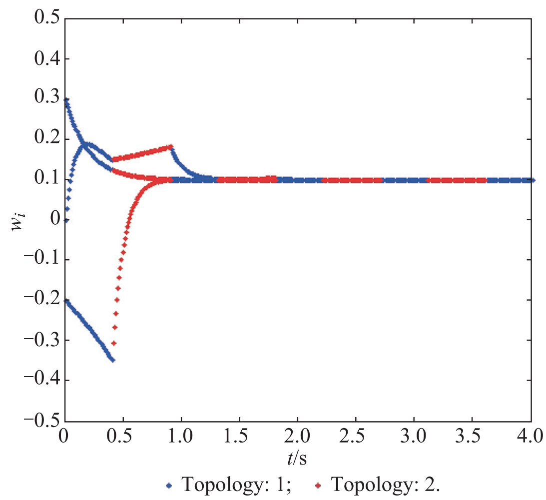


Fig. 2 Pitch rate of three aircraft with a periodic switch between two topologies

Now we can consider a multiple system as a switching system, so that each of the topologies represent an unstable subsystem. As a result, the subsystems of the switching system are obtained by using the topologies (13) as follows.

$$\text{Topology 1}: \boldsymbol{A}_1 = \begin{bmatrix} -2a_1 & 0 & 0 \\ b_2 & (2a_2 - b_2) & 0 \\ 0 & 0 & 2a_3 \end{bmatrix}$$
$$\text{Topology 2}: \boldsymbol{A}_2 = \begin{bmatrix} -2a_1 & 0 & 0 \\ 0 & 2a_2 & 0 \\ b_3 & 0 & (2a_3 - b_3) \end{bmatrix} \tag{14}$$

Using the values given in [15], $b_2 = b_3 = 10, a_1 = 5, a_2 = a_3 = 1$ the subsystems are obtained as follows:

$$\boldsymbol{A}_1 = \begin{bmatrix} -10 & 0 & 0 \\ 10 & -8 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \quad \boldsymbol{A}_2 = \begin{bmatrix} -10 & 0 & 0 \\ 0 & 2 & 0 \\ 10 & 0 & -8 \end{bmatrix}. \tag{15}$$

Since both subsystems are lower triangular, the eigenvalues of the subsystems will be the same as that of the main matrix, which in both matrices are equal to -10, -8, and 2. As a result, both topologies are unstable.

By introducing a stable $\boldsymbol{A}_3$ subsystem and adding it to other subsystems, a virtual switching system is formed. It should be noted that there is no limit to the choice of a stable subsystem, and we can consider each sustainability matrix as a stable subsystem.

$$\boldsymbol{A}_3 = \begin{bmatrix} -10 & 0 & 0 \\ -10 & -8 & 0 \\ 0 & 0 & -2 \end{bmatrix} \tag{16}$$

This subsystem with the eigenvalues of -10, -8, and -2 is stable. The matrices $\boldsymbol{Q}_i$ are considered as follows:

$$\boldsymbol{Q}_1 = \boldsymbol{Q}_2 = \boldsymbol{I}_2, \quad \boldsymbol{Q}_3 = 10^5 \boldsymbol{Q}_1. \tag{17}$$

Then the switching table $\mathcal{C}_k^i$ is calculated.

Finally, by using the result of the switching table, a stable switching rule has been obtained for the switching system. Given the initial state $\boldsymbol{x}_0 = [1 \ \ 1 \ \ 1]$, the key sequence and final cost of the switching system are obtained as follows:

$$I^* = \{1 \ \ 2 \ \ 1 \ \ 2 \ \ 1 \ \ 2 \ \ 1 \cdots\}$$
$$\mathcal{T}^* = \{0.04 \ \ 0.06 \ \ 0.14 \ \ 0.05 \ \ 0.06 \ \ 0.05 \ \ 0.1 \cdots\}$$
$$J_N^* = 0.037\,2.$$

The switching system using the switching law obtained from the switch table is shown in Fig. 3. It is observed that $\Omega_i = (w_i - w^*)^2$ in three aircraft converges to the origin. Since $\Omega_i = (w_i - w^*)^2$ and $w^* = 0.1$, we conclude that the pitch rate of all three aircraft w_i converges to the optimum 0.1.

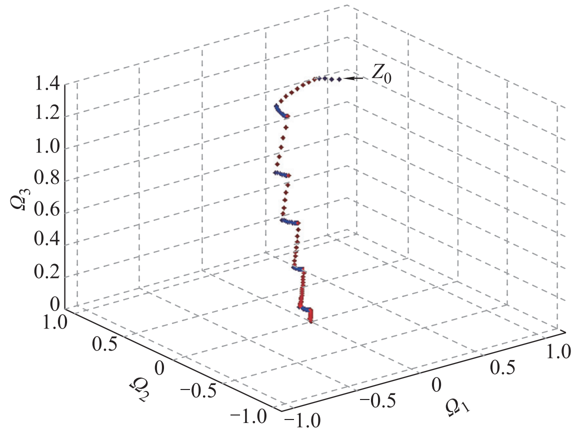


Fig. 3 Optimal path of the switching system for aircrafts

As shown in Fig. 3, the stable subsystem $\boldsymbol{A}_3$ does not appear and only two unstable subsystems have been used to stabilize the switching system. Consequently, using Theorem 3, we can conclude that the original system $\{\boldsymbol{A}_i\}_{i\in\mathcal{S}}$ has the potential for global asymptotic stability.

Now, in order to compare the performance of the optimal control method in unstable switching systems using the method presented in [15], again, the stable switching rule for the aircraft problem considering the simplified form of the topologies, with the initial conditions is calculated ($\boldsymbol{x}_0 = [0.3 \quad 0 \quad -0.2]$). The state of the switching system with the optimal control strategy is shown in Fig. 4. Comparing the results of Fig. 4 with Fig. 2, we conclude that the pitch rates of all three aircraft converge to the final value within 1.5 s in the periodic method obtained in Fig. 2, while as shown in Fig. 4. the complete convergence rate of all three pitch rates is obtained within 0.6 s. As a result, the proposed method has a better performance compared to the method presented in [15].

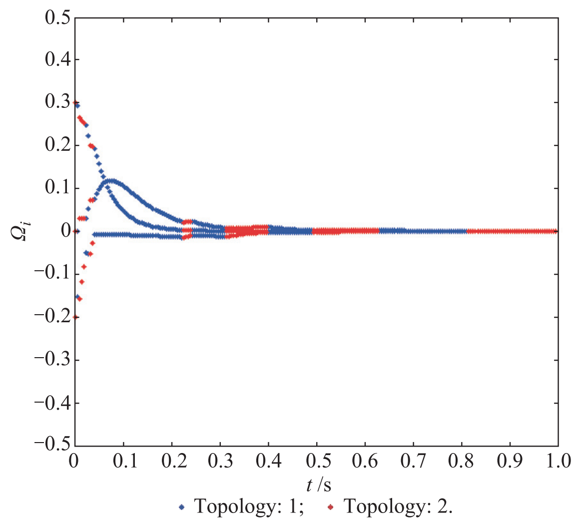


Fig. 4 States of all aircraft by using optimal control method between two unstable topologies

4. Conclusions

We apply the optimal control of switched systems with all unstable modes for an aircraft team problem as a class of multi-agent systems. This method is successfully applied to the problem with an optimal switching law. According to the reduced time for stability, the transmitted information between the aircraft is reduced and it is important in practical situations. Future works can consider a systematic criterion to determine the weighting matrix associated to the virtual stable dynamics.

References

[1] LIBERZON D, MORSE A S. Basic problems in stability and design of switched systems. IEEE Control Systems, 1999, 19(5): 59 – 70.

[2] LIBERZON D. Switching in systems and control. Boston: Birkhäuser, 2003.

[3] BRANICKY M S. Multiple Lyapunov functions and other analysis tools for switched and hybrid systems. IEEE Trans. on Automatic Control, 1998, 43(4): 475 – 482.

[4] HESPANHA J P, MORSE A S. Stability of switched systems with average dwell-time. Proc. of the 38th IEEE Conference on Decision and Control, 1999: 2655 – 2660.

[5] MORSE A S. Supervisory control of families of linear set-point controllers-part I: exact matching. IEEE Trans. on Automatic Control, 1996, 41(10): 1413 – 1431.

[6] YE H, MICHEL A N, HOU L. Stability theory for hybrid dynamical systems. IEEE Trans. on Automatic Control, 1998, 43(4): 461 – 474.

[7] HUANG S, XIANG Z. Finite-time stabilisation of a class of switched nonlinear systems with state constraints. International Journal of Control, 2018, 91(6): 1300 – 1313.

[8] LI S, GUO J, XIANG Z. Global stabilization of a class of switched nonlinear systems under sampled-data control. IEEE Trans. on Systems, Man, and Cybernetics: Systems, 2018, 2381 – 2385.

[9] LI S, GUO J, XIANG Z. Sampled-data adaptive prescribed performance control of a class of nonlinear systems. Neurocomputing, 2018, 283: 282 – 292.

[10] LIAN J, LI C, XIA B. Sampled-data control of switched linear systems with application to an F-18 aircraft. IEEE Trans. on Industrial Electronics, 2017, 64(2): 1332 – 1340.

[11] MAO J, GUO J, XIANG Z. Sampled-data control of a class of uncertain switched nonlinear systems in nonstrict-feedback form. International Journal of Robust and Nonlinear Control, 2018, 28(3): 918 – 939.

[12] XIE G, WANG L. Periodic stabilizability of switched linear control systems. Automatica, 2009, 45(9): 2141 – 2148.

[13] YANG H, JIANG B, COCQUEMPOT V. Stabilization of switched nonlinear systems with unstable modes. Berlin: Springer International Publishing, 2014.

[14] YANG H, JIANG B, COCQUEMPOT V. A survey of results and perspectives on stabilization of switched nonlinear systems with unstable modes. Nonlinear Analysis: Hybrid Systems, 2014, 13(10 – 11): 45 – 60.

[15] YANG H, YANG B, ZHAO Z L, et al. Stabilization of switched nonlinear systems with all unstable modes: application to multi-agent systems. IEEE Trans. on Automatic Control, 2011, 56(9): 2230 – 2235.

[16] OLFATI-SABER R, MURRAY R M. Consensus problems in networks of agents with switching topology and time-delays. IEEE Trans. on Automatic Control, 2004, 49(9): 1520 – 1533.

[17] SHI G, HONG Y. Global target aggregation and state agreement of nonlinear multi-agent systems with switching topologies. Automatica, 2009, 45(5): 1165 – 1175.

[18] ZHAO J, HILL D J, LIU T. Synchronization of complex dynamical networks with switching topology: a switched system point of view. Automatica, 2009, 45(11): 2502 – 2511.

[19] BEMPORAD A, GIUA A, SEATZU C. Synthesis of state-feedback optimal controllers for continuous-time switched linear systems. Proc. of the 41st IEEE Conference on Decision and Control, 2002: 3182 – 3187.

[20] CORONA D. Optimal control of linear affine hybrid automata. Cagliari, Italy: University of Cagliari, 2005.

[21] CORONA D, GIUA A, SEATZU C. Stabilization of switched systems via optimal control. Nonlinear Analysis: Hybrid Systems, 2014, 11(1): 1 – 10.

[22] GIUA A, SEATZU C, VAN DER MEE C. Optimal control of autonomous linear systems switched with a pre-assigned finite sequence. Proc. of the International Symposium on Intelligent Control, 2001: 144 – 149.

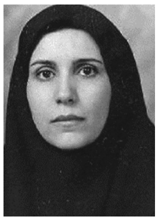
[23] DING X C. Real-time optimal control of autonomous switched systems. Atlanta, USA: Georgia Institute of Technology, 2009.

[24] KHALIL H K. Nonlinear systems. New Jewsey: Prentice Hall, 2002.

Biographies

TAVAKOLIFAR Dariush was born in 1975. He received his B.S. degree in electronics engineering from Razi University, Kermanshah, Iran, in 2000 and M.S. degree in control engineering from South Tehran Branch, Islamic Azad University (IAU), Tehran, in 2004. He is currently a Ph.D. candidate in control engineering in the Department of Electrical and Computer Engineering, Tehran Science and Research Branch, IAU, Tehran, Iran. Currently, he is an instructor in the Department of Engineering, Islamic Azad University. His current research interests include switched systems, multi-agent systems, large scale systems, and model reduction.
E-mail: dtavakolifar@gmail.com

KHALOOZADEH Hamid was born in 1965. He received his B.S. degree in control engineering from Sharif University of Technology, Tehran, Iran, in 1990, M.S. degree in control engineering from K. N. Toosi University of Technology, Tehran, in 1993, and Ph.D. degree in control engineering from Tarbiat Modarres University, Tehran, in 1998. Currently, he is a professor in the Department of Systems and Control, Faculty of Electrical Engineering, K. N. Toosi University of Technology. He is the director of the Industrial Control Center of Excellence, K. N. Toosi University of Technology. His research interests include stochastic estimation and control, system identification, optimal control, and time series analysis.
E-mail: h_khaloozadeh@kntu.ac.ir



AMJADIFARD Roya was born in 1968. She received her B.S. degree in electrical engineering from the Ferdowsi University of Mashhad, Mashhad, Iran, in 1990, M.S. degree in control engineering, K. N. Toosi University of Technology, Tehran, Iran, in 1995, and Ph.D. degree in control engineering from Tarbiat Modares University, Tehran, in 2004. Her current research interests include singularly perturbed systems control, robust control, fuzzy control, and neural networks.
E-mail: amjadifard@khu.ac.ir