Clustering-based maintainability demonstration for complex systems with a mixed maintenance time distribution

WU Zhenya* and HAO Jianping

Department of Equipment Command and Management, Shijiazhuang Campus of Army Engineering University,
Shijiazhuang 050003, China

Abstract: During maintainability demonstration, the maintenance time for complex systems consisting of mixed technologies generally conforms to a mixture distribution. However existing maintainability standards and guidance do not explain explicitly how to deal with this situation. This paper develops a comprehensive maintainability demonstration method for complex systems with a mixed maintenance time distribution. First of all, a K-means algorithm and an expectation-maximization (EM) algorithm are used to partition the maintenance time data for all possible clusters. The Bayesian information criterion (BIC) is then used to choose the optimal model. After this, the clustering results for equipment are obtained according to their degree of membership. The degree of similarity for the maintainability of different kinds of equipment is then determined using the projection method. By using a Bootstrap method, the prior distribution is obtained from the maintenance time data for the most similar equipment. Then, a test method based on Bayesian theory is outlined for the maintainability demonstration. Finally, the viability of the proposed approach is illustrated by means of an example.

Keywords: complex system, maintainability demonstration, mixture distribution model, Bayesian test.

DOI: 10.21629/JSEE.2019.06.20

1. Introduction

Maintainability demonstration is a formal process conducted by a product developer and an end-customer to determine whether specified maintainability requirements have been achieved [1]. The mean time to repair (MTTR) is a key metric for describing system maintainability and the main index that is drawn upon during maintainability demonstration.

Maintainability demonstration is typically implemented according to relevant standards or specifications that contain detailed test methods and procedures. According to the

MIL-HDBK-470A handbook, there are two possible test plans for MTTR demonstration. One of these relies on the assumption that maintenance time data can be adequately described using a lognormal distribution. The other uses a central limit theorem and is distribution-free. The distributions typically used in maintainability analysis are normal, lognormal, and exponential [1–3]. Lognormal distribution is the most commonly-used for the maintainability analysis of products [4–8]. It applies to most maintenance tasks and repair actions that are comprised of several subsidiary tasks of unequal frequency and time duration, such as those relating to computer and data systems, telecommunications equipment, control systems and consumer electronics where there are consistent technologies in use.

However, in special cases, a multi-modal distribution, also known as a mixture distribution, can provide a better representation of maintenance time data samples when there is more than one time dependent factor [7]. A random variable has a mixture distribution if its probability density function can be expressed by a convex combination of various other probability density functions [9]. This kind of multi-modal distribution of maintenance times would indicate that several distributions are present in the data. In other words, several repair actions (for several basic types of defect) might be included in one set of maintenance times. This would then yield a histogram catering for several means [10,11]. Complex systems using mixed technologies, such as aircraft flight control, microprocessor-controlled mechanical equipment, etc., are typically comprised of different types of equipment, so they are likely to exhibit a multi-modal distribution. When the maintenance time conforms to a mixture distribution, two problems can arise during MTTR demonstration:

(i) As well as determining if a specification has been met, the objectives of maintainability demonstration include improving product maintainability by uncovering design problems. The population mean of a mixture distribu-

Manuscript received December 26, 2018.

*Corresponding author.

This work was supported by the National Defense Pre-research Funds (9140A27010215JB34422).

tion is the weighted mean of the component distributions' mean values, so the MTTR of a complex system incorporates the combined effect of all the equipment's components. That being the case, if a maintainability demonstration is undertaken for the whole system, the root causes of poor maintainability are hard to find should it fail the test.

(ii) If maintainability demonstration is performed for each kind of equipment containing consistent technology separately, this will not only ensure a single distribution pattern, but also reflect the maintainability of the product clearly. However, according to MIL-HDBK-470A, the number of samples should not be smaller than 30. It is almost impossible to obtain enough samples for maintainability demonstration at an equipment level during operational tests and evaluation because the tests are expensive.

Generally, the problem of insufficient samples can be dealt with by using probabilistic and statistical approaches, such as Bayesian techniques [12,13], Bootstrap methods [14,15], and Dempster-Shafer (D-S) evidence theory [16–18]. A Bayesian approach enables available relevant information to be used for prior knowledge modeling. Possible sources of relevant information include engineering design and test data, operating data in different environments, and operating experiences with similar equipment. In this regard, Zhang et al. [12] proposed a reliability analysis method based on an interval-valued triangular fuzzy Bayesian network. Yuan et al. [13] also developed a reliability evaluation methodology for complex systems using dynamic object-oriented Bayesian networks (DOOBNS). Bootstrap methods also offer a solution to the problem of insufficient sample data and are best-suited to situations where no prior information is available. Li et al. [14] proposed a Bootstrap method for characterizing the uncertainty in probabilistic models and its effect on geotechnical reliability. Guo et al. [15], meanwhile, used Bootstrapping

as a supplementary method for estimating the remaining life probability interval. The D-S evidence theory is really an extension of the Bayesian approach. It makes explicit any lack of information concerning a proposition's probability by separating firm support for the proposition from its plausibility. To this end, Miao et al. [16] presented a D-S evidence theory-based integrative verification method to solve the problem of combining lab and field data. Somewhat differently, to reduce incorrect diagnosis results, Yang et al. [17] used an amendment based on the D-S evidence theory to solve conflicts and determine which fault had contributed more to apparent features or symptoms. Gong et al. [18] also presented a new fault diagnosis method for the main coolant system of a nuclear power plant that is based on the D-S evidence theory.

The idea underlying the research reported here is that, for the component equipment in complex systems, there are not only differences, but also similarities in their maintenance processes. Thus, rather than having to either perform maintainability demonstration on the whole system or on each component, a compromise is available. This involves grouping the equipment into clusters according to their maintenance time distributions and repair actions. Maintainability demonstration can then be performed for each cluster after selecting the representative equipment. The basic idea of equipment clustering is shown in Fig. 1. Within each cluster, the introduction of relevant information may mean that the maintainability demonstration will require little in the way of additional testing. More importantly, by using this approach, the maintainability demonstration for complex systems can be decomposed into demonstrations for several clusters of equipment. Thus, a clear picture concerning the maintainability of the whole system can be constructed according to the demonstration results for each cluster.

Fig. 1 Equipment clustering analysis

The main purpose of this research is to develop a combined method for complex system MTTR demonstration. The proposed method consists of three main parts: (i) A Gaussian mixture distribution clustering model that

can partition equipment using consistent technologies into clusters. Equipment in the same cluster will have a similar maintenance time distribution. (ii) Projection to calculate the extent to which equipment in the same cluster has

a similar degree of maintainability. On the basis of this, the equipment most similar to the selected representative equipment is chosen as a source of relevant information for prior distribution. (iii) Testing based on the Bayesian theory to provide the MTTR demonstration.

The remainder of the paper is organized as follows: In Section 2, we provide some background regarding lognormal distribution and maintenance time, mixture distribution models, expectation-maximization (EM) algorithms and projection methods. Section 3 proposes a Gaussian mixture distribution clustering model based on an EM algorithm, which is used to handle the equipment clustering problem. Section 4 establishes the projection method for calculating the degree of maintainability similarity between equipment, then lays out a Bootstrap method for obtaining the prior distribution. This forms the basis of a proposed MTTR demonstration method based on the Bayesian theory. Section 5 uses an example to validate the proposed approach. Some overall conclusions are put forward in Section 6.

2. Background

2.1 Lognormal distribution and maintenance time

As mentioned in the Section 1, the use of multiple technologies results in a mixture distribution of the maintenance time in complex systems. Therefore, where the equipment using consistent technology can be identified, the maintenance time for this equipment will usually conform to a single mode distribution, such as a lognormal distribution, normal distribution or exponential distribution [1–3]. However, drawing upon field data, some research has suggested that lognormal distribution is a better descriptor for maintenance time than other distributions [4–8]. Almog pointed out that, because repair time includes diagnostic, correction, and verification tasks, that themselves take time, a maintenance time distribution must have a value of zero at time $t = 0$, increase to its maximum value rapidly and then gradually decrease towards zero over longer periods of time [4]. With an exponential distribution it is possible to suggest that the maximum number of repairs can be made in zero time. Thus, it can be shown heuristically that exponential distribution is not an appropriate model, even if, when maintenance times are clustered or grouped into intervals, e.g., histograms, the results may appear to fit an exponential distribution. Bovaird and Zagor [6] suggested two possible explanations for the lognormal distribution of maintenance time. The first explanation is that lognormal distribution can frequently be seen to associate with two kinds of phenomena [19,20]. One relates to growing organisms; the other

relates to partitioning processes, e.g., placing objects in logically-derived categories. The location and correction of a fault for electronic equipment is, to a large extent, a human partition process where objects are placed in logical categories. The categories in question are functional assemblies or components, with failures being categorized according to the failed component. The second explanation is that most experienced maintenance personnel are familiar with the nature and repair of the most common kinds of equipment faults. Occasionally, however, an unfamiliar trouble occurs or an inexperienced technician is called upon to repair the equipment. These repairs require more time. Therefore, this type of maintenance exhibits a pattern of repair times that cluster around a relatively low value, but with a distribution that shows a tail to the right, i.e., the distribution is unimodal but skewed to the right. As a member of the class of such curves, lognormal distribution describes this situation very well over the observed range of variates, i.e., maintenance times.

In conclusion, it is not surprising that the maintenance time for equipment with consistent technologies conforms to a lognormal distribution for most cases. For simplicity, in our research, the maintenance times of equipment with consistent technologies will all be assumed to conform to lognormal distributions. Thus, the maintenance time data for a complex system can be thought of as conforming to a Gaussian mixture distribution after natural logarithmic transformation.

2.2 Mixture distribution models

Let us suppose that the distribution of a random variable X can be represented by a probability density function in the form [21] of

$$f(x) = \lambda_1 f_1(x) + \cdots + \lambda_K f_K(x),$$

$$\lambda_j > 0; \quad \sum_{j=1}^K \lambda_j = 1 \quad (1)$$

and

$$f_j(x) \geq 0, \quad \int f_j(x) dx = 1, \quad j = 1, 2, \dots, K.$$

In such a case, x is said to have a finite mixture distribution and $f(x)$, as defined by (1), is a finite mixture density function. The parameter λ_j ($j = 1, 2, \dots, K$) denotes the mixing weights and $f_j(x)$ ($j = 1, 2, \dots, K$) denotes the component densities of the mixture.

2.3 EM algorithm

The EM algorithm was originally proposed as a general method for obtaining maximum likelihood estimates in the presence of missing data [22]. Its use for the estimation of

mixture models has been studied in detail by McLachlan and Basford [23] and Redner and Walker [24].

Consider a population whose distribution has a mixture density of the form

$$f(x; \theta) = \sum_{j=1}^K \alpha_j f_j(x; \theta) \quad (2)$$

where $f_j(x; \theta)$ ($j = 1, 2, \dots, K$) are component densities, θ is a parameter or parameter set and $\sum_{j=1}^K \alpha_j = 1$.

Then, for N observations from this mixture distribution, the “complete” data would be $\mathbf{y}_i = (x_i, \mathbf{z}_i)$ ($i = 1, 2, \dots, N$), where $\mathbf{z}_i = (z_{i0}, z_{i1}, \dots, z_{iK})$, such that

$$z_{ij} = \begin{cases} 1, & \text{the } i\text{th observation is in the } j\text{th cluster} \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

The vector $\mathbf{z}_i$ has a multinomial distribution with the parameters $(1; \mathbf{a})$, where $\mathbf{a} = (\alpha_1, \alpha_2, \dots, \alpha_K)$. This leads to a “complete-data log-likelihood”, namely,

$$l(\mathbf{y}; \theta, \mathbf{a}) = \sum_{i=1}^N \sum_{j=1}^K z_{ij} \{\ln \alpha_j + \ln f_j(x_i; \theta)\}. \quad (4)$$

Step E (the expectation) requires the computation of $\hat{z}_{ij} = E(z_{ij} | x_1, \dots, x_N, \theta, \mathbf{a})$, which is the posterior probability that x_i is in the j th cluster. Maximum likelihood estimates of θ and $\mathbf{a}$ are obtained in step M (the maximization), which consists of maximizing the expected complete-data log-likelihood, namely,

$$l^*(\mathbf{y}; \theta, \mathbf{a}) = \sum_{i=1}^N \sum_{j=1}^K \hat{z}_{ij} \{\ln \alpha_j + \ln f_j(x_i; \theta)\}. \quad (5)$$

This process is iterated to convergence.

2.4 Projection method

The projection method was originally proposed by Wang [25]. Here, the value of a projection is calculated based on the product of the norm and the cosine of the angle between two vectors. This reflects not only direction but also distance. This method has recently attracted a lot of attention [26–28]. In what follows, we will briefly introduce some of the principal concepts associated with the projection method.

Definition 1 Let $\mathbf{p} = (p_1, p_2, \dots, p_n)$ be a vector, then

$$\|\mathbf{p}\| = \sqrt{\sum_{i=1}^n p_i^2} \quad (6)$$

is called the module of vector $\mathbf{p}$.

Definition 2 Let $\mathbf{p} = (p_1, p_2, \dots, p_n)$ and $\mathbf{q} = (q_1, q_2, \dots, q_n)$ be two vectors, then

$$\mathbf{p} \cdot \mathbf{q} = \sum_{i=1}^n p_i q_i \quad (7)$$

is called the inner product between vectors $\mathbf{p}$ and $\mathbf{q}$.

Definition 3 Let $\mathbf{p} = (p_1, p_2, \dots, p_n)$ and $\mathbf{q} = (q_1, q_2, \dots, q_n)$ be two vectors, then

$$\text{prj}_{\mathbf{q}}(\mathbf{p}) = \|\mathbf{p}\| \cos(\mathbf{p}, \mathbf{q}) = \|\mathbf{p}\| \times \frac{\mathbf{p} \cdot \mathbf{q}}{\|\mathbf{p}\| \times \|\mathbf{q}\|} = \frac{\mathbf{p} \cdot \mathbf{q}}{\|\mathbf{q}\|} \quad (8)$$

is called the projection of the vector $\mathbf{p} = (p_1, p_2, \dots, p_n)$ from $\mathbf{q} = (q_1, q_2, \dots, q_n)$.

The projection is illustrated in Fig. 2. In general, the bigger the value of $\text{prj}_{\mathbf{q}}(\mathbf{p})$, the bigger the degree of similarity between vectors $\mathbf{p}$ and $\mathbf{q}$.

Fig. 2 Projection of vector $\mathbf{p}$ from $\mathbf{q}$

3. Gaussian mixture distribution clustering model for complex system equipment

In this section, a K-means algorithm is first of all used to obtain initial partitions of the maintenance time data for all possible clusters. Then, an EM algorithm is used to improve the clustering for each of the Gaussian mixture distribution models. Once the EM iterations converge, a Bayesian information criterion (BIC) approach is used to choose the optimal model. This leads to the final equipment clustering result according to degrees of membership. The basic clustering model process is shown in Fig. 3.

Fig. 3 Gaussian mixture distribution clustering model process

3.1 Obtaining the initial partitions using a K-means algorithm

Suppose G equipment using consistent technology has been classified in a complex system through analysis of design information. Let $D = \{x_1, \dots, x_N\}$ denote the maintenance time data for the whole system, and $D^{(g)} = \{x_1^{(g)}, x_2^{(g)}, \dots, x_{n_g}^{(g)}\}$ denote the equipment sample g ($g = 1, 2, \dots, G$), where n_g is the sample size and $N = \sum_{g=1}^G n_g$.

The above maintenance time data has already been subjected to natural logarithmic transformation.

The K-means clustering algorithm is one of the oldest and most widely-used clustering algorithms [29,30]. It is a prototype-based, simple partition-clustering algorithm that attempts to find K non-overlapping clusters. In this section, a K-means clustering algorithm is used to obtain initial partitions of the maintenance time data for all possible clusters. The clustering steps are as follows:

Step 1 Determining the minimum and maximum number of clusters $K_{\min}$ and $K_{\max}$.

The minimum and maximum number of clusters $K_{\min}$ and $K_{\max}$ can be determined from the histogram of sampled maintenance time data or according to expert experience.

Step 2 Choosing k ($k = K_{\min}, \dots, K_{\max}$) points $c_1, c_2, \dots, c_k$ from the maintenance time data at random as the initial centroids of clusters $W_1, W_2, \dots, W_K$, respectively.

Step 3 Partitioning the maintenance time data into clusters.

Here, the Euclidean distance between the maintenance time data points and each centroid needs to be calculated. If $d_{ij}(x_i, c_j) < d_{im}(x_i, c_m)$, where $d_{ij}(x_i, c_j)$ is the Euclidean distance between data point x_i and centroid c_j and $m = 1, 2, \dots, k; j = 1, 2, \dots, k; j \neq m; i = 1, 2, \dots, N$, then, data point x_i belongs to cluster W_j .

Step 4 Calculating the new centroids of each cluster, i.e.,

$$c_j^* = \frac{1}{n_j} \sum_{x \in W_j} x, \quad j = 1, 2, \dots, k \quad (9)$$

where n_j is the number of data points belonging to cluster W_j .

Step 5 Repeating Steps 3 and 4 until no more changes are made.

Step 6 Obtaining the optimal cluster partitions.

To avoid getting local minima, Steps 2 to 4 are repeated several times and the values of the within-group sum-of-

squares for each cluster partition are compared, i.e.,

$$E = \sum_{j=1}^k \sum_{i=1}^{n_j} d_{ij}^2(x_i, c_j). \quad (10)$$

The cluster partitions with the smallest value of within-group sum-of-squares are chosen to be the optimal ones.

3.2 Improving the clustering using an EM algorithm

In this section, an EM algorithm is used to further improve the clustering based on the Gaussian mixture distribution model. The steps are as follows:

Step 1 Calculating initial parameters of each component distribution.

The initial parameters for each component's distribution can be calculated as follows:

$$\hat{\theta}_j^0 = \frac{\sum_{x \in W_j} x}{n_j}, \quad j = 1, 2, \dots, k \quad (11)$$

$$\hat{\sigma}_j^0 = \sqrt{\frac{\sum_{x \in W_j} (x - \hat{\theta}_j^0)^2}{n_j}}, \quad j = 1, 2, \dots, k \quad (12)$$

$$\hat{\alpha}_j^0 = \frac{n_j}{N}, \quad j = 1, 2, \dots, k. \quad (13)$$

Step 2 Calculating the posterior probabilities of the data points.

The posterior probabilities for the maintenance time data points can be obtained using

$$\hat{\gamma}_{ij} = \frac{\alpha_j f(x_i; \theta_j, \sigma_j)}{\sum_{j=1}^k \alpha_j f(x_i; \theta_j, \sigma_j)}, \quad i = 1, 2, \dots, N; j = 1, 2, \dots, k \quad (14)$$

where $\hat{\gamma}_{ij}$ represents the estimated posterior probability that point x_i belongs to the i th component distribution and $f(x_i; \theta_j, \sigma_j)$ is the normal density for the i th component distribution at x_i .

Step 3 Calculating the parameters of the new iteration.

The posterior probability indicates the likelihood of a point belonging to each of the separate component densities. This estimated posterior probability can be used to obtain a weighted update of the parameters for each component of the new iteration. These are

$$\hat{\theta}_j = \frac{\sum_{i=1}^N \hat{\gamma}_{ij} x_i}{\sum_{i=1}^N \hat{\gamma}_{ij}}, \quad j = 1, 2, \dots, k \quad (15)$$

$$\hat{\sigma}_j^2 = \frac{\sum_{i=1}^N \hat{\gamma}_{ij}(x_i - \hat{\theta}_j)^2}{\sum_{i=1}^N \hat{\gamma}_{ij}}, \quad j = 1, 2, \dots, k \quad (16)$$

$$\hat{\alpha}_j = \frac{\sum_{i=1}^N \hat{\gamma}_{ij}}{N}, \quad j = 1, 2, \dots, k. \quad (17)$$

Step 4 Repeating steps 2 and 3 until the estimates converge.

3.3 Choosing the number of components

When mixture distribution models are used to provide a model-based clustering, referred to as a “direct application” by Titterton et al. [9], the choice of the number of components must be carefully and critically addressed. Available methods for estimating the number of components include optimizing Akaike information criteria (AIC) and BIC, estimating the number through non-parametric maximum likelihood, hypothesis testing and Bayesian approaches with entropy distances [31]. In this case, BIC is used to choose the number of components, as follows [32]:

$$\text{BIC} = -\frac{1}{N} \sum_{i=1}^N \ln \sum_{j=1}^k \alpha_j f(x_i; \theta_j, \sigma_j) + \frac{k \ln N}{2N}. \quad (18)$$

The smaller the BIC value, the better the model. The mixture distribution model with the smallest BIC value is chosen as the optimal model.

3.4 Partitioning equipment into clusters

Let $f(x; \theta) = \sum_{j=1}^k \alpha_j f(x; \theta_j, \sigma_j)$ denote the Gaussian mixture distribution model obtained above. The degree of membership of the kind of equipment g to a component distribution j is defined as

$$\hat{\lambda}_{gj} = \sum_{x_i \in D^{(g)}} \hat{\gamma}_{ij}, \quad g = 1, 2, \dots, G;$$

$$i = 1, 2, \dots, N; \quad j = 1, 2, \dots, k. \quad (19)$$

The equipment g then belongs to the cluster $W_{j^{(g)}}$, i.e.,

$$j^{(g)} = \arg \max_j \hat{\lambda}_{gj}. \quad (20)$$

All equipment in the same cluster is considered to have a similar maintenance time distribution. However, if there is only one kind of equipment in a cluster, no relevant information can be used to obtain a prior distribution. In that

case, a traditional non-Bayesian method is the only way to perform a maintainability demonstration.

4. MTTR demonstration based on Bayesian theory

In this section, a projection method is used to calculate the degrees of similarity for the maintainability of different kinds of equipment in the same cluster. This is followed by a Bootstrap method to obtain the prior distribution from the maintenance time data for equipment most similar to the selected representative equipment. After that, a Bayesian test method can be used to perform an MTTR demonstration.

4.1 Obtaining the prior distribution based on degrees of similarity

As stated in Section 1, equipment with a similar maintenance time distribution and repair actions can be seen as a source of relevant information when performing a maintainability demonstration. After the clustering analysis described in Section 3, equipment in the same cluster is considered to have a similar maintenance time distribution. However, it is difficult to obtain the similarity between their repair actions. Another way of proceeding, in view of this, is to compare the maintainability characteristics of equipment according to projection-based maintainability indices. Wani and Gandhi [33] previously defined the attributes of mechanical systems in terms of their design, personnel and support features. These maintainability attributes are listed in Table 1.

Table 1 Maintainability attributes for mechanical systems

Feature	Number	Maintainability attribute
Design	1	Accessibility
	2	Disassembly/assembly
	3	Standardization
	4	Simplicity
	5	Identification
	6	Diagnosability
	7	Modularization
	8	Tribo-concepts
Personnel	9	Personnel including ergonomics
	10	System environment
Logistic	11	Tools and test equipment
Support	12	Documentation

Each attribute in the table can be assigned a value on a scale, e.g., 0 to 4, according to its associated system design features and the specifications given in MIL-HDBK-472 [34]. The higher the attribute value, the higher its maintainability and the better it is from a maintenance point of view. Therefore, if an attribute has a value of 4, it can be seen to have maximum maintainability. We need to study the sys-

tem design information and maintenance procedures carefully to determine the maintainability attributes values for each equipment.

Let $\mathbf{H}_i = (h_1^{(i)}, h_2^{(i)}, \dots, h_{12}^{(i)})$ ($i = 1, 2, \dots, G$) denote the attribute value vector for equipment i . $\mathbf{w} = (w_1, w_2, \dots, w_{12})^T$ is the weight vector of its attributes, where $w_k \in [0, 1]$ ($k = 1, 2, \dots, 12$) and $\sum_{k=1}^{12} w_k = 1$.

Weight vectors can be obtained in a number of ways, for instance, by using analytic hierarchy process (AHP) methods or entropy weights. Then, according to (8), the projection D_{ij} of equipment i from j can be calculated as follows:

$$D_{ij} = \frac{\sum_{k=1}^{12} w_k^2 h_k^{(i)} h_k^{(j)}}{\sqrt{\sum_{k=1}^{12} w_k^2 (h_k^{(j)})^2}}, \quad i = 1, 2, \dots, G; \\ j = 1, 2, \dots, G; i \neq j. \quad (21)$$

The bigger the projection value, the greater the maintainability similarity of the equipment. After this, the maintenance time data from the equipment with the largest projection value can be used to obtain the prior distribution.

4.2 MTTR demonstration based on Bayesian theory

Tests in which prior information is incorporated into decision criteria by means of Bayes theorem are called Bayesian tests. A Bayesian approach provides an efficient solution to the integration of different sources of data. It combines the associated probability distributions of each source of data into a joint posterior distribution [35]. Harold [36] previously suggested using Bayesian maintainability-demonstration tests based on controls applied to accepted product maintainability. In condensed form, this method can be described as follows:

Let $X \sim \text{LogN}(\theta, \sigma^2)$ denote the maintenance time distribution of a specified kind of equipment. The variance σ^2 will either be known from prior information or a reasonably precise estimate can be obtained. Let $\{X_1, X_2, \dots, X_N\}$ denote the maintenance time sample collected from field data. Assuming that the desired maintainability level for the specified equipment is $\text{MTTR} = \mu_0$ and the minimum acceptable value is $\text{MTTR} = \mu_1$, a hypothesis test can be defined in the following terms:

$$\begin{cases} H_0 : \theta = \theta_0 = \ln \mu_0 - \frac{1}{2}\sigma^2 \\ H_1 : \theta = \theta_1 = \ln \mu_1 - \frac{1}{2}\sigma^2 \end{cases}. \quad (22)$$

If the mean maintenance time is μ_0 , the probability of acceptance will be $1 - \alpha$. If the product is accepted, the prob-

ability that its mean maintenance time is greater than μ_1 will be β_b . These two requirements can be written as

$$P[T_n \leq T^* | \theta = \theta_0] = 1 - \alpha \quad (23)$$

$$P[\theta > \theta_1 | T_n \leq T^*] = \beta_b \quad (24)$$

where T_n is the mean of the natural logarithm of n observed maintenance time, i.e., $T_n = \sum_{i=1}^n \ln X_i / n$, with T^* being the preselected critical value for deciding whether the equipment passes, such that $T_n \leq T^*$ means it will pass and $T_n > T^*$ means it will fail.

The requirement expressed by (23) is the usual producer's risk control. The requirement expressed by (24) offers a consumer assurance that the maintainability level of the equipment at least meets the minimum requirement. The notation β_b is used to distinguish the Bayesian risk from the classical Beta risk. The consumer's best interests are served by the more direct approach of assuring acquisition of satisfactory products, rather than controlling against likelihood of having to accept a poor product.

Let $N(\bar{\theta}, v^2)$ denote the prior distribution of θ . From this we can get

$$Y = \frac{\bar{\theta} - \theta_1}{v} \quad (25)$$

$$Z = \frac{\theta_1 - \theta_0}{v}. \quad (26)$$

The sample size can be found using the following equation:

$$n = \frac{K\sigma^2}{(\theta_1 - \theta_0)^2} \quad (27)$$

where the value of K for all combinations of α, β_b, Y and Z can be determined according to the tables in [36]. If $K = 0$, the table indicates that the prior distribution will already meet the Bayesian risk requirement, thus obviating the need for testing. After this, given n, T^* can be obtained using

$$T^* = \theta_0 + Z_\alpha \frac{\sigma}{\sqrt{n}}. \quad (28)$$

5. Case study

In this section, we describe the implementation of a maintainability demonstration in a complex mechanical system. Nine kinds of equipment using consistent technology are classified. Table 2 shows the maintenance time data for each kind of equipment according to the field data and time estimations from the maintenance staff. The values in brackets are the natural logarithms of the maintenance time data.

Table 2 Equipment maintenance time records

Number	A	B	C	D	E	F	G	H	I
1	17(2.83)	35(3.56)	13(2.56)	16(2.77)	31(3.43)	20(3.00)	31(3.43)	19(2.94)	45(3.81)
2	9(2.19)	39(3.66)	21(3.04)	14(2.64)	28(3.33)	11(2.40)	16(2.77)	28(3.33)	47(3.85)
3	29(3.37)	21(3.04)	28(3.33)	15(2.71)	23(3.14)	18(2.89)	30(3.40)	23(3.14)	42(3.74)
4	11(2.40)	13(2.56)	19(2.94)	16(2.77)	33(3.50)	20(3.00)	16(2.77)	25(3.22)	41(3.71)
5	20(3.00)	11(2.40)	18(2.89)	15(2.71)	37(3.61)	8(2.08)	16(2.77)	28(3.33)	36(3.58)
6	10(2.30)	17(2.83)	33(3.50)	12(2.48)	29(3.37)	9(2.20)	37(3.61)	23(3.14)	41(3.71)
7	11(2.40)	19(2.94)	12(2.48)	16(2.77)	38(3.64)	13(2.56)	9(2.20)	25(3.22)	31(3.43)
8	25(3.22)	10(2.30)	19(2.94)	36(3.58)	29(3.37)	10(2.30)	20(3.00)	28(3.33)	32(3.47)
9	10(2.30)	15(2.71)	23(3.14)	19(2.94)	38(3.64)	32(3.47)	19(2.94)	33(3.50)	34(3.53)
10	15(2.71)	—	23(3.14)	—	39(3.66)	33(3.50)	22(3.09)	27(3.30)	34(3.53)
11	—	—	18(2.89)	—	39(3.66)	11(2.40)	—	36(3.58)	34(3.53)
12	—	—	17(2.83)	—	32(3.47)	18(2.89)	—	28(3.33)	—
13	—	—	20(3.00)	—	23(3.14)	—	—	—	—

Mean = 23.62 min(3.07); Variance $\approx 97.61 \text{ min}^2(0.20)$

The frequency histograms for the maintenance time data and the values after natural logarithmic transformation are shown in Fig. 4.

The histograms indicate that there may be at least two modes in the maintenance time distribution, which means it does not conform to lognormal or other single mode distributions. Additionally, none of the equipment meets the minimum sample size requirement, i.e., at least 30 samples. Therefore, as mentioned in Section 1, traditional approaches are not applicable here.

(a) Maintenance time data

(b) Maintenance time data after natural logarithmic transformation

Fig. 4 Frequency histograms of maintenance time data and after natural logarithmic transformation

5.1 Clustering analysis

5.1.1 Clustering the data using the K-means algorithm

After observing the frequency histograms, we can specify the possible number of clusters, i.e., $k = 2, 3, 4$. The clustering results for each of these, using the K-means algorithm, are shown in Fig. 5.

(a) $k=2$

Fig. 5 Clustering results using the K-means algorithm

The centroids and boundaries for each cluster partition are listed in Table 3.

Table 3 Centroids and boundaries for each cluster partition

Model	Centroid				Boundary			
	1	2	3	4	1	2	3	4
$k = 2$	2.68	3.44	—	—	[2.07 3.05]	[3.09 3.86]	—	—
$k = 3$	2.38	2.95	3.52	—	[2.07 2.64]	[2.70 3.22]	[3.29 3.86]	—
$k = 4$	2.36	2.79	3.06	3.52	[2.07 2.57]	[2.63 2.90]	[2.94 3.22]	[3.29 3.86]

5.1.2 Improving the clustering using the EM algorithm

In the above section, the initial partitions for the maintenance time data are obtained using the K-means algorithm.

Next, the Gaussian mixture model is used to obtain an initial estimate for the parameters of each component's distribution according to (11)–(13), as shown in Table 4.

Table 4 Initial parameters for each component's distribution using the K-means algorithm

Model	Component distribution			
	1	2	3	4
$k = 2$	$N(2.68, 0.08), \hat{\alpha}_1^0 = 0.49$	$N(3.44, 0.04), \hat{\alpha}_2^0 = 0.51$	—	—
$k = 3$	$N(2.38, 0.02), \hat{\alpha}_1^0 = 0.20$	$N(2.95, 0.02), \hat{\alpha}_2^0 = 0.40$	$N(3.52, 0.02), \hat{\alpha}_3^0 = 0.40$	—
$k = 4$	$N(2.36, 0.02), \hat{\alpha}_1^0 = 0.18$	$N(2.79, 0.01), \hat{\alpha}_2^0 = 0.18$	$N(3.06, 0.01), \hat{\alpha}_3^0 = 0.24$	$N(3.52, 0.02), \hat{\alpha}_4^0 = 0.40$

The estimated model returned by the EM algorithm is shown in Table 5.

Table 5 Improved parameters for each component's distribution using the EM algorithm

Model	Component distribution			
	1	2	3	4
$k = 2$	$N(2.84, 0.14), \hat{\alpha}_1 = 0.67$	$N(3.53, 0.02), \hat{\alpha}_2 = 0.33$	—	—
$k = 3$	$N(2.34, 0.02), \hat{\alpha}_1 = 0.16$	$N(2.91, 0.04), \hat{\alpha}_2 = 0.42$	$N(3.50, 0.03), \hat{\alpha}_3 = 0.42$	—
$k = 4$	$N(2.38, 0.02), \hat{\alpha}_1 = 0.19$	$N(2.75, 0.01), \hat{\alpha}_2 = 0.07$	$N(2.97, 0.02), \hat{\alpha}_3 = 0.30$	$N(3.49, 0.03), \hat{\alpha}_4 = 0.44$

5.1.3 Choosing the number of components

Table 6 shows the BIC values for each model, according to (18).

Table 6 BIC values for each model

Model	$k = 2$	$k = 3$	$k = 4$
BIC	0.580 1	0.564 4*	0.577 7

The model with three components has the smallest BIC value, so this is the optimal model for describing the maintenance time distribution of the complex system.

5.1.4 Partitioning the equipment into clusters

In the above section, the model with three components is chosen as the optimal model to describe the maintenance time distribution. Drawing upon (19), the degree of membership to each component distribution for different kinds of equipment is established as illustrated in Table 7.

Table 7 Membership degrees of equipment to each component distribution

Number	A	B	C	D	E	F	G	H	I
1	4.91*	2.38	1.21	0.95	0	5.31*	1.01	0	0
2	3.67	4.60*	9.58*	7.04*	2.00	4.69	5.98*	4.43	0.05
3	1.42	2.02	2.21	1.00	11.00*	2.00	3.02	7.57*	10.95*

Then, (20) is used to obtain the clustering results, as follows:

$$\begin{cases} W_1 = \{A, F\} \\ W_2 = \{B, C, D, G\} \\ W_3 = \{E, H, I\} \end{cases}.$$

$$w = [0.02 \ 0.10 \ 0.12 \ 0.04 \ 0.10 \ 0.10 \ 0.06 \ 0.08 \ 0.18 \ 0.04 \ 0.02 \ 0.14].$$

Thus, according to (21), the equipment projections for C , D and G from B are as follows:

$$D_{CB} \approx 0.79; \quad D_{DB} \approx 0.72; \quad D_{GB} \approx 0.59.$$

This result shows that equipment C has the most similar maintainability to B . Thus, the prior distribution of θ could be obtained from the maintenance time data of equipment C by using the Bootstrap method, as follows:

$$\pi_C(\theta) = N(2.976, 0.005).$$

5.2.2 MTTR demonstration

Let us now assume that the desirable maintainability index level for equipment B is $MTTR = 20$ min and the minimum acceptable value is $MTTR = 25$ min. If the mean maintenance time is 20 min, there will be a 95% probability of acceptance. It will also be 95% assuring that the accepted product will not exhibit a mean repair time greater than 25 min. An estimation procedure provides an estimate

It can be seen from the results that each kind of equipment has at least one other kind in the same cluster, which means the maintenance time data from other equipment can be used to help perform the maintainability demonstration.

5.2 MTTR demonstration based on Bayesian theory

5.2.1 Calculating the degree of maintainability similarity using projection method

Let us assume that we want to perform a maintainability demonstration for cluster W_2 , with equipment B being selected as the representative kind of equipment. After studying their design details and maintenance procedures, the maintainability attribute value vectors for the equipment in cluster W_2 are as follows:

$$H_B = [1 \ 1 \ 2 \ 4 \ 1 \ 2 \ 2 \ 4 \ 3 \ 4 \ 3 \ 3]$$

$$H_C = [1 \ 2 \ 4 \ 4 \ 3 \ 2 \ 4 \ 1 \ 2 \ 4 \ 2 \ 3]$$

$$H_D = [2 \ 1 \ 4 \ 1 \ 3 \ 2 \ 3 \ 1 \ 3 \ 4 \ 4 \ 1]$$

$$H_G = [1 \ 1 \ 2 \ 1 \ 3 \ 4 \ 3 \ 1 \ 2 \ 4 \ 2 \ 1].$$

A weight vector is obtained, drawing upon expert knowledge by means of AHP:

of 0.2 for σ^2 . Therefore, according to (22), the hypothesis test can be defined as

$$H_0 : \theta = \theta_0 = 2.90$$

$$H_1 : \theta = \theta_1 = 3.12$$

with the test requirements being

$$P[T_n \leq T^* | \theta = 2.90] = 0.95$$

$$P[\theta > 3.12 | T_n \leq T^*] = 0.05.$$

Referring to (25) and (26), the inputs needed to use the tables in [36] are as follows:

$$\alpha = 0.05$$

$$\beta_b = 0.01$$

$$Y = \frac{\bar{\theta} - \theta_1}{v} = \frac{2.976 - 3.12}{0.07} = -2.021$$

$$Z = \frac{\theta_1 - \theta_0}{v} = \frac{3.12 - 2.90}{0.07} = 3.156.$$

To be conservative, the next higher tabular entries of $Y = -1$ and $Z = 4$ are used, giving a result of $K = 5.840$. In that case,

$$n = \frac{K\sigma^2}{(\theta_1 - \theta_0)^2} = \frac{5.840 \times 0.2}{(3.12 - 2.90)^2} = 24.$$

Therefore, the critical value is

$$T^* = \theta_0 + Z_\alpha \sigma / \sqrt{n} = 2.896 + 1.645 \times 0.447 / 4.899 = 3.046.$$

Next, on top of the existing nine maintenance samples, an additional random sample of 15 maintenance actions is observed. The sample mean is computed by using

$$T_n = \frac{\sum_{i=1}^n \ln X_i}{n}.$$

If $T_n \leq 3.046$, the equipment would be accepted. Otherwise, it would be rejected.

From the above results, we can see that, by using a Bayesian method to process the maintenance time data from similar equipment, an MTTR demonstration can be performed with fewer equipment samples than a traditional approach. By repeating this procedure for each cluster, an overall picture of the maintainability of the whole system can be obtained.

6. Conclusions

This paper presents a comprehensive method for dealing with the problem of how to provide an effective MTTR demonstration for a complex system. The basic idea involves partitioning the component equipment into clusters, then using relevant information from similar equipment to perform a maintainability demonstration for representative equipment in each cluster. A Gaussian mixture distribution clustering model and a projection method are proposed to ensure similarity in the maintenance time distribution and repair actions, respectively. The statistical samples are first used for the equipment clustering analysis, then for Bayesian tests. From the case study presented in Section 5, it can be seen that, by performing a maintainability demonstration for each cluster using a Bayesian method, fewer samples will be needed than the case using a traditional approach and a clearer picture of the system maintainability can be established.

In conclusion, the method we propose offers advantages over traditional ones when tackling the problem of maintainability demonstration for a complex system. However, in complex systems, the maintenance time distribution for equipment with consistent technology does not

always conform to a lognormal distribution. In that case, there is a need for future research to examine how to do clustering analysis when the maintenance time data conforms to different types of distribution. There is also a need to explore whether, for different types of equipment, the current maintainability index is truly capable of reflecting all the maintainability characteristics related to time distribution.

References

- [1] U.S. Department of Defense. Designing and developing maintainable products and systems. MIL-HDBK-470A, 1997.
- [2] DHILLON B S. Engineering maintainability. Houston: Gulf Professional Publishing, 1999.
- [3] EBELING C E. An introduction to reliability and maintainability engineering. New York: McGraw-Hill, 1997.
- [4] ALMOG R. A study of the application of the lognormal distribution to corrective maintenance repair time. Monterey, California: Naval Postgraduate School, 1979.
- [5] CAMOZU E. A study of the application of the lognormal and gamma distributions to corrective maintenance repair time data. Monterey, California: Naval Postgraduate School, 1982.
- [6] BOVAIRD R L, ZAGOR H L. Lognormal distribution and maintainability in support systems research. Naval Research Logistics, 1961, 8(4): 343–356.
- [7] International Electrotechnical Commission. Maintainability of equipment. IEC 60706-3, 2006.
- [8] KLINE M B. Suitability of the lognormal distribution for corrective maintenance repair times. Reliability Engineering, 1984, 9(2): 65–80.
- [9] TITTERINGTON D M, SMITH A F M, MAKOV U E. Statistical analysis of finite mixture distributions. Chichester: John Wiley & Sons, 1985.
- [10] SMITH D J. Reliability engineering. Bath: Pitman, 1972.
- [11] SMITH D J. Reliability, maintainability and risk: practical methods for engineers. Oxford: Elsevier, 2011.
- [12] ZHANG R J, ZHANG L L, WANG N N, et al. Reliability evaluation of a multi-state system based on interval-valued triangular fuzzy Bayesian networks. International Journal of System Assurance Engineering and Management, 2016, 7(1): 16–24.
- [13] YUAN X B, CAI B P, MA Y P, et al. Reliability evaluation methodology of complex systems based on dynamic object-oriented Bayesian networks. IEEE Access, 2018, 6: 11289–11300.
- [14] LI D Q, TANG X S, PHOON K K. Bootstrap method for characterizing the effect of uncertainty in shear strength parameters on slope reliability. Reliability Engineering & System Safety, 2015, 140: 99–106.
- [15] GUO J B, WANG C X, CABRERA J, et al. Improved inverse Gaussian process and bootstrap: degradation and reliability metrics. Reliability Engineering & System Safety, 2018, 178: 269–277.
- [16] MIAO Q, LIU L, FENG Y, et al. Complex system maintainability verification with limited samples. Microelectronics Reliability, 2011, 51(2): 294–299.
- [17] YANG H, HASSAN S G, WANG L, et al. Fault diagnosis method for water quality monitoring and control equipment in aquaculture based on multiple SVM combined with D-S evidence theory. Computers and Electronics in Agriculture, 2017, 141: 96–108.

- [18] GONG Y J, SU X Y, QIAN H, et al. Research on fault diagnosis methods for the reactor coolant system of nuclear power plant based on D-S evidence theory. *Annals of Nuclear Energy*, 2018, 112: 395–399.
- [19] LIMPET E, ABBT M, STAHEL W A. Log-normal distributions across the sciences: keys and clues. *BioScience*, 2001, 51(5): 341–352.
- [20] AITCHISON J, BROWN J A C. *The lognormal distribution*. Cambridge: Cambridge University Press, 1957.
- [21] MCLACHLAN G J, PEEL D. *Finite mixture models*. New York: John Wiley & Sons, 2000.
- [22] DEMPSTER A P, LAIRD N M, RUBIN D B. Maximum likelihood from incomplete data via the EM algorithm. *Journal of the Royal Statistical Society*, 1977, 39(1): 1–38.
- [23] MCLACHLAN G J, BASFORD K E. *Mixture models: inference and applications to clustering*. New York: Marcel Dekker, 1998.
- [24] REDNER R A, WALKER H F. Mixture densities, maximum likelihood and the EM algorithm. *SIAM Review*, 1984, 26(2): 195–239.
- [25] WANG Y M. A projection method for multiindices decision making. *Journal of Systems Engineering and Electronics*, 1998, 9(3): 1–7.
- [26] XU Z S. On method for uncertain multiple attribute decision making problems with uncertain multiplicative preference information on alternatives. *Fuzzy Optimization and Decision Making*, 2005, 4(2): 131–139.
- [27] YUE Z L. Approach to group decision making based on determining the weights of experts by using projection method. *Applied Mathematical Modelling*, 2012, 36(7): 2900–2910.
- [28] ZHENG G Z, JING Y Y, HUANG H X, et al. Application of improved grey relational projection method to evaluate sustainable building envelope performance. *Applied Energy*, 2010, 87(2): 710–720.
- [29] FORGY E. Cluster analysis of multivariate data: efficiency versus interpretability of classifications. *Biometrics*, 1965, 21: 768–780.
- [30] LLOYD S. Least squares quantization in PCM. *IEEE Trans. on Information Theory*, 1982, 28(2): 129–137.
- [31] RAY S. *Distance-based model-selection with application to analysis of gene expression data*. Pennsylvania, USA: The Pennsylvania State University, 2003.
- [32] SCHWARZ G. Estimating the dimension of a model. *Annals of Statistics*, 1978, 6(2): 461–464.
- [33] WANI M F, GANDHI O P. Development of maintainability index for mechanical systems. *Reliability Engineering & System Safety*, 1999, 65(3): 259–270.
- [34] U.S. Department of Defense. *Maintainability prediction*. MIL-HDBK-472, 1966.
- [35] DURRANT-WHYTE H. Consistent integration and propagation of disparate sensor observations. *Proc. of the IEEE International Conference on Robotics and Automation*, 1986: 1464–1469.
- [36] HAROLD S. *Maintainability prediction and demonstration techniques*. RADC-TR-69-356, New York: Rome Air Development Center, 1970.

Biographies

WU Zhenya was born in 1990. He received his B.S. degree in missile engineering from Air Force Engineering University, Xi'an, Shaanxi, China, in 2013, and M.S. degree in equipment management from Air Force Engineering University, in 2016. He is currently pursuing his Ph.D. degree in armament science and technology at Shijiazhuang Campus of Army Engineering University. His current research interests include maintainability demonstration and statistics.

E-mail: wzy9012@163.com

HAO Jianping was born in 1969. He is a professor of Army Engineering University. He received his M.S. and Ph.D. degrees from Mechanical Engineering College, Shijiazhuang, China. His research mainly focuses on maintainability and maintenance engineering. Since 2016, he has established a research group and has begun to study demonstration of complex system maintainability and theory

of equipment general quality characteristics.

E-mail: jianping_hao001@sina.com