SAGE-based algorithm for DOA estimation and array calibration in the presence of sensor location errors

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Abstract: The direction of arrival (DOA) estimation problem in the presence of sensor location errors is studied and an algorithm based on space alternating generalized expectation-maximization (SAGE) is presented. First, the narrowband case is considered. Based on the small perturbation assumption, this paper proposes an augmentation scheme so as to estimate DOA and perturbation parameters. The E-step and M-step of the SAGE algorithm in this case are derived. Then, the algorithm is extended to the wideband case. The wideband SAGE algorithm is derived in frequency domain by jointing all frequency bins. Simulation results show that the algorithm achieves good convergence and high parameter estimation precision.

Keywords: direction of arrival (DOA), array calibration, space alternating generalized expectation-maximization (SAGE), wideband signal.

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1. Introduction

The direction of arrival (DOA) estimation problem plays a key role in sonar, radar, communications systems and radio surveillance [1–4]. Various algorithms have been proposed for solving this problem. However, most of them rely on ideal array model. In fact, the array calibration errors are inevitable, such as sensor location errors and mutual coupling. The methods based on an ideal model may deteriorate significantly in the presence of array errors [5,6].

Various array calibration methods have been proposed with respect to array errors. They can generally be categorized into off-line [7–10] and on-line [11–15] calibration methods. The former one requires calibration sources, which is not fit to the realistic situations. The latter one is also known as self-calibration. It does not require any calibration sources. In [11], a new DOA estimation and blind calibration method was presented in the presence of mutual

coupling. In [12], a self-calibration algorithm in the presence of mutual coupling and gain/phase perturbation was presented. Both of them require multi-dimensional parameter searching. At the same time, the convergence is bad. In [15], a DOA estimation and array self-calibration method was presented to mutual coupling, gain/phase perturbation and array location errors. It cannot require multi-dimensional parameter searching, and the convergence is well. However, this algorithm is not suitable for deterministic signals.

For avoiding array errors, some researchers proposed DOA estimation techniques which were less sensitive to array errors [16–20]. In [16], a novel DOA estimation method based on multiple signal classification (MUSIC) was presented. It proposed an optimal weightings method that ignored the finite sample effects of noise. In [18], a more general analysis was introduced to achieve better performance against correlated signals. In [19], the statistical DOA estimation properties of MUSIC and estimating signal parameters via rotational invariance techniques (ESPRIT) in the presence of array errors were studied. However, the all above methods still suffer from imperfect array models. Therefore, the estimation precision of these methods is limited.

Although many DOA estimation and array calibration methods have been proposed, most of them concern only narrowband signals. Recently, a modified nominal transformation method was presented [21], which can realize wideband DOA estimation under mutual coupling. However, it requires the accurate measurement of an actual array manifold at some reference frequency and preliminary DOA estimation. In [22], a correlation domain RELAX DOA estimation method for wideband signals was presented. It can achieve accurate results in the presence of array errors by using recursion iterative matrix operators, but it cannot realize array calibration.

The rest of this paper is organized as follows. In Sec-

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tion 2, the approach for narrowband DOA estimation and array calibration with sensor location errors are described. In Section 3, the proposed method is extended to wideband cases. In Section 4, the algorithm efficiency is illustrated by simulation results. Following a discussion on the results of the experiments, our conclusions is presented in Section 5.

2. Narrowband DOA estimation and array calibration with sensor location errors

2.1 Narrowband model in the presence of sensor location errors

Suppose that the signals DOA is $\vartheta = [\vartheta_1, \dots, \vartheta_K]^T$, and the number of uniform linear array elements is M . We take the first array sensor as the reference. The array model in the presence of sensor location errors is given as follows:

$$\mathbf{x}(t) = \sum_{k=1}^K \mathbf{a}'(\vartheta_k) s_k(t) + \mathbf{v}(t) = \mathbf{A}'(\vartheta) \mathbf{s}(t) + \mathbf{v}(t) \quad (1)$$

where $\mathbf{A}'(\vartheta) = [\mathbf{a}'(\vartheta_1), \mathbf{a}'(\vartheta_2), \dots, \mathbf{a}'(\vartheta_K)]$ is the array manifold matrix, and $\mathbf{a}'(\vartheta_k) = [1, e^{j2\pi(d+\tilde{d}_1)\sin\vartheta/\lambda}, \dots, e^{j2\pi((M-1)d+\tilde{d}_{M-1})\sin\vartheta/\lambda}]^T$ is the perturbed responding vector, λ represents the signal wavelength, d is inter-element spacing, and $\tilde{d}_1, \tilde{d}_2, \dots, \tilde{d}_{M-1}$ are location errors. The observe vector is $\mathbf{x}(t) = [x_1(t), x_2(t), \dots, x_M(t)]^T$. The signal vector $\mathbf{s}(t) = [s_1(t), s_2(t), \dots, s_K(t)]^T$. $s_k(t)$ denotes the k th signal. The noise vector is $\mathbf{v}(t) = [v_1(t), v_2(t), \dots, v_M(t)]^T$. It is independent, identically complex normally distributed with zero mean and covariance matrix $\sigma^2 \mathbf{I}$, where σ is an unknown noise spectral parameter. The sensor location errors are assumed to exist along the array axes and thus do not destroy the linear geometry of the array.

In order to facilitate the DOA and perturbation parameters information extraction, we established a new array model in the presence of sensor location errors in [15],

$$\mathbf{x}(t) = \mathbf{A}(\vartheta) \mathbf{s}(t) + \mathbf{Q}(t) \tilde{\mathbf{d}} + \mathbf{v}(t) \quad (2)$$

where $\mathbf{A}(\vartheta) = [\mathbf{a}(\vartheta_1), \dots, \mathbf{a}(\vartheta_K)]$ is the array manifold matrix, and $\mathbf{a}(\vartheta_k) = [e^{j\varphi_{k,1}}, \dots, e^{j\varphi_{k,M}}]^T$ is the array responding vector. $\varphi_{k,m} = 2\pi(m-1)d\sin\vartheta_k/\lambda$, $\tilde{\mathbf{d}} = [\tilde{d}_1, \dots, \tilde{d}_{M-1}]^T$ stands for the sensor location errors vector. Let $\Psi = \text{diag}([0, \tilde{d}_1, \dots, \tilde{d}_{M-1}]^T)$, $\Phi(\vartheta) = \mathbf{A}(\vartheta) \text{diag}\left(\frac{j2\pi}{\lambda} [\sin\vartheta_1, \dots, \sin\vartheta_K]^T\right)$, we can get that,

$$\mathbf{Q}(t) \tilde{\mathbf{d}} = \Psi \Phi(\vartheta) \mathbf{s}(t). \quad (3)$$

By taking the differentiation of both sides of (3) with respect to $\tilde{d}_p$ ($p = 1, \dots, M-1$), we can conclude that,

$[\mathbf{Q}(t)]_{:,p} = \mathbf{G}_p \Phi(\vartheta) \mathbf{s}(t)$, and $\mathbf{G}_p = \partial \Psi / \partial \tilde{d}_p$ has its $(p+1, p+1)$ th element being the only nonzero value of 1.

2.2 Narrowband SAGE-based algorithm for DOA estimation and array calibration

The standard maximum likelihood (ML) estimation method can achieve high accuracy results, but it needs multi-dimensional search and the cost of the computational complexity is enormous. The multi-dimensional maximization search can be simplified to one dimensional search by the space alternating generalized expectation-maximization (SAGE) algorithm, which follows a more flexible optimization scheme [23,24]. In this section, a DOA estimation and sensor location errors calibration method using the SAGE algorithm is studied.

Each iteration of SAGE consists of several cycles. A cycle consists of an E-step followed by an M-step. Let Ω summarize all unknown parameters $\Omega = \{\vartheta, \{\mathbf{s}(t_n)\}_{n=1}^N, c, \sigma^2\}$. We divide Ω into K subsets $\Omega_k = \{\vartheta_k, \{\mathbf{s}_k(t_n)\}_{n=1}^N, c, \sigma^2\}$ ($k = 1, 2, \dots, K$, $n = 1, 2, \dots, N$). Correspondingly, we let $\Omega_k(t) = \{\vartheta_k, s_k(t), c, \sigma^2\}$. Each subset corresponds to one source signal. We can construct the augmented data associated with the k th cycle as follows:

$$\mathbf{y}_k(t) = \mathbf{a}(\vartheta_k) s_k(t) + \mathbf{Q}_k(t) c + \mathbf{v}(t) \quad (4)$$

where $[\mathbf{Q}_k(t)]_{:,p} = \mathbf{G}_{k,p} \phi(\vartheta_k) s_k(t)$, $\phi(\vartheta_k) = \mathbf{a}(\vartheta_k) \frac{j2\pi}{\lambda} \sin\vartheta_k$, $\mathbf{G}_{k,p}$ has its $(p+1, p+1)$ th element being the only nonzero value of 1. $\mathbf{v}(t)$ is the total noise contained in the array output $\mathbf{x}(t)$, and $\mathbf{v}(t) \sim \mathcal{N}(\mathbf{0}, \sigma^2 \mathbf{I}_M)$. In the following, the E-step and M-step of DOA estimation are derived using augmented data in (4).

According to (4) and the distribution of $\mathbf{v}(t)$, the likelihood of $\mathbf{y}_k(t)$ is given by

$$p(\mathbf{y}_k(t) | \Omega_k(t)) = |\pi \sigma^2 \mathbf{I}_M|^{-1} \exp \left\{ -\frac{1}{\sigma^2} \|\mathbf{y}_k(t) - \mathbf{a}'(\vartheta_k) s_k(t)\|_2^2 \right\}. \quad (5)$$

Because the noise parts in $\mathbf{y}_1(t), \mathbf{y}_2(t), \dots, \mathbf{y}_K(t)$ are the same and $\mathbf{y}_k(t)$ ($t = t_1, t_2, \dots, t_N$) are independent for different time instances, the unified likelihood of $\{\mathbf{y}_1(t_n), \mathbf{y}_2(t_n), \dots, \mathbf{y}_K(t_n)\}_{n=1}^N$ is the mean of the likelihood of $\{\mathbf{y}_k(t_n)\}_{n=1}^N$ for different k values.

$$L = \frac{1}{K} \sum_{n=1}^N \sum_{k=1}^K \ln p(\mathbf{y}_k(t_n) | \Omega_k(t_n)) = -MN \ln \pi - MN \ln \sigma^2 - \frac{1}{K \sigma^2} \sum_{n=1}^N \sum_{k=1}^K \|\mathbf{y}_k(t_n) - \mathbf{a}(\vartheta_k) s_k(t_n) - \mathbf{Q}_k(t_n) \tilde{\mathbf{d}}\|_2^2 \quad (6)$$

The SAGE algorithm updates each unknown parameter by the iterative method. The iterative process of the SAGE algorithm includes an E-step and an M-step. Assume $\Omega^{(q-1)}$ is the estimate of the $(q-1)$ th iteration, the E-step and M-step of the q th iteration are given as follows.

E-step:

Calculate

$$\mathcal{F}(\Omega^{(q)}; \Omega^{(q-1)}) = E[L|\{\mathbf{x}(t_n)\}_{n=1}^N, \Omega^{(q-1)}] \quad (7)$$

where $E[\cdot]$ stands for the expectation operator, and it is equivalent to computing the following conditional expectations:

$$\begin{aligned} \mathbf{y}_k^{(q)}(t) &= E[\mathbf{y}_k(t)|\mathbf{x}(t), \Omega^{(q-1)}] = \\ &\mathbf{a}'(\vartheta_k^{(q-1)})s_k^{(q-1)}(t) + \\ &\rho[\mathbf{x}(t) - \mathbf{A}'(\tilde{\mathbf{d}}^{(q-1)}, \vartheta^{(q-1)})\mathbf{s}^{(q-1)}(t)] \\ \mathbf{R}_{y_k}^{(q)} &= \frac{1}{N}. \end{aligned} \quad (8)$$

$$\begin{aligned} E \left[\sum_{n=1}^N \mathbf{y}_k(t_n) \mathbf{y}_k^H(t_n) | \{\mathbf{x}(t_n)\}_{n=1}^N, \Omega^{(q-1)} \right] = \\ \frac{1}{N} \sum_{n=1}^N \mathbf{y}_k^{(q)}(t_n) (\mathbf{y}_k^{(q)}(t_n))^H. \end{aligned} \quad (9)$$

$\mathbf{A}'(\tilde{\mathbf{d}}^{(q-1)}, \vartheta^{(q-1)})$ stands for the perturbed responding matrix, which can express the dependence on sensor location errors.

M-step:

The unknown parameters Ω are updated by maximizing $\mathcal{F}(\Omega^{(q)}; \Omega^{(q-1)})$. Calculate the differentiations of $\mathcal{F}(\Omega^{(q)}; \Omega^{(q-1)})$ to those unknown parameters, and assume that is equivalent to zero.

$$\vartheta_k^{(q)} = \arg \max_{\theta} \frac{(\mathbf{a}'(\tilde{\mathbf{d}}^{(q-1)}, \theta))^H \mathbf{R}_{y_k}^{(q)} \mathbf{a}'(\tilde{\mathbf{d}}^{(q-1)}, \theta)}{\|\mathbf{a}'(\tilde{\mathbf{d}}^{(q-1)}, \theta)\|_2^2} \quad (10)$$

$$s_k^{(q)}(t_n) = \frac{(\mathbf{a}'(\tilde{\mathbf{d}}^{(q-1)}, \vartheta_k^{(q)}))^H \mathbf{y}_k^{(q)}(t_n)}{\|\mathbf{a}'(\tilde{\mathbf{d}}^{(q-1)}, \vartheta_k^{(q)})\|_2^2} \quad (11)$$

$$(\sigma^2)^{(q)} = \frac{1}{KMN}.$$

$$E \left[\sum_{n=1}^N \sum_{k=1}^K \|\mathbf{y}_k(t_n) - \mathbf{a}'(\vartheta_k) s_k(t_n)\|_2^2 \right] \quad (12)$$

$$\tilde{\mathbf{d}}^{(q)} = \left\{ E \left[\sum_{n=1}^N \sum_{k=1}^K \mathbf{Q}_k^H(t_n) \mathbf{Q}_k(t_n) \right] \right\}^{-1}.$$

$$E \left[\sum_{n=1}^N \sum_{k=1}^K \mathbf{Q}_k^H(t_n) (\mathbf{y}_k(t_n) - \mathbf{a}'(\vartheta_k) s_k(t_n)) \right] \quad (13)$$

As can be seen in (10) and (11), the unknown parameters ϑ_k and $\{s_k(t_n)\}_{n=1}^N$ are updated mainly relying on the corresponding augmented data $\{\mathbf{y}_k(t_n)\}_{n=1}^N$. As can be seen in (12) and (13), the update processes of the parameters $\tilde{\mathbf{d}}$ and σ^2 depend on $\{\mathbf{y}_1(t_n), \dots, \mathbf{y}_K(t_n)\}_{n=1}^N$. It demonstrates all signal components model including the parameters $\tilde{\mathbf{d}}$ and σ^2 . Thus, we divide the iteration process into two parts: sequential update of the parameters ϑ_k and $\{s_k(t_n)\}_{n=1}^N$, unified update of $\tilde{\mathbf{d}}$ and σ^2 using all array output.

3. Extension to wideband cases

For wideband cases, the frequency dependence of the array manifold must be considered. Most array calibration methods cannot deal with wideband cases. In this section, an extension of the SAGE-base algorithm to wideband cases is described. First, similar to the narrowband case, in order to facilitate the DOA and perturbation parameters information extraction, a new array model is established in the frequency domain. Then, in each frequency bin, the augmented data is constructed according to the above description in the narrowband case. Because the noise parts in different frequency bins are considered independent, a jointly likelihood function for all frequency bins can be established easily. Finally, the E-step and M-step for the unknown parameters estimation are derived based on the jointly likelihood function.

3.1 Wideband model in the presence of sensor location errors

Suppose that K unknown and deterministic wideband signals impinge onto an M element linear array from directions of $\vartheta = [\vartheta_1, \dots, \vartheta_K]^T$. By applying the discrete time Fourier transform, we can express the array output with sensor location errors as follows in the frequency domain:

$$\begin{aligned} \mathbf{X}(f) &= \mathbf{A}'(\vartheta, f) \mathbf{S}(f) + \mathbf{V}(f) = \\ &\mathbf{A}(\vartheta, f) \mathbf{S}(f) + \mathbf{Q}(f) \mathbf{c} + \mathbf{V}(f) \end{aligned} \quad (14)$$

where $\mathbf{A}(\vartheta, f) = [\mathbf{a}(\vartheta_1, f), \dots, \mathbf{a}(\vartheta_K, f)]$, $\mathbf{A}'(\vartheta, f) = [\mathbf{a}'(\vartheta_1, f), \dots, \mathbf{a}'(\vartheta_K, f)]$, and $\mathbf{a}(\vartheta, f) = [1, e^{j2\pi f d \sin \vartheta / c}, \dots, e^{j2\pi f (M-1) d \sin \vartheta / c}]^T$, the perturbed responding vector

$$\mathbf{a}'(\vartheta, f) =$$

$$[1, e^{j2\pi f (d + \tilde{d}_1) \sin \vartheta / c}, \dots, e^{j2\pi f ((M-1)d + \tilde{d}_M) \sin \vartheta / c}]^T,$$

$f = f_1, \dots, f_N$ represents the frequency bin. c is the velocity of light. d is inter-element spacing, and $\tilde{\mathbf{d}} = [\tilde{d}_1, \dots, \tilde{d}_{M-1}]^T$ stands for the perturbation vector. The following equation holds for $\mathbf{Q}(f)$ and $\tilde{\mathbf{d}}$,

$$\begin{aligned} \mathbf{Q}(f) \tilde{\mathbf{d}} &= [\mathbf{A}'(\vartheta, f) - \mathbf{A}(\vartheta, f)] \cdot \mathbf{S}(f) = \\ &\boldsymbol{\Psi} \boldsymbol{\Phi}(\vartheta, f) \mathbf{S}(f) \end{aligned} \quad (15)$$

where $\Psi = \text{diag}([0, \tilde{d}_1, \dots, \tilde{d}_{M-1}]^T)$, $\Phi(\vartheta, f) = \mathbf{A}(\vartheta, f) \text{diag}\left(\frac{j2\pi f}{c} [\sin \vartheta_1, \dots, \sin \vartheta_K]^T\right)$. By taking the differentiation of both sides of (15) with respect to $\tilde{d}_p$ ($p = 1, \dots, M-1$), we can conclude that, $[\mathbf{Q}(f)]_{:,p} = \mathbf{G}_p \Phi(\vartheta, f) \mathbf{s}(f)$, and $\mathbf{G}_p = \partial \Psi / \partial \tilde{d}_p$ has its $(p+1, p+1)$ th element being the only nonzero value of 1.

3.2 Wideband DOA estimation algorithm in the presence of sensor location errors

Similar to the narrowband case, for applying the SAGE algorithm, we can construct augmented data associated with the k th signal components as follows:

$$\mathbf{Y}_k(f) = \mathbf{a}(\vartheta_k, f) S_k(f) + \mathbf{Q}_k(f) \tilde{\mathbf{d}} + \mathbf{V}(f) \quad (16)$$

where $[\mathbf{Q}_k(f)]_{:,p} = \mathbf{G}_{k,p} \phi(\vartheta_k, f) S_k(f)$, and $\mathbf{G}_{k,p}$ has its $(p+1, p+1)$ th element being the only nonzero value of 1, $\phi(\vartheta_k, f) = \mathbf{a}(\vartheta_k, f) \frac{j2\pi f}{c} \sin \vartheta_K$, $\mathbf{V}(f) \sim \mathcal{N}(\mathbf{0}, \sigma^2 \mathbf{I}_M)$. According to (16) and the distribution of $\mathbf{V}(f)$, the likelihood of $\mathbf{Y}_k(f)$ is given by

$$p(\mathbf{Y}_k(f) | \Omega_k(f)) = |\pi \sigma^2 \mathbf{I}_M|^{-1}.$$

$$\exp \left\{ -\frac{1}{\sigma^2} \|\mathbf{Y}_k(f) - \mathbf{a}'(\vartheta_k, f) S_k(f)\|_2^2 \right\} \quad (17)$$

where $\Omega_k(f)$ is the subset of all unknown parameters set $\Omega = \{\vartheta, \{S(f_n)\}_{n=1}^N, c, \sigma^2\}$, and correspondingly, $\Omega(f) = \{\vartheta, S(f), c, \sigma^2\}$. Because the noise parts in $\mathbf{Y}_1(f), \dots, \mathbf{Y}_K(f)$ are the same and $\mathbf{Y}_k(f_1), \dots, \mathbf{Y}_k(f_n)$ are independent for all k values, the unified likelihood of $\{\mathbf{Y}_1(f_n), \dots, \mathbf{Y}_K(f_n)\}_{n=1}^N$ is the mean of the likelihood of $\{\mathbf{Y}_k(f_n)\}_{n=1}^N$ for different k values.

$$L = \frac{1}{K} \sum_{n=1}^N \sum_{k=1}^K \ln p(\mathbf{Y}_k(f_n) | \Omega_k(f_n)) = -MN \ln \pi - MN \ln \sigma^2 - \frac{1}{K \sigma^2}.$$

$$\sum_{n=1}^N \sum_{k=1}^K \|\mathbf{Y}_k(f_n) - \mathbf{a}(\vartheta_k) S_k(f_n) - \mathbf{Q}_k(f_n) \tilde{\mathbf{d}}\|_2^2 \quad (18)$$

In the following, based on (18), we derive the E-step and M-step of the SAGE algorithm for wideband DOA estimation and array calibration. Given the estimate of the $(q-1)$ th iteration $\Omega^{(q-1)}$, the q th iteration consists of the following steps.

E-step:

Calculate

$$\mathcal{F}(\Omega^{(q)}; \Omega^{(q-1)}) = \mathbb{E}[L | \{\mathbf{X}(f_n)\}_{n=1}^N, \Omega^{(q-1)}]. \quad (19)$$

It is equivalent to computing the following conditional expectation:

$$\begin{aligned} \mathbf{Y}_k^{(q)}(f) &= \mathbb{E}[\mathbf{Y}_k(f) | \mathbf{X}(f), \Omega^{(q-1)}] = \\ &\mathbf{a}'(\vartheta_k^{(q-1)}, \tilde{\mathbf{d}}, f) S_k^{(q-1)}(f) + \mathbf{X}(f) - \\ &\mathbf{A}'(\tilde{\mathbf{d}}^{(q-1)}, \vartheta^{(q-1)}, f) \mathbf{S}^{(q-1)}(f). \end{aligned} \quad (20)$$

$\mathbf{A}'(\tilde{\mathbf{d}}^{(q-1)}, \vartheta^{(q-1)}, f)$ stands for the perturbed responding matrix, which emphasizes the dependence on sensor location errors parameters.

M-step:

The unknown parameters Ω are updated by maximizing $\mathcal{F}(\Omega^{(q)}; \Omega^{(q-1)})$, which can be realized by setting its differentiation to those parameters to zero.

$$\begin{aligned} \vartheta_k^{(q)} &= \arg \max_{\vartheta} \sum_{f=f_1}^{f_N} \frac{(\mathbf{a}'(\tilde{\mathbf{d}}^{(q-1)}, \vartheta, f))^H \mathbf{Y}_k^{(q)}(f)}{\|\mathbf{a}'(\tilde{\mathbf{d}}^{(q-1)}, \vartheta, f)\|_2^2} \\ &(\mathbf{Y}_k^{(q)}(f))^H \mathbf{a}'(\tilde{\mathbf{d}}^{(q-1)}, \vartheta, f) \end{aligned} \quad (21)$$

$$\begin{aligned} S_k^{(q)}(f_n) &= \frac{(\mathbf{a}'(\tilde{\mathbf{d}}^{(q-1)}, \vartheta_k^{(q)}, f_n))^H \mathbf{Y}_k^{(q)}(f_n)}{\|\mathbf{a}'(\tilde{\mathbf{d}}^{(q-1)}, \vartheta_k^{(q)}, f_n)\|_2^2} \\ (\sigma^2)^{(q)} &= \frac{1}{KM N}. \end{aligned} \quad (22)$$

$$\mathbb{E} \left[\sum_{n=1}^N \sum_{k=1}^K \|\mathbf{Y}_k(f_n) - \mathbf{a}'(\vartheta_k, \tilde{\mathbf{d}}, f) S_k(f_n)\|_2^2 \right] \quad (23)$$

The iteration process is similar to the narrowband case. First, the parameters ϑ_k and $S_k(f_n)$ are sequentially updated for different k values. Then, the parameters $\tilde{\mathbf{d}}$ and σ^2 are updated using all augmented data.

4. Simulation results

In this section, we carry out simulations to demonstrate the performance of the proposed method. In the first simulation, the problem of DOA estimation and sensor location errors calibration for narrowband signals is considered. Then, the wideband case is considered in the second simulation. For all simulations, if the increase of the data likelihood function is smaller than 10^{-4} , the iterative processes of the algorithm are terminated. The max number of iterations is 200.

For evaluating the performance of the algorithm, the average root-mean-square error (RMSE) is considered. It is defined as follows:

$$\text{RMSE}_{\vartheta} = \sqrt{\sum_{w=1}^W \|\hat{\vartheta}^w - \vartheta\|_2^2 / (K \times W)} \quad (24)$$

where W stands for the number of Monte Carlo experiments. $\hat{\vartheta}^w$ is the estimated DOA vector in the w th simulation. ϑ is the true DOA vector. In the simulation, W is set to 500. Similarly, the RMSE is used for evaluating the

precision of sensor location errors estimates. It is defined as follows:

$$\text{RMSE}_{\tilde{\mathbf{d}}} = \sqrt{\sum_{w=1}^W \|\tilde{\mathbf{d}}^w - \tilde{\mathbf{d}}\|_2^2 / W}. \quad (25)$$

The parameters $\tilde{\mathbf{d}}^w$ is the estimated sensor location error vector in the w th simulation, and $\tilde{\mathbf{d}}$ is the true location error vector.

4.1 Statistical performance for narrowband cases

In the first experiment, we suppose that two equal-power deterministic narrowband signals impinge onto an 8-element linear array from directions of -10° and 30° , and the nominal geometry of the array is uniform with inter-element spacing equaling half-wavelength of the incident signals. The snapshot number is fixed at 200. The sensor location errors are modeled as independent, identically distributed Gaussian random. The standard deviations vary from 0 to $0.2d$. The initial sensor location errors estimate is given by $\mathbf{d}^{[0]} = [0, 0, 0, 0, 0, 0, 0, 0]^T$. The initial DOA estimate is given by $\boldsymbol{\vartheta}^{[0]} = [-5^\circ, 20^\circ]$. Through computing the least squares estimation for $\boldsymbol{\vartheta} = \boldsymbol{\vartheta}^{[0]}$ and $\mathbf{d} = \mathbf{d}^{[0]}$, we can get the initial signal vector $\mathbf{s}^{[0]}(t)$.

In Fig. 1, the results of 100 Monte Carlo trails of our method are displayed when the signal-to-noise ratio (SNR) is 5 dB. The abscissa and ordinate are the degrees of ϑ_1 and ϑ_2 respectively. From the figure, we can see that the estimation results are distributed around the true value. It demonstrates that our method has good convergence and robustness. For comparison, the DOA estimation RMSE of our method, the ML method [14] and the iterative modification Toeplitz approximation method (IMTAM) [25] are given in Fig. 2. As can be seen, when the SNR increases from -10 dB to 30 dB, our method can get higher precision of parameter estimation.

Fig. 1 DOA estimation results of two narrowband signals obtained by 100 Monte Carlo trails

Fig. 2 RMSE of narrowband DOA estimates of different methods against SNR

For illustrating the array calibration performance of our method, we fix the sensor location errors $0, 0.05d, 0.11d, 0.08d, 0.06d, -0.07d, -0.15d, -0.04d$. Fig. 3 shows the RMSE of sensor location errors of different methods against SNR. We can easily observe that the ML method (no array calibration result is available from IMTAM) fails to estimate the sensor location error accuracy, while our method provides reasonable results. This shows that our method can achieve a better performance than the ML method in the presence of sensor location errors.

Fig. 3 RMSE of narrowband sensor location errors of different methods against SNR

4.2 Statistical performance for wideband cases

In the second experiment, the statistical performance of our method for wideband cases is considered. We assume that two deterministic wideband linear frequency modulation (LFM) signals impinge onto the array from directions of -10° and 20° respectively. The central frequency of the first signal is 100 MHz with a bandwidth of 40%. The central frequency of the second signal is 150 MHz with a bandwidth of 50%. The initial phases of the two signals are $\pi/3$. The chirp period is

0.1 μ s and the sampling frequency is 1 GHz. The spacing between elements is half-wavelength of a 150 MHz sinusoid. Other parameters are kept the same as the first simulation.

When the SNR increases from -10 dB to 30 dB, the RMSE of DOA estimation of our method, Standard Wide-EM [26] and CD-WRELAX [22] are given in Fig. 4. As can be seen in Fig. 4, the performance of the Standard Wide-EM algorithm is the worst. The CD-WRELAX algorithm gets a better performance than the Standard Wide-EM algorithm, and our method gets the best performance. Our method can jointly estimate DOA and sensor location error parameters. It outperforms the wide DOA estimation without array calibration. Fig. 5 shows the RMSE of sensor location errors. Because the existing methods cannot estimate sensor location errors in the wideband case, we only give the sensor location errors RMSE of our method. The sensor location errors can be estimated accurately using our method.

Fig. 4 RMSE of wideband DOA estimation of different methods against SNR

Fig. 5 RMSE of wideband sensor location errors against SNR

5. Conclusions

In this paper, a method for DOA estimation and sensor location errors calibration is presented. Both the narrowband

case and the wideband case are considered. The method has guaranteed convergence and high precision. Moreover, we can jointly estimate DOA estimation and sensor location errors for wideband signals, which is not reported in public literature. Numerical experiments show that the proposed method can achieve a better performance than the existing methods.

References

- [1] CALROS E, TAKADA J. ICA based blind source separation applied to radio surveillance. *IEICE Transactions on Communications*, 2003, E86-B(12): 3491–3497.
- [2] TORIBIO R, SAILLARD J, POULIGUEN P. Identification of radar targets in resonance zone: E-pulse techniques. *Journal of Electromagnetic Waves and Applications*, 2003, 17(12): 1723–1725.
- [3] LIU H, XU G. A subspace method for signature waveform estimation in synchronous CDMA systems. *IEEE Trans. on Communications*, 1996, 44(10): 1345–1354.
- [4] KAYHAN A S, AMIN M G. Spatial evolutionary spectrum for DOA estimation and blind signal separation. *IEEE Trans. on Communications*, 2000, 48(3): 791–798.
- [5] ROLLER C, WASYLKIWSKY W. Effects of mutual coupling on super-resolution DF in linear arrays. *Proc. of the IEEE International Conference on Acoustics, Speech, and Signal Processing*, 1992, 5: 257–260.
- [6] LIU Z M, HUANG Z T, ZHOU Y Y. Bias analysis of MUSIC in the presence of mutual coupling. *IET Signal Processing*, 2009, 3(1): 74–84.
- [7] CLEMENTS J, LO J. Recursive direction finding in the presence of sensor array uncertainties. *Proc. of the IEEE International Conference on Acoustics, Speech, and Signal Processing*, 1993: 308–311.
- [8] FISTAS N, MANIKAS A. A new general global array calibration method. *Proc. of the IEEE International Conference on Acoustics, Speech, and Signal Processing*, 1994: 73–76.
- [9] ROCKAH Y, SCHULTHEISS P M. Array shape calibration using sources in unknown locations-part II: near-field sources and estimator implementation. *IEEE Trans. on Acoustics, Speech, and Signal Processing*, 1987, 35(6): 724–735.
- [10] SMITH J, LEUNG Y, CANTONI A. Broadband eigenvector methods for towed array shape estimation with a single source. *Proc. of the IEEE International Conference on Acoustics, Speech, and Signal Processing*, 1996: 3193–3196.
- [11] LIU Z M, HUANG Z T. DOA estimation with uniform linear arrays in the presence of mutual coupling via blind calibration. *Signal Processing*, 2009, 89(7): 1446–1456.
- [12] WEISS A J, FRIEDLANDER B. Array shape calibration using eigenstructure methods. *Signal Processing*, 1991, 22(3): 251–258.
- [13] CHUNG P J, WAN S. Array self-calibration using sage algorithm. *Proc. of the IEEE International Conference on Sensor Array and Multichannel Signal Processing Workshop*, 2008: 165–169.
- [14] WEISS A J, FRIEDLANDER B. Array shape calibration using sources in unknown locations—a maximum likelihood approach. *IEEE Trans. on Acoustic, Speech, and Signal Processing*, 1989, 37(12): 251–258.
- [15] LIU Z M, ZHOU Y Y. A unified framework and sparse Bayesian perspective for direction-of-arrival estimation in the presence of array imperfections. *IEEE Trans. on Signal Processing*, 2013, 61(15): 3786–3798.

- [16] SWINDLEHURST A L, KAILATH T. A performance analysis of subspace-based methods in the presence of model errors — Part I: The MUSIC algorithm. *IEEE Trans. on Signal Processing*, 1992, 40(7): 1758–1774.
- [17] VIBERG M, SWINDLEHURST A L. Analysis of the combined effects of finite samples and model errors on array processing performance. *IEEE Trans. on Signal Processing*, 1994, 42(11): 3073–3083.
- [18] JANSSEN M, SWINDLEHURST A L, OTTERSTEN B. Weighted subspace fitting for general array error models. *IEEE Trans. on Signal Processing*, 1998, 46(9): 2484–2498.
- [19] AKANGA S, STOICA P. Finite sample and modeling error effects on ESPRIT and MUSIC direction estimators. *IEE Proceedings of Radar, Sonar, Navigation*, 1994: 249–255.
- [20] WAX M, SHEINVALD J. Direction finding of coherent signals via spatial smoothing for uniform circular arrays. *IEEE Trans. on Antennas Propagation*, 1994, 42(5): 613–620.
- [21] HONG T D, RUSSER P. An analysis of wideband direction-of-arrival estimation for closely-spaced sources in the presence of array model errors. *IEEE Microwave Wireless Components Letters*, 2003, 13(8): 314–316.
- [22] CHEN L D, ZHANG C, TAO H M, et al. Approach for wideband direction-of-arrival estimation in the presence of array model errors. *Journal of Systems Engineering and Electronics*, 2009, 20(1): 69–75.
- [23] JEFFREY F A, ALFRED H O. Space-alternating generalized expectation-maximization algorithm. *IEEE Trans. on Signal Processing*, 1994, 42(10): 2664–2676.
- [24] PEI J C, JOHANN F B. Comparative convergence analysis of EM and SAGE algorithms in DOA estimation. *IEEE Trans. on Signal Processing*, 2001, 49(12): 2940–2948.
- [25] CHEN Y M, LEE J H, YEH C C, et al. Bearing estimation without calibration for randomly perturbed arrays. *IEEE Trans. on Signal Processing*, 1991, 39(1): 194–197.
- [26] CADALLI N, ARIKAN O. Wideband maximum likelihood di-

rection finding and signal parameter estimation by using the tree-structured EM algorithm. *IEEE Trans. on Signal Processing*, 1999, 47(1): 201–206.

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